IC/2003/40

ON PION-NUCLEUS SCATTERING, PAPER-II

L. Begum

Sangita Haque

W. Nahar

S. Nasmin Rahman

and

Md. A. Rahman

Available at. http://www.ictp.trieste.it/~pub_off IC/2003/40

United Nations Educational Scientific and Cultural Organization
and
International Atomic Energy Agency

THE ABDUS SALAM INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

ON PION-NUCLEUS SCATTERING, PAPER- II

L. Begum, Sangita Haque and W. Nahar
Department of Physics, University of Dhaka, Dhaka, Bangladesh,

S. Nasmin Rahman
Industrial Physics Division, B.C.S.I.R., Dhaka, Bangladesh

and

Md. A. Rahman[1]
Department of Physics, University of Dhaka, Dhaka, Bangladesh
and
The Abdus Salam International Centre for Theoretical Physics, Trieste, Italy.

Abstract

This is the continuation of our earlier paper-I. We study the elastic scattering of pions from several nuclei between ^{12}C to ^{208}Pb on either side of the Delta resonance and inelastic scattering leading to the lowest 2^+ collective states in ^{12}C, $^{24,\,26}Mg$ and ^{28}Si. Both the elastic and inelastic scattering angular distributions are well reproduced by the strong absorption model. The deformation parameters obtained are in excellent agreement with previous studies. The ratio $(G_n(\pi)/N)/\ (G_p(\pi)/Z)$ for most of the levels covered in the present work is nearly unity, in good agreement with previous results obtained from DWIA analyses.

MIRAMARE – TRIESTE
June 2003

---

[1] Regular Associate of the Abdus Salam ICTP.

1. Introduction

The pion-nucleus scattering is a useful laboratory to understand various nuclear properties such as neutron and proton radii and densities of nuclei. At around 200 MeV pion energy the fundamental pion-nucleon interaction is featured by strong and broad p-wave resonances, termed as the Δ(3,3) or Δ or simply the (3,3) resonance. With the pion beam facilities at LAMPF, SIN, CERN, TRIUMF and other laboratories, a large number of experiments have been carried out on the pion-nucleus interaction. Theoretical studies on the problem proceed along the following lines. One is the 'zero range approximation' for the pion-nucleon interaction, which allows one to work in the co-ordinate space. Another is the 'momentum space approach', where one constructs the optical model potential for the pion-nucleus scattering in the momentum space. Third is the 'Δ – hole model', where it is assumed that the dominant mode of interaction of pions with nuclei is to excite a target nucleon to a Δ, with a corresponding hole in the nucleon state.

The elastic scattering, as well as inelastic scattering to selected states (namely the lowest 2^+ and 3^- collective states) in some nuclei were previously studied by some of the present authors [1] using the generalized diffraction model of Frahn and Venter [2]. A reasonable good description of both elastic and inelastic scattering data are possible

The present work is an extension of these and we intend to study the elastic scattering of pions from several nuclei between ^{12}C to ^{208}Pb atop and on either side of the Delta resonance and inelastic scattering to the lowest 2^+ collective states in ^{12}C, ^{24,26}Mg and ^{28}Si. Recently Peterson [3] has carried out a systematic study of the transition matrix elements as extracted from the inelastic scattering of pions to natural parity states in a large number of even-even nuclei based on the DWIA method. It would be interesting to compare these results with the results obtained from an altogether different approach namely the geometrical model SAM. The various angular distribution data on the elastic and inelastic scattering are taken from [4-11].

2. Mathematical Formalism

We use the strong absorption model of Frahn and Venter [2]. The scattering function η_ℓ of the model is parametrized directly and a closed form expression is obtained for the differential cross of the elastic scattering using suitable approximations [12]. The parameters of the model are T, Δ and μ for the description of the elastic scattering. Potgieter and Frahn [13] have obtained expressions for the inelastic scattering amplitude as the first derivative of the elastic scattering amplitude following the pioneering work of Austern and Blair [14]. The parameters T, Δ and μ are fixed from the elastic scattering and the only free parameter is the normalization constant of the theory to the experiment. The deformation lengths $\beta_L R$ and the deformation parameter β_L of multipolarity L are extracted from the normalization constant.

Table I. The best fit SAM parameters for π^+.

Nucleus	E_π	SAM parameters			Derived quantities						
	MeV	T	Δ	$\frac{\mu}{4\Delta}$	R (fm)	d (fm)	$\lambdabar$ (fm)	R-$\lambdabar$ (fm)	σ_r (mb)	$\frac{\sigma_r}{\pi R^2}$	r_0 (fm)
^{12}C	80	3.00	0.42	0.22	4.08	0.56	1.32	2.76	661.00	1.27	1.45
^{12}C	100	3.15	0.45	0.25	3.82	0.54	1.18	2.64	581.00	1.27	1.32
^{12}C	160	4.15	0.80	0.08	3.96	0.75	0.93	3.03	730.00	1.48	1.40
^{12}C	162	4.15	0.80	0.05	3.93	0.75	0.93	3.00	720.00	1.48	1.39
^{13}C	80	3.05	0.40	0.23	4.14	0.53	1.32	2.82	670.00	1.25	1.43
^{13}C	162	4.10	0.77	0.05	3.89	0.72	0.93	2.96	691.00	1.47	1.35
^{14}C	80	3.10	0.50	0.21	4.20	0.67	1.32	2.88	740.00	1.34	1.43
^{16}O	114.3	3.7	0.64	0.20	4.18	0.71	1.11	3.07	751.00	1.37	1.37
^{16}O	160	4.7	0.81	0.08	4.20	0.76	0.93	3.27	890.00	1.41	1.46
^{16}O	240	5.6	0.95	0.17	4.35	0.73	0.76	3.59	821.00	1.39	1.43
^{24}Mg	180	5.47	0.78	0.07	4.90	0.70	0.88	4.20	1000.0	1.32	1.43
^{26}Mg	180	5.55	0.75	.10.0	4.97	0.66	0.88	4.09	1010.0	1.29	1.42
^{28}Si	180	5.9	0.90	0.05	5.29	0.79	0.88	4.41	1181.0	1.35	1.48
^{28}Si	291	7.25	1.00	0.15	5.10	0.70	0.69	4.41	3700.0	1.8	1.42
^{58}Ni	291	10.0	1.00	0.18	7.03	0.70	0.69	6.34	6201.0	1.6	1.60
^{208}Pb	291	14.90	1.10	0.18	10.56	0.76	0.69	9.87	12670.0	1.44	1.60

Table II. The best fit SAM parameters for π^-.

Nucleus	E_π	SAM parameters			Derived quantities						
	MeV	T	Δ	$\frac{\mu}{4\Delta}$	R (fm)	d (fm)	$\bar{\lambda}$ (fm)	R-$\bar{\lambda}$ (fm)	σ_r (mb)	$\frac{\sigma_r}{\pi R^2}$	r_0 (fm)
^{12}C	100	3.2	0.45	0.27	3.79	0.54	1.18	2.61	591.00	1.31	1.3
^{12}C	164	4.30	0.88	0.057	3.99	0.82	0.92	3.07	781.00	1.56	1.4
^{12}C	230	4.80	0.90	0.09	3.77	0.71	0.78	2.99	670.00	1.49	1.3
^{12}C	260	5.05	0.95	0.105	3.74	0.71	0.73	3.01	651.00	1.5	1.3
^{12}C	280	5.10	0.99	0.121	3.64	0.71	0.71	2.93	621.00	1.51	1.2
^{13}C	80	3.24	0.39	0.263	4.28	0.52	1.32	2.96	731.00	1.27	1.4
^{13}C	162	4.27	0 77	0.055	3 99	0.72	0.93	3.06	740.00	1.48	1.3
^{14}C	80	3.24	0.48	0.271	4.39	0.64	1.32	3.07	771.00	1.28	1.4
^{14}C	164	4.6	0.83	0.033	4.27	0.78	0.92	3.35	850.00	1.49	1.4
^{16}O	160	4.55	0.75	0.083	4.26	0.71	0.93	3.33	820.00	1.43	1.39
^{16}O	240	5.8	0.93	0.113	4.45	0.72	0.76	3.69	871.00	1.41	1.46
^{24}Mg	180	5.7	0.85	0.044	5.01	0.75	0.88	4.13	1100.00	1.39	1.47
^{26}Mg	180	5.8	0.85	0.06	5.1	0.75	0.88	4.22	1130.00	1.38	1.46
^{28}Si	180	5.9	0.90	0.003	5.18	0.79	0.88	4.3	1190.0	1.41	1.45
^{28}Si	291	7.25	1.1	0.136	5.03	0.77	0.69	4.34	3801.0	1.88	1.40
^{58}Ni	291	9.7	1.2	0.156	6.68	0.84	0.69	5.99	6151.0	1.76	1.50
^{208}Pb	291	15.5	1.1	0.159	9.63	0.70	0.69	8.94	13640	1.48	1.60

3. Results and Discussion

3.1 Elastic scattering

The SAM analysis of the angular distribution data for the elastic scattering of π^+ and π^- probes are presented respectively in tables I and II. Comparisons of the experiment with model predictions are shown in figs.1-3. The model employed reproduces almost all the diffractive oscillations of all the target nuclei between ^{12}C and ^{208}Pb up to all measured angular range except for minor mismatch between the experiment and theory in a few cases of light nuclei like ^{12}C and ^{16}O beyond 100^0 or so.

We determine the quantities like interaction radius 'R', the surface diffuseness 'd' and the reaction cross section σ_r. These are also presented in tables I and II.

The standard nuclear radius parameter $r_0 = (R/A^{1/3})$ turns out to be fairly constant over the entire range of mass number being

$$r_o = 1.44 \pm 0.07 \text{ fm, for } \pi^+$$

$$= 1.42 \pm 0.07 \text{ fm, for } \pi^-$$

The diffuseness 'd' of the nuclear surface remains practically constant at the value d = 0.71 ± 0.09 fm both for π^+ and π^- probes. σ_r is calculated from the SAM parameters using the relation:

$$\sigma_r = \frac{\pi T^2}{k^2}\left[1 + \frac{2\Delta}{T} + \frac{\pi^2}{3}\left(\frac{\Delta}{T}\right)^2 - \frac{1}{3}\left(\frac{\mu}{\Delta}\right)^2\left(\frac{\Delta}{T}\right)\right] \qquad (1)$$

These are also shown in tables I and II. The mass number dependence of σ_r, as found in the present work, is then looked into. The reaction cross section increases as the mass number of the target increases, the projectile energy remaining constant. It is quite interesting to note that the σ_r values for π^+ and π^- at 180 MeV and 291 MeV for the target nuclei $^{24,\,26}$Mg, ^{28}Si, ^{58}Ni and ^{208}Pb are rather indistinguishable from each other. Somewhat higher values of σ_r from π^- probes is in accord with more absorptive nature of π^- than π^+. The quantity $\frac{\sigma_r}{\pi R^2}$ in the present work is sensibly constant at the value 1.44 (being the mean value of 1.41 for π^+ and 1.47 for π^- probes). Here again the estimated values of σ_r at 180 MeV for π^- are consistently higher than those for π^+ (tables I and II) and the difference between π^- and π^+ cross section i.e. the quantity $(\sigma_r(\pi^-) - \sigma_r(\pi^+))$ systematically increases with an increase in the mass number (i.e. with an increase in the neutron excess of the nuclei). This feature is in good agreement with [18] and with the known (3,3) dominance around 180 MeV. This gets further supported by an appreciable damping of the diffraction oscillation in the measured elastic angular distribution off the resonance (i.e. at $E_\pi = 260$ and 280 MeV for ^{12}C as in fig.1.) as against prominent diffraction oscillation around the resonance as for ^{24,26}Mg (fig.1). So the simple model used in the present analyses with just three parameters is quite successful in reproducing the observed features in the angular distributions.

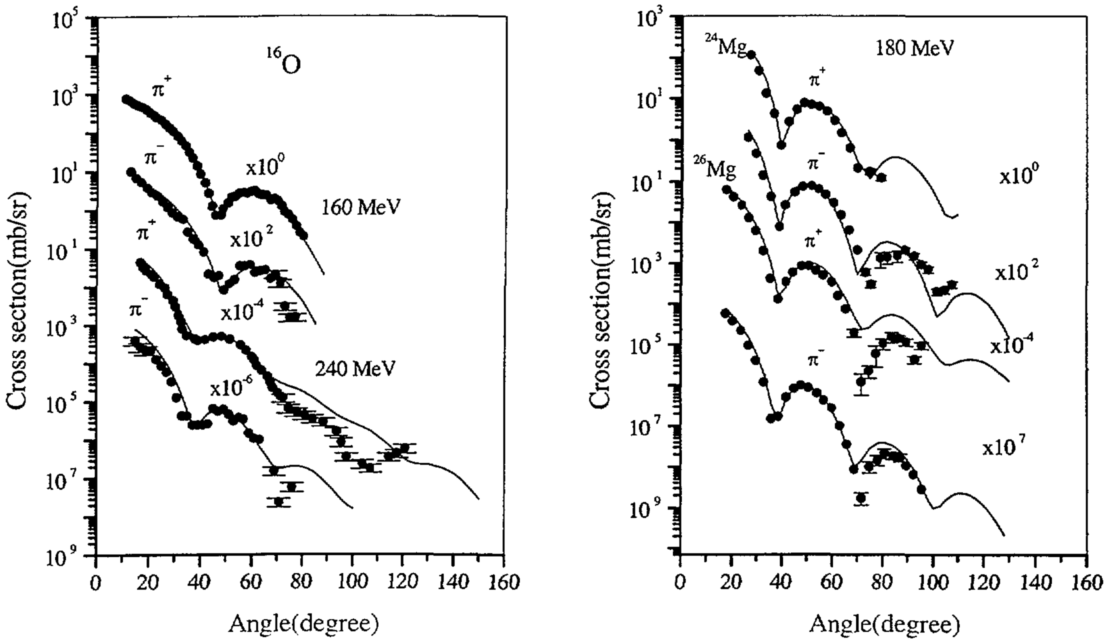

Fig 1. SAM fit to the elastic scattering of pions from different nuclei.

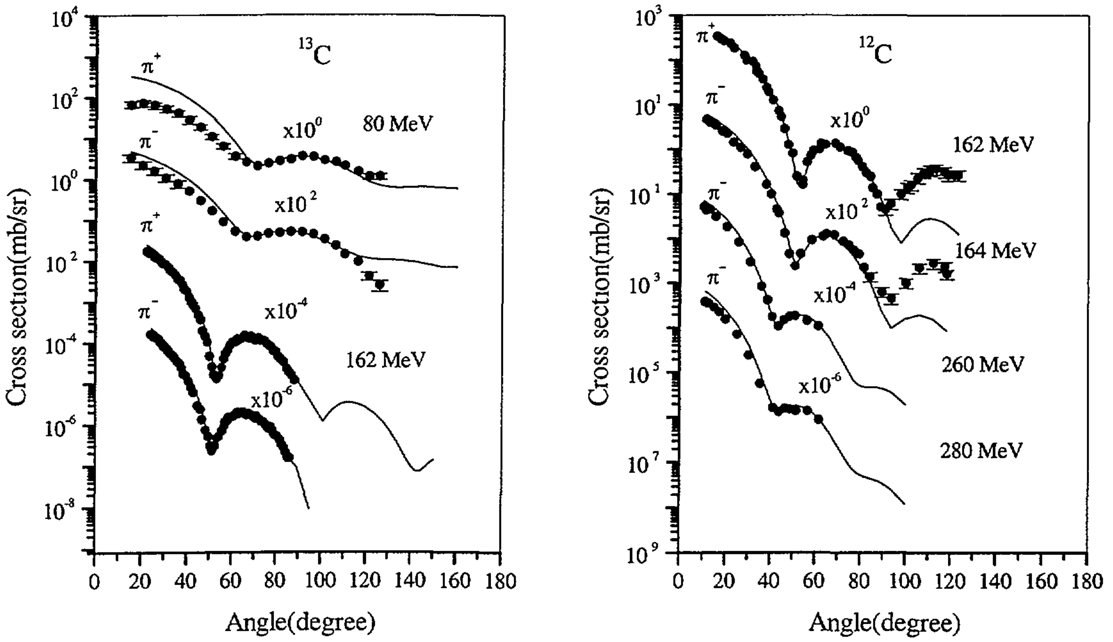

Fig 2. SAM fit to the elastic scattering of pions from different nuclei.

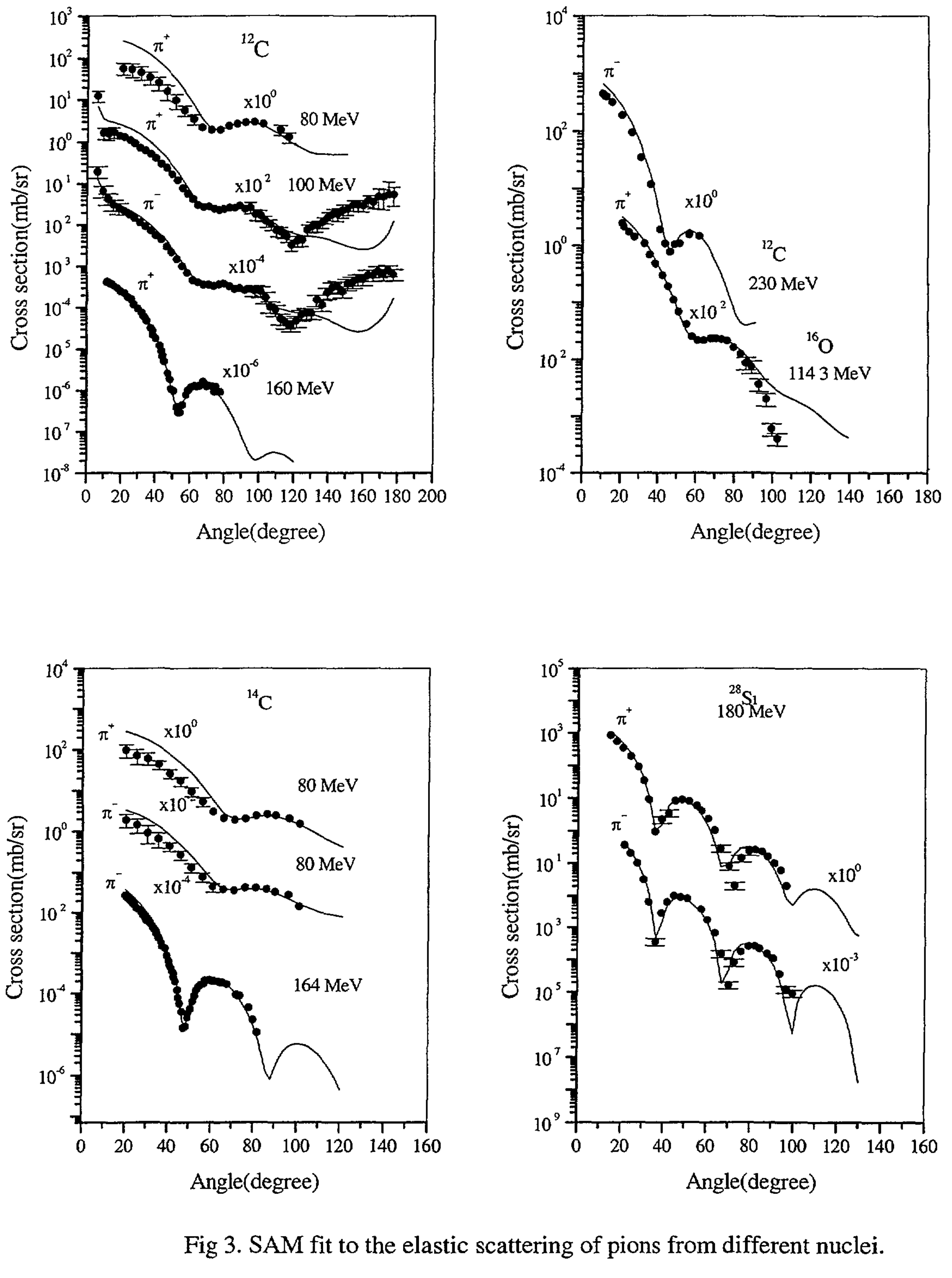

Fig 3. SAM fit to the elastic scattering of pions from different nuclei.

3.2 Inelastic scattering.

Angular distribution data [4-11] for the inelastic scattering of pions exciting to the lowest 2^+ collective states in ^{12}C, ^{24,26}Mg and ^{28}Si are reproduced by the SAM parameters as obtained from the relevant elastic scattering analyses (tables I and II). The quality of fit to the inelastic scattering data are shown in figs. 4-5. The deformation parameters are summarized in table III with the relevant values adopted by Raman et al.[15]. The normalization procedure is followed as in paper-I. A reasonable good fit is obtained in most of the cases covering all oscillations in the angular distributions. This indicates the fact that a dominant collective mode of excitation of the level. This also adds to the reliability of the SAM analyses. The extracted values of the deformation parameter should be meaningful for levels with a good fit, since the formalism is strictly valid for single excitation [14]. The deformation parameters extracted in the present work are in reasonably good agreement with other values from other formalism (table III).

The proton and neutron deformation parameters β_p and β_n respectively are obtained from the $\beta(\pi^+)$ and $\beta(\pi^-)$ values given in table III using the relations [8] at pion energies around the (3,3) resonance. From β_p and β_n, the proton and neutron matrix elements M_p and M_n respectively are then extracted and therefrom the ratios of these to the single particle matrix elements, termed as $G_p(\pi)$ and $G_n(\pi)$ respectively are obtained. The ratio $(G_n(\pi)/N)/(G_p(\pi)/Z)$ is approximately the same and is in agreement with previous results obtained from the DWIA [8] within errors for all of the levels studied in the present work (table IV), suggesting thereby their isoscalar mode of excitation.

Table III. Deformation parameters for the lowest 2^+ state.

Nucleus	E_x MeV	E_π MeV	Deformation parameter, β_2			
			(a)	(b)	(c)	(d)
^{12}C	4.4	100	0.34	0.42	0.592	
^{24}Mg	1.4	180	0.92	0.85	0.608	0.82
^{26}Mg	1.81	180	0.54	0.50	0.428	
^{28}Si	1.78	180	0.63	0.53	*0.49 ± 0.06(π^+) 0.46 ± 0.05(π^-)	

(a) Present work from the inelastic scattering of π^+
(b) Present work from the inelastic scattering of π^-
(c) Adopted values : ref. [15]
*ref. [16]
(d) Previous work : DWBA calculation ; ref. [17]

Table. IV(a) Proton and neutron deformation parameters and transition matrix elements extracted from the inelastic scattering of 180 MeV pions leading to the lowest 2^+ state.

Nucleus	β_p	β_n	$G_p(\pi)$		$G_n(\pi)$		$(G_n/N)/(G_p/Z)$	
	(a)	(a)	(a)	(b)	(a)	(b)	(a)	(b)
^{24}Mg	0.96	0.82	7.27	5.12	6.21	4.61	0.85	0.90
^{26}Mg	0.61	0.45	4.62	3.55	3.97	3.64	0.74	0.88
^{28}Si	0.68	0.48	6.00	3.80	4.24	3.80	0.71	1.00

(a) From SAM analysis (Present work)
(b) From DWIA analysis (Peterson [3] and references therein)

(b)

Nucleus	M_p		M_n		$G_0(\pi)$		$G_0(\alpha)$
	(a)	(b)	(a)	(b)	(a)	(b)	(b)
^{24}Mg	32.95	23	28.15	21	13.48	9.73	8.83
^{26}Mg	22.09	16.8	16.29	17.4	8.59	7.19	7.51
^{28}Si	30.18	19.1	21.30	19.1	10.24	7.60	7.08

(a) From SAM analysis (Present work)
(b) From DWIA analysis (Peterson [3] and references therein)

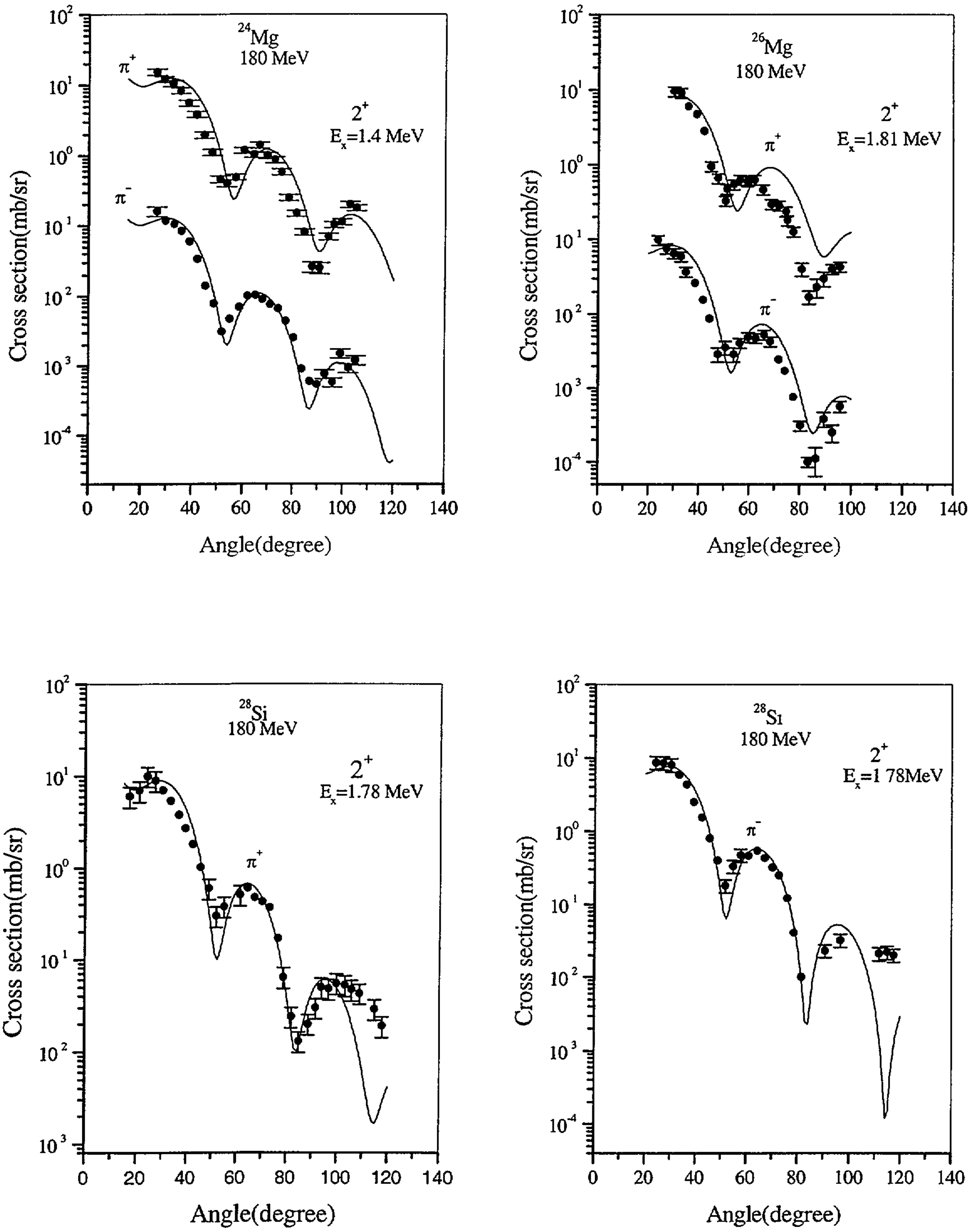

Fig 4. SAM fit to the inelastic scattering of pions leading to the lowest 2^+ states.

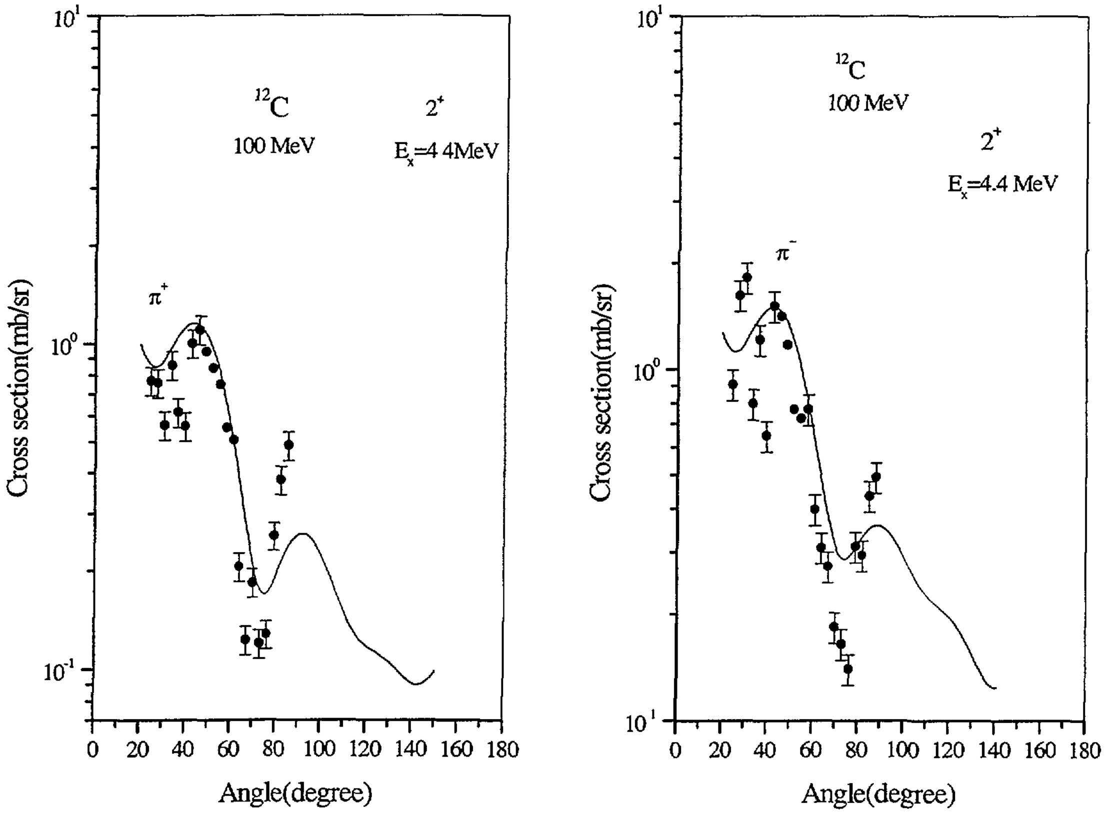

Fig 5. SAM fit to the inelastic scattering of pions leading to the lowest 2^+ states.

4. Conclusion

The present work gives an account of the elastic and inelastic scattering of pions (π^+ and π^-) from several nuclei between ^{12}C to ^{208}Pb at around the Delta resonance, within the framework of the generalized diffraction model of Frahn and Venter. The experimental angular distributions are reasonably well described by the model employed. Though the interaction radius 'R' varies with mass number and energies, but the standard nuclear radius 'r_0'remains fairly constant over the entire range of mass number and so is the diffuseness. The deformation parameters extracted are in good agreement with other values obtained from other formalisms. The ratio $(G_n(\pi)/N)/(G_p(\pi)/Z)$ at Delta resonance is reasonably constant for the levels studied in the present work indicating thereby their isoscalar mode of excitation.

ACKNOWLEDGMENTS

One of the authors (Md.A.Rahman) acknowledges continued support from the Abdus Salam International Centre for Theoretical Physics, Trieste, Italy. L. Begum gratefully acknowledges financial support from the Bose Center for Advanced Study and Research in Natural Sciences, University of Dhaka. Part of this work was done within the framework of the Associateship Scheme of the Abdus Salam ICTP. Financial support from the Swedish International Developing Cooperation Agency is acknowledged.

REFERENCES

[1] RAHMAN Md. A., SEN GUPTA H.M.and RAHMAN M., *Phys. Rev. C,**41** (1990) 2305.*

[2] FRAHN W.E., Fundamentals in Nuclear Theory (International Atomic Energy Agency, Vienna), p-1(1967); FRAHN W.E. and VENTER R.H., *Ann.Phys. (N.Y.), **24** (1963) 243.*

[3] PETERSON R. J., *Phys. Rev. C, **48** (1993) 1128.*

[4] ANTOUK L.E. et al., *Nucl. Phys. A, **420** (1984) 435.*

[5] BLECHER M et al., *Phys. Rev. C, 28 (183) 2034.*

[6] GIBSON E. F. et al., *Nucl. Phys. A,377 (1982) 377.*

[7] LUNKE C. et al., *Phys. Lett. B,**78** (1978) 201.*

[8] MAILET J. P. et al., *Nucl. Phys. A,**316** (1979) 277.*

[9] OSET E. and WEISE W., *Nucl. Phys. A,**329** (1979) 375.*

[10] SEESTROM MORRIS S.J. et al., *Phys. Rev. C, **28** (1983) 1301.*

[11] WIEDNER et al., *Phys. Lett. B, **97** (1980) 37.*

[12] VENTER R..H., *Ann. Phys. (N.Y.), **25** (1963) 405.*

[13] POTGIETER J. M. and FRAHN W. E. , *Nucl. Phys. , **88** (1966) 434.*

[14] AUSTERN N. and BLAIR J . S. , *Ann. Phys. (N. Y.), **33** (1965) 15.*

[15] RAMAN S. , MALARKEY C.H. , MILNER W.T. , NESTOR JR. C W. and STELSON P. H. , *At. Data Nucl. Data Tables, **36** (1987) 1.*

[16] RAMAN et al., *Phys. Rev. C , **41** (1990).*

[17] WIEDNER C. A. et al., *Phys. Rev. C , **28** (1983) 2165.*

[18] SATCHLER G. R., *Nucl. Phys. A ,**540** (1992) 533.*