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HEAVY-ION NUCLEUS SCATTERING

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Abstract

Heavy ion-nucleus scattering is an excellent laboratory to probe high spin phenomena, exotic nuclei and for the analysis of various exit channels. The Strong Absorption Model or the generalized diffraction models, which are semi-classical in nature, have been employed in the description of various heavy ion-nucleus scattering phenomena with reasonable success. But one needs to treat the deflection function (scattering angles) quantum mechanically in the Wave Mechanical picture for the appropriate description of the heavy-ion nucleus scattering phenomena. We have brought the mathematics for the cross-section of the heavy-ion nucleus scattering to an analytic expression taking account of the deflection function (scattering angles) quantum mechanically. ^{9}Be, ^{16}O, ^{20}Ne and ^{32}S heavy-ion beams elastic scattering from ^{28}Si, ^{24}Mg and ^{40}Ca target nuclei at various projectile energies over the range 20-151 MeV have been analysed in terms of the 2-parameter formalism of the present quantal formulation and from the Strong Absorption Model for comparison. Typical fits are shown and the nuclear parameters obtained from the analyses of both approaches are presented.

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I. Introduction

There are several types of nuclear interactions, such as elastic scattering, inelastic scattering, absorption etc. Of them heavy-ion interaction has been playing a very important and significant role in nuclear Physics since the beginning of the subject. The interest in this direction is gaining momentum because of the advent of special types of accelerators, capable of producing fairly intense, high energy beams of HEAVY-IONS, e.g. nuclear projectiles whose masses exceed that of alpha particles ($Z \geq 3$, $A \geq 5$); these projectiles can be as heavy as uranium or even beyond.

The interaction of heavy ion projectiles with medium weight nuclei is characterized by several features, which offer interesting possibilities for experimental and theoretical investigations. On the experimental side, heavy- ion beams in an energy range spanning on either side of the Coulomb barrier together with high resolution particle detection systems are now readily available.

Semi-classical theories of heavy ion collisions have been developed [1-4] due to the fact that heavy ions, under certain conditions, behave almost like classical particles. It turns out that the region of semi-classical behaviour is essentially restricted to energies below the Coulomb barrier and for large value of the Sommerfeld parameter. Semi-classical theories of heavy-ion reactions are mainly valid [5-7] below and near the Coulomb barrier where both the real nuclear potential and the absorption are weak. Quantal effects become increasingly important at energies above the Coulomb barrier. The real nuclear potential becomes stronger and the absorption increases because of the opening of many non-elastic channels. Well above the Coulomb barrier we enter into an essentially quantal regime in which the semi-classical description becomes inadequate. In such situations one may come across the quantal analogues of optical phenomena such as rainbow and glory scattering [8].

The present work has been undertaken with an aim to investigate the applicablility of the quantal formulation of nuclear scattering to analyse the heavy ions like ^{9}Be,^{16}O,^{20}Ne and ^{32}S elastically scattered from,^{24}Mg,^{28}Si ,and ^{40}Ca target nuclei at various projectile energies over the range 20-151 MeV [9-13] in terms of the 2-parameter formalism of the present model. The interaction radius R and surface diffuseness d are estimated from the parameters T and Δ respectively.

II. Mathematical Formalism

The scattering formalism in the strong absorption phenomena has been semi-classically developed and brought to an analytic expression for the cross section for elastic scattering [14], as well as for the inelastic scattering with multiple mode of collective excitations in nuclei [15]. Only the salient mathematical points are shortly sketched below; for details we refer to the references already cited.

These semi-classical models due to Frahn, Venter and Potgieter [14-16] directly parameterize the scattering matrix η_ℓ in terms of the orbital angular momentum expressed in partial expansion of the projectile beams and avoids the potential concept as used in the optical model. Corresponding to a certain particular value of the orbital angular momentum of the projectile, called the cut-off or critical angular momentum, just grazing the nuclear surface, has got the highest contribution to the scattering processes. The scattering function η_ℓ is given by:

$$|\eta_l| = 0 \qquad l<T$$

$$|\eta_l| = 0 \qquad l>T \tag{1}$$

where T is the cut-off angular momentum.

It is possible to arrive at a complete analytical formulation for the parameterized S-matrix model for any form of η_l with or without Coulomb interaction if one writes the scattering function η_l in terms of real and imaginary parts:

$$\mathrm{Re}\,\eta_l\left(-2i\sigma_l\right) = g(t) + (1 - g(t))$$

and $$\mathrm{Im}\,\eta_l\left(-2i\sigma_l\right) = \frac{\mu dg(t)}{dt} \tag{2}$$

The σ_l is the usual Coulomb phase shift for the l-th partial wave. The g(t)'s are continuously differentiable functions of the angular momentum (t = l+1/2). The analytical treatment of η_l is independent of any functional form of g(t). The only practical requirement is that $\frac{dg(t)}{dt}$ should possess a simple Fourier transformation. The Coulomb scattering angle θ_c corresponding to the cut-off angular momentum T can be expressed semi-classically through the relation:

$$\theta_c = 2arctg\left(\frac{n}{T}\right)$$

here n is the Sommerfeld parameter.

The classical cross section for a range db of impact parameters scattering into the solid angle element $d\Omega = \sin\theta d\theta d\phi$ is

$$d\sigma_{cl} = bdbd\phi = b\frac{db}{d\theta}\,d\theta d\phi = \frac{b}{\sin\theta}\frac{db}{d\theta}d\,\Omega \tag{3}$$

and the classical differential cross section becomes :

$$\frac{d\sigma_{cl}}{d\Omega} = \sigma_{cl}(\theta) = \frac{b}{\sin\theta}\frac{db}{d\theta} \tag{4}$$

Now to determine the classical differential cross section one requires the knowledge of the impact parameter b as a function of scattering angle θ, i.e. b(θ). We define the inverse, "the classical deflection function" Θ (b), such that

$$\sigma_{el} = \frac{b}{\sin\theta}\frac{1}{\Theta'(b)} \tag{5}$$

the prime denotes the first derivative.

The deflection function $\Theta(b)$ is determined by the scattering potential U(r) through the classical expression:

$$\Theta(b)=\pi-2b\int_{r_{min}}^{\infty} dr\frac{1}{r^2}\left[1-\frac{U(r)}{E}-\frac{b^2}{r^2}\right]^{-\frac{1}{2}} \tag{6}$$

where, r_{min} is the radius of the closest approach to the scatterer.

Now the semi-classical relation between the orbital angular momentum *l* and the impact parameter b is:

$$l(l+1)\approx\left(l+\frac{1}{2}\right)^2\rightarrow(kb)^2; \qquad l+\frac{1}{2}=\lambda\rightarrow kb \tag{7}$$

and the semi-classical expression for the phase shifts (JWKB approximation) [17] is

$$\delta_{cl}(\lambda)=\frac{1}{2}\pi\lambda-k\int_{r_{min}}^{\infty} dr r\frac{d}{dr}\left[1-\frac{U(r)}{E}-\left(\frac{\lambda}{kr}\right)^2\right]^{\frac{1}{2}} \tag{8}$$

On differentiating the expression (8) w.r.t. λ yields

$$\frac{d}{d\lambda}2\delta_{cl}(\lambda)=\pi-2\frac{1}{k}\int_{r_{min}}^{\infty} dr\frac{1}{r^2}\left[1-\frac{U(r)}{E}-\left(\frac{\lambda}{kr}\right)^2\right]^{-\frac{1}{2}} \tag{9}$$

Now comparing with eqn. (6) and on replacing λ by kb, eqn. (9) reduces to the classical deflection $\Theta(b)$. One therefore defines, in general, the function

$$\Theta(\lambda)=\frac{d}{d\lambda}2\delta(\lambda) \tag{10}$$

and keeps in mind its simple meaning in the semi-classical approximation.

The heavy ion angular distribution is cast usually in the form $\frac{\sigma(\theta)}{\sigma_R(\theta)}$. Dividing the amplitude f (θ) by the Rutherford scattering amplitude $f_R(\theta)$:

$$f_R(\theta)=-\frac{n}{2k}\frac{1}{\left(\sin\frac{1}{2}\theta\right)^2}e^{i2\left[\delta(0)-\ln\left(\sin\frac{1}{2}\theta\right)\right]} \tag{11}$$

The ratio of the amplitude becomes:

$$\frac{f(\theta)}{f_R(\theta)}=1+A(\theta)e^{i\alpha(\theta)}\left\{-\frac{1}{2}\operatorname{erfc}\left[-e^{i\frac{\pi}{4}}u(\theta)\right]e^{i[u(\theta)]^2}-B(\theta)e^{i\beta(\theta)}\right\} \qquad \theta\le\theta_c$$

$$\frac{f(\theta)}{f_R(\theta)}=A(\theta)e^{i\alpha(\theta)}\left\{\frac{1}{2}\operatorname{erfc}\left[-e^{i\frac{\pi}{4}}u(\theta)\right]e^{i[u(\theta)]^2}-B(\theta)e^{i\beta(\theta)}\right\} \qquad \theta\ge\theta_c \tag{12}$$

In the above expression for the ratio of the amplitudes, the effects of the real nuclear phase shifts δ_o has been disregarded, i.e. to set δ_o=0 for the sake of simplicity.

The expressions for A(θ), B(θ), α(θ), β(θ), u(θ) and θ_c in eqn. (12) have the following forms:

$$A(\theta)=\frac{\sin\frac{1}{2}\theta}{\sin\frac{1}{2}\theta_c}\left(\frac{\tan\frac{1}{2}\theta}{\tan\frac{1}{2}\theta_c}\right)^{1/2} F[\Delta(\theta-\theta_c)] \tag{13}$$

$$B(\theta)=\frac{1}{(\pi n)^{1/2}}\frac{\sin\frac{1}{2}\theta_c}{\theta+\theta_c}\frac{F[\Delta(\theta+\theta_c)]}{F[\Delta(\theta-\theta_c)]} \tag{14}$$

$$\alpha(\theta)=2n\left[\frac{\theta_c-\theta}{2\tan\frac{1}{2}\theta_c}+\ln\left(\frac{\sin\frac{1}{2}\theta}{\sin\frac{1}{2}\theta_c}\right)\right] \tag{15}$$

$$\beta(\theta)=2T\theta+\frac{\pi}{4}=\frac{2n\theta}{\tan\frac{1}{2}\theta_c}+\frac{\pi}{4} \tag{16}$$

$$u(\theta)=n^{\frac{1}{2}}\frac{\theta-\theta_c}{2\sin\frac{1}{2}\theta_c} \tag{17}$$

where,

$$\theta_c=\Theta_R(T)=2\arctan\left(\frac{n}{T}\right) \tag{18}$$

A detailed discussion of the properties of eqn. (12) has been given elsewhere [14]. We are summarizing the main features here. The two terms in the braces in eqn. (12) correspond to Fresnel and the positive branch of Fraunhofer diffraction scattering, respectively. In the " dark " region $\theta \geq \theta_c$, we concentrate our discussion to angles large enough compared to θ_c so that the error function eqn. (12) may be replaced by its asymptotic expression [14]. The cross section ratio may then be written in the form:

$$\frac{\sigma(\theta)}{\sigma_R(\theta)}=\frac{\bar{\sigma}(\theta)}{\sigma_R(\theta)}\left\{1+\frac{2c(\theta)}{1+[c(\theta)]^2}\sin(2T\theta)\right\} \qquad \theta \geq \theta_c \tag{19}$$

where

$$\frac{\bar{\sigma}(\theta)}{\sigma_R(\theta)}=\frac{1}{\pi n}\left(\sin\frac{1}{2}\theta\right)^2\frac{\tan\frac{1}{2}\theta}{\tan\frac{1}{2}\theta_c}\left\{\frac{F[\Delta(\theta-\theta_c)]}{\theta-\theta_c}\right\}^2\left\{1+[c(\theta)]^2\right\} \tag{20}$$

and

$$c(\theta)=\frac{F[\Delta(\theta+\theta_c)]/(\theta+\theta_c)}{F[\Delta(\theta-\theta_c)]/(\theta-\theta_c)} \tag{21}$$

This shows that in the "dark" region the oscillations are purely of Fraunhofer type. However, these oscillations are damped by a factor determined by c(θ) which, for the specific form factor is given by:

$$F(\Delta x)=\frac{\pi\Delta x}{\sinh(\pi\Delta x)}$$

and c(θ) assumes the form:

$$c(\theta)=\frac{\sinh[\pi\Delta(\theta-\theta_c)]}{\sinh[\pi\Delta(\theta+\theta_c)]}\cong e^{-2\pi\Delta\theta c}$$

$$\approx e^{-4\pi(\Delta/T)n} \tag{22}$$

The scattering cross section incorporating the above mathematical steps and discussions in the quantal formulation becomes:

$$\sigma(\theta)=\sigma_R(\theta)\times\left(4\pi\frac{\nabla^2}{u}\right)\left(\sin\frac{\theta}{2}\right)^2\left(\frac{\tan\frac{\theta}{2}}{\tan\frac{\theta_c}{2}}\right)$$

$$\times(e-2\pi\Delta(\theta-\theta c))[1+e-8\pi(\Delta/T)$$

$$+2e^{(-4\pi\Delta n/T)}\sin(2T\theta)] \tag{23}$$

III. Results and Discussion

The analytical expressions (23) have been used to calculate the elastic scattering of cross sections for the interacting pairs $^{16}O+^{24}Mg$, $^{16}O+^{40}Ca$ and $^{32}S+^{40}Ca$ at various incident energies between 28 and 150 MeV, taking the advantage of quantal formulation. The same experimental data have been analysed using analytic expressions for the elastic scattering cross section as quoted in the ref. [14)]. This is the so-called Strong Absorption Model (SAM) analyses. The parameters of the two approaches, the standard nuclear radius r_0 and the interaction radius R, extracted from these analyses have been presented in **Tables** 1 and 2 respectively. Some typical angular distribution fit to the experimental data from both approaches are shown in **Figs**.1-3. The quality of angular distribution fit is satisfactory in the forward angles throughout the analyses. There is however a slight deviation relatively at large angles between the theory including quantal deflection and experiment. The extracted standard nuclear radius r_0 from quantal formulation has a somewhat higher mean value of 1.62 fm than a mean value of 1.54 fm from the usual SAM analyses. The following observations are worth mentioning from a comparison of the theoretical prediction in the quantal picture and the experiment.

I. The scattering cross section $\sigma(\theta)\propto\Delta^2/n$ for a given scattering angle.

II. For a given Δ/T, the damping of the Fraunhofer oscillation dampen exponentially on Sommerfeld parameter n (Coulomb damping).

III. The cross section at large angle, in the case of strong Coulomb damping, decreases with increasing angle (a strong exponential decrease) with a logarithmic slope proportional to the smoothing width Δ.

Table 1 Summary of elastic scattering parameters from quantal formulation

Incident particle + Target Nucleus	E_{lab} (Mev)	T	Δ	d (fm)	R (fm)	r_0 (fm)
$^{16}O+^{24}Mg$	28	6.00	0.35	0.06	8.76	1.62
$^{16}O+^{24}Mg$	29	5.90	0.35	0.05	8.46	1.57
$^{16}O+^{24}Mg$	30	7.00	0.4	0.07	8.39	1.55
$^{16}O+^{24}Mg$	33	10.20	0.4	0.09	8.35	1.55
$^{16}O+^{40}Ca$	40	10.00	0.4	0.054	8.79	1.48
$^{16}O+^{40}Ca$	55.6	22.00	0.6	0.11	8.44	1.42
$^{16}O+^{40}Ca$	74.4	44.00	1.20	0.22	10.61	1.78
$^{32}S+^{40}Ca$	100	31.00	0.60	.064	10.27	1.56
$^{32}S+^{40}Ca$	120	60.00	1.00	0.12	12.14	1.84
$^{32}S+^{40}Ca$	151	80.00	1.00	0.11	12.58	1.90

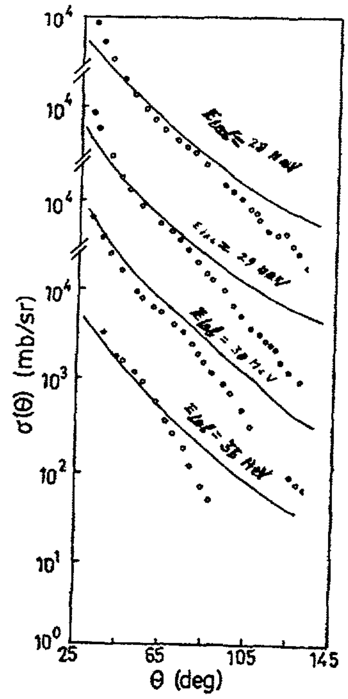

Fig. 1 Elastic scattering of ^{16}O from ^{24}Mg (quantal analyses)

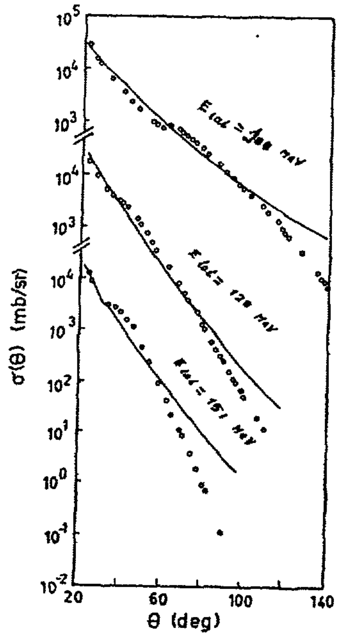

Fig. 3 Elastic scattering of ^{32}S from ^{40}Ca (quantal analyses)

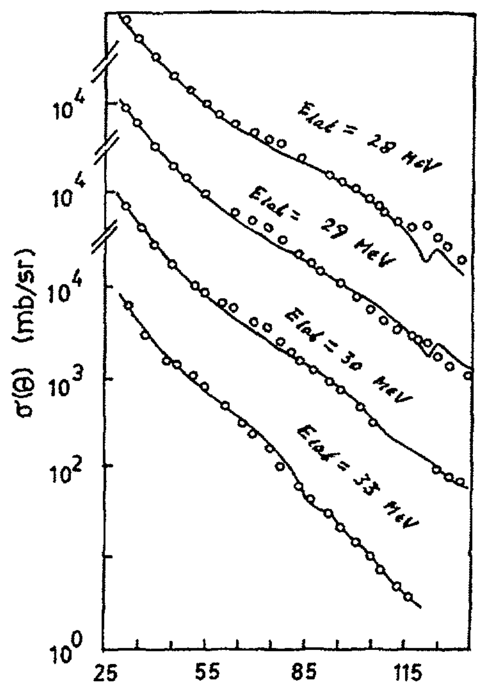

Fig. 2 Elastic scattering of ^{16}O from ^{24}Mg (SAM analyses)

Table 2 Summary of elastic scattering parameters from Strong Absorption Model formalism

Incident particle + Target Nucleus	E_{lab} (Mev)	T	Δ	μ	d (fm)	R (fm)	r_0 (fm)
$^{16}O+^{24}Mg$	28	6	1.6	0.001	0.27	8.76	1.62
$^{16}O+^{24}Mg$	29	5.9	0.2	0.1	0.03	8.46	1.57
$^{16}O+^{24}Mg$	30	7	0.7	0.60	0.13	8.39	1.55
$^{16}O+^{24}Mg$	33	10.2	0.5	0.50	0.12	8.35	1.55
$^{16}O+^{40}Ca$	40	10	0.1	0.30	0.13	8.79	1.48
$^{16}O+^{40}Ca$	55.6	22	0.1	0.50	0.18	8.44	1.42
$^{16}O+^{40}Ca$	74.4	36	1.2	0.30	0.21	9.27	1.56
$^{32}S+^{40}Ca$	100	31	0.1	0.50	0.11	10.27	1.56
$^{32}S+^{40}Ca$	120	50	1.0	0.50	0.12	10.94	1.66
$^{32}S+^{40}Ca$	151	53	1.0	0.30	0.11	9.57	1.45

IV. Conclusion

The heavy-ions ^{16}O, ^{20}Ne and ^{32}S scattered from ^{24}Mg, ^{28}Si and ^{40}Ca target nuclei elastically, are analysed both from quantal formulation and from the conventional Strong Absorption Model approaches. The standard nuclear radius r_0 extracted from the quantal description, though it has a somewhat higher mean value of 1.62 fm than the mean value of 1.54 fm from the SAM analyses, is within allowable uncertainties. The agreement between the three characteristic observations of the scattering cross section in the quantal picture and the experimental data is encouraging. Further works in this direction are being carried out.

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