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ON PION-NUCLEUS SCATTERING, PAPER- I

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Abstract

The strong absorption model of Frahn and Venter is used to study the elastic scattering of pions at energies below and above the Delta resonance from a number of nuclei between ^{14}N and ^{152}Sm. A reasonable good fit is obtained in each case over the entire angular range except for a few cases of 65 and 80 MeV data where clearly the strong absorption conditions are not perfectly satisfied. The best fit parameters values of the model and their systematics are discussed. The inelastic scattering of pions is then studied. Pions leading to the lowest 2^+ and 3^- collective states in some nuclei are analyzed. Again a good account of the inelastic scattering processes is possible without any readjustment of any elastic scattering parameters. The relevant deformation parameters are extracted. The proton and neutron matrix elements M_p and M_n are extracted from the proton and neutron deformation parameters β_p and β_n respectively.

1. Introduction

Pion-Nucleus scattering is one of the most important means to provide the information about nuclear properties in the ground and excited states. The pion-nucleus scattering is a good tool, to know the information about the proton and neutron radii and densities of nuclei. The pion is a boson with no spin and can exist in all three charge states viz. positive, negative and neutral (i.e. π^+, π^- and π^0). Pion's properties such as spin, mass and charge states have given it the correct behavior of an Yukawa particle.

The pion-nucleus scattering around 200 MeV is characterized by a strong and broad p-wave resonance of width 125 MeV in the fundamental pion-nucleon interaction, termed as the Δ (3,3) or simply the (3,3) resonance. The pion incident on a nucleon forms a resonance as $\pi^+ + P \rightarrow N^*$ $(= \Delta^{++}) \rightarrow \pi^+ + P$ at pion incident energy at around 200 MeV. This is the so-called Δ^{++} resonance in an $\ell_p=1$ state with J=3/2 and I=3/2. Its width is ~ 125 MeV and its life time is ~ 5×10^{-24}sec.; which is too small to be measured directly. In the region of the Δ (3,3) resonance and beyond it, the pion-nucleon interaction is strongly absorptive and the high energy pions thus have wave length shorter than the characteristic dimension of the target nucleus (<0.5 fm. or so). The interaction can thus be described to a first approximation as a diffraction process. The angular distributions of pions and various projectiles under appropriate conditions indeed display the above diffraction oscillations and these have been successfully accounted for by the diffraction models[1-4]. Strong absorption phenomena greatly simplify the theoretical formalism in analyzing the elastic scattering problem without requiring any knowledge of the absorption mechanism and naturally lead to analytic expressions for the elastic scattering amplitude.

2. Mathematical Formalism

The elastic scattering amplitude of a spin-zero pion incident on a spin-zero target nucleus is in terms of the usual partial wave expansion:

$$f(\theta) = \frac{i}{2k}\sum_{\ell=0}^{\infty}(2\ell+1)(1-\eta_\ell)P_\ell(\cos\theta) \tag{1}$$

where the scattering function , $\eta_\ell = \exp(2i\delta_\ell)$, η_ℓ is a sub matrix of the full scattering matrix S(ℓ) and δ_ℓ is the phase-shift of the ℓ-th partial wave. The condition $|\eta_\ell(\ell)| << 1$ or $\eta_\ell = 0$ for $\ell<\ell'$ holds for the strong absorption of the incident pions and the condition $|\eta_\ell| \geq 1$ or $\eta_\ell=1$ for $\ell \geq \ell'$ refers to the scattering of the incident pions.

The scattering cross-section is:

$$\sigma_{sc} = \frac{\pi}{k^2}\sum_{\ell=0}^{\infty}(2\ell+1)(|1-\eta_\ell|^2) \tag{2}$$

and the reaction cross-section is:

$$\sigma_r = \frac{\pi}{k^2}\sum_{\ell=0}^{\infty}(2\ell+1)(1-|\eta_\ell|^2) \tag{3}$$

which for spin-zero charged particle becomes:

$$\sigma_r = \frac{\pi T^2}{k^2}[1+\frac{2\Delta}{T}+\frac{\pi^2}{3}(\frac{\Delta}{T})^2-\frac{1}{3}(\frac{\mu}{\Delta})^2(\frac{\Delta}{T})] \tag{4}$$

A complete analytical formulation is possible for the parametrized S-matrix model of η_ℓ in ℓ-space with or without Coulomb interaction [1], if one assumes the scattering function η_ℓ split into real and imaginary parts:

$$\eta_\ell \exp(-2i\sigma_\ell) = g(t) + i\mu\frac{dg(t)}{dt} \tag{5}$$

The g(t)'s are continuously differentiable functions of the angular momentum $t(=\ell+1/2)$, which is characterized by critical angular momentum $T^{\pm}(=L\pm1/2)$ and rounding parameter $\Delta^{\pm}$ in the ℓ-space. Two of the most important parameters, the cut-off angular momentum T and the rounding parameter Δ are related respectively to the interaction radius R and the surface diffuseness d through the well known expressions given by:

$$T = kR[1-\frac{2n}{kR}]^{1/2} \tag{6}$$

and

$$\Delta = kd\left[1-\frac{n}{kR}\right]\left[1-\frac{2n}{kR}\right]^{-1/2} \tag{7}$$

Here k and n are respectively the wave number and the Coulomb parameter. The analytic expression for the elastic scattering amplitude $f(\theta)$ is obtained after performing consistent approximations as in refs.[1-6] starting from eqns.(1) and (5) ; $f(\theta)$ has the form:

$$f(\theta)=\frac{T}{K}(\frac{\theta}{\sin\theta})^{1/2}[\frac{\pi\Delta\theta}{\sinh(\pi\Delta\theta)}][\frac{iJ_1(T\theta)}{\theta}+\mu J_0(T\theta)] \tag{8}$$

In the situation of strong absorption the SAM formalism can be extended to non-elastic processes, mainly the inelastic scattering leading to various collective modes of excitation. In conjunction with the Austern-Blair relation, the amplitude for the inelastic scattering is expressed in terms of the elastic scattering matrix. Once the elastic scattering cross section is obtained in terms of the model parameters like T, Δ and μ, the inelastic scattering amplitude can be expressed as the first derivative of the elastic scattering amplitude [7,8]. The resulting expressions for the inelastic scattering cross sections can be evaluated once the scattering function η_ℓ as a function of ℓ is known from elastic scattering. The differential cross sections for single excitation of multiple order L in the Austern-Blair approximation is given by:

$$\frac{d\sigma}{d\Omega}(0\rightarrow L) = \sum_{M=-L}^{L}\left|f_{LM}(\theta)\right|^2 \tag{9}$$

where

$$f_{LM}(\theta) = \frac{i}{2}(2L+1)^{\frac{1}{2}}C_L\sum_{\ell,\ell'}i^{\ell-\ell'}(2\ell'+1)^{\frac{1}{2}}e^{i(\sigma_\ell+\sigma_{\ell'})}(\frac{\partial\eta_{\bar{\ell}}}{\partial\ell})$$

$$<\ell'L00|\ell0><\ell'L,-MM|\ell0>Y_{\ell'}^{-M}(\theta,0) \tag{10}$$

Finally, the inelastic differential cross-section is given by:

$$\frac{d\sigma}{d\Omega}(0\to L)=|C_L|^2(2L+1)(\frac{T^2}{16\pi})(\frac{\theta}{\sin\theta})\{(H_-+H_+)^2\sum_{M=-L}^{L}[\alpha_{LM}(\theta)J_{|M|}(T\theta)$$

$$-\beta_{LM}(\theta)J_{|M|-1}(T\theta)]^2+(H_--H_+)^2\sum_{M=-L}^{L}[\alpha_{LM}(\theta)J_{|M|-1}(T\theta)+\beta_{LM}(\theta)J_{|M|}(T\theta)]^2\} \tag{11}$$

The functions $\alpha_{LM}(\theta)$ and $\beta_{LM}(\theta)$ are the elements of the rotation matrix and has the property:

$\alpha_{LM}(\theta)=0$; if (L+M) is odd

$\beta_{LM(\theta)}=0$; if (L+M) is even

The coefficients C_L are the reduced nuclear matrix elements. For single excitation, the nuclear matrix elements C_L can be expressed in terms of the deformation distance δ_L by,

$$C_L=\delta_L/(2L+1)^{1/2} \tag{12}$$

The functions $\alpha_{2M}(\theta)$ and $\beta_{2M}(\theta)$ corresponding to L=2 state have the following forms:

$$\alpha_{2,\pm2}(\theta)=(\frac{1}{4})(\frac{3}{2})^{1/2}(1+\cos\theta),\beta_{2,\pm2}(\theta)=0,$$

$$\alpha_{2,\pm1}(\theta)=0,\beta_{2,\pm1}(\theta)=(\frac{1}{2})(\frac{3}{2})^{1/2}\sin\theta,$$

$$\alpha_{2,0}(\theta)=(\frac{1}{4})(3\cos\theta-1),\beta_{2,0}=0$$

Eqn.(11) for the inelastic scattering cross-section for the L=2 state becomes:

$$\frac{d\sigma}{d\Omega}(0\to 2)=\delta_2^2(\frac{T^2}{64\pi})(\frac{\theta}{\sin\theta})[(H_-+H_+)^2\{[\frac{1}{4}(3\cos\theta-1)^2+3\sin^2\theta]J_0^2(T\theta)$$

$$+\frac{3}{4}(1+\cos\theta)^2J_2^2(T\theta)\}+4(H_--H_+)^2J_1^2(T\theta)] \tag{13}$$

For octupole excitation (L =3) the functions $\alpha_{3M}(\theta)$ and $\beta_{3M}(\theta)$ corresponding to L=3 state are as follows:

$$\alpha_{3,\pm3}(\theta)=0.466\cos\frac{3\theta}{2}+0.4191\cos\frac{\theta}{2},\beta_{3,\pm3}(\theta)=0$$

$$\alpha_{3,\pm2}(\theta)=0,\beta_{3,\pm2}(\theta)=0.114\sin\frac{3\theta}{2}+0.3421\sin\frac{\theta}{2}$$

$$\alpha_{3,\pm1}(\theta)=0.1805\cos\frac{3\theta}{2}-0.108\cos\frac{\theta}{2},\ \beta_{3,\pm1}=0$$

$$\alpha_{3,0}(\theta)=0,\ \beta_{3,0}(\theta)=0.069\sin\frac{3\theta}{2}-0.375\sin\frac{\theta}{2}$$

Then eqn.(11) for the inelastic scattering cross-section for the L=3 state reduces to:

$$\frac{d\sigma}{d\Omega}(0\to 3)=\delta_3^2(\frac{T^2}{64\pi})(\frac{\theta}{\sin\theta})[(H_--H_+)^2\{J_0^2(T\theta)(2\alpha_{31}^2(\theta)+\beta_{30}^2(\theta))$$

$$+J_2^2(T\theta)(2\alpha_{33}^2(\theta)+2\beta_{32}^2(\theta))\}+(H_-+H_+)^2\{J_1^2(T\theta)$$

$$\times(2\alpha_{31}^2(\theta)+2\beta_{32}^2(\theta)+\beta_{30}^2(\theta))+J_3^2(T\theta)2\alpha_{33}^2(\theta)\}] \tag{14}$$

where,

$$H_{\pm} = [1+\mu(\theta_c \pm \theta)]F[\Delta(\theta_c \pm \theta)]$$

Eqns. (13) and (14) depend on the four parameters viz. T, Δ, μ and δ_L. The SAM parameters T, Δ and μ are obtained from the analysis of the elastic scattering cross-section and the deformation length, $\delta_L=\beta_L R$ is determined by normalizing the SAM cross section with experimental ones. The deformation parameter β_L is related to the deformation length δ_L through the expression:

$$\delta_L = (T/k)\beta_L \qquad (15)$$

The proton and neutron deformation parameters, denoted respectively by β_p and β_n, are obtained from the $\beta(\pi^+)$ and $\beta(\pi^-)$ values using the following relations *[9,10]* which are strictly valid at pion energies around the Δ - resonance,

$$Z\beta_p = [3\beta(\pi^+) - \beta(\pi^-)]A/4$$

and

$$N\beta_n = [3\beta(\pi^-) - \beta(\pi^+)]\,A/4 \qquad (16)$$

The proton and neutron matrix elements, M_p and M_n respectively, are then extracted from the above values of β_p and β_n and from there the ratios of these to the single particle matrix elements, denoted respectively by $G_p(\pi)$ and $G_n(\pi)$ are obtained, as in ref. [9]. The various terms are defined in ref. [9].

3. Results and Discussion

3.1 Elastic scattering

The result of the SAM analyses of the angular distribution data [14-19] for the elastic scattering of π^+ and π^- are summarized respectively in tables I and II. Some typical fits are illustrated in figs.1-3.

The analytical expression (8) has been used to calculate the elastic scattering cross-sections for $\pi^{\pm}$ incident on the target nuclei ^{14}N at 162 MeV, ^{28}Si at 130, 164 and 226 MeV, ^{40}Ca at 65, 80 and 180 MeV and $^{42,44,48}Ca$ at 180 MeV. The angular distribution data for the elastic scattering of $\pi^{\pm}$ from the target nuclei covered in the present work are available over the angular range 10^0-130^0. The fit obtained to the experimental data is excellent in most cases. The SAM fit is usually poor at low energies, for example, the fit is poor for ^{40}Ca at 65 and 80 MeV compared to quality of fit achieved at relatively higher energies. Generally, it is observed that the SAM description to the data becomes increasingly better as both target mass and projectile energy increase.

The interaction radius R and the surface diffuseness d are calculated from the best fit parameters using relations 6 and 7 and these are shown in tables I and II. The standard nuclear radius is obtained from the relation $R = r_0 A^{1/3}$ and is presented in tables I and II. The standard nuclear radius 'r_0' is approximately constant at the value given by

r_0 = 1.49±0.06 fm, for π^+
= 1.49±0.09 fm, for π^-

Table I. The best fit SAM parameters for π^+.

Nucleus	E_π (MeV)	SAM Parameters T	Δ	$\mu/4\Delta$	Derived quantities R (fm)	$\lambda\!\!\!^-$ (fm)	(R-$\lambda\!\!\!^-$) (fm)	d (fm)	σ_r (mb)	$\sigma_r/\pi R^2$	r_o (fm)
^{14}N	162	4.2	0.75	0.06	3.98	0.93	3.05	0.71	711.00	1.43	1.35
^{28}Si	130	4.9	0.83	0.045	5.20	1.04	4.16	0.87	1171.00	1.39	1.46
^{28}Si	164	5.6	0.90	0.056	5.27	0.92	4.35	0.84	1191.00	1.37	1.48
^{28}Si	226	6.48	0.95	0.158	5.18	0.79	4.39	0.75	1110.00	1.32	1.45
^{40}Ca	65	4.0	0.60	0.23	6.13	1.46	4.64	0.88	1451.00	1.24	1.55
^{40}Ca	80	4.1	0.70	0.21	5.63	1.32	4.31	0.93	1300.00	1.31	1.43
^{40}Ca	164	6.3	0.95	0.092	5.94	0.92	5.02	0.88	1470.00	1.33	1.50
^{40}Ca	180	6.8	0.93	0.108	6.11	0.88	5.23	0.82	1510.00	1.29	1.55
^{40}Ca	241	7.75	0.97	0.232	6.00	0.76	5.24	0.74	1400.00	1.24	1.52
^{42}Ca	180	6.8	0.90	0.111	6.11	0.88	5.23	0.80	1491.00	1.28	1.52
^{44}Ca	180	6.8	0.90	0.111	6.10	0.88	5.22	0.80	1491.00	1.28	1.50
^{48}Ca	180	7.0	0.93	0.089	6.28	0.88	5.40	0.82	1590.00	1.29	1.51
^{152}Sm	180	9.7	0.99	0.333	8.82	0.88	7.94	0.87	2720.00	1.11	1.50

Table II. The best fit SAM parameters for π^-.

Nucleus	E_π (MeV)	SAM Parameters T	Δ	$\mu/4\Delta$	Derived quantities R (fm)	$\bar{\lambda}$ (fm)	$R-\bar{\lambda}$ (fm)	d (fm)	σ_r (mb)	$\sigma_r/\pi R^2$	r_o (fm)
^{14}N	162	4.30	0.75	0.077	4.01	0.93	3.08	0.71	740.00	1.47	1.37
^{28}Si	130	4.95	0.7	0.139	5.09	1.04	4.05	0.73	1120.00	1.38	1.43
^{28}Si	164	5.6	0.85	0.009	5.15	0.92	4.23	0.79	1171.00	1.41	1.44
^{28}Si	226	6.28	0.8	0.103	4.93	0.79	4.14	0.63	1010.00	1.33	1.38
^{40}Ca	65	4.2	0.5	0.375	5.98	1.46	4.52	0.74	1440.00	1.28	1.51
^{40}Ca	80	4.2	0.45	0.35	5.41	1.32	4.09	0.60	1151.00	1.26	1.37
^{40}Ca	163.3	6.5	0.85	0.074	5.96	0.92	5.04	0.79	1510.00	1.35	1.51
^{40}Ca	180	6.86	0.93	0.013	5.99	0.88	5.11	0.82	1541.00	1.37	1.52
^{40}Ca	205	7.4	0.65	0.25	6.07	0.82	5.25	0.54	1390.00	1.19	1.54
^{40}Ca	241	7.75	0.99	0.106	5.88	0.76	5.12	0.76	1440.00	1.33	1.49
^{42}Ca	180	7.0	0.95	0.027	6.12	0.88	5.24	0.84	1610.00	1.36	1.53
^{44}Ca	180	7.15	1.0	0.012	6.25	0.88	5.37	0.88	1690.00	1.37	1.58
^{48}Ca	180	7.3	0.95	0.027	6.38	0.88	5.5	0.84	1721.00	1.34	1.53
^{152}Sm	180	10.85	1.05	0.035	9.34	0.88	8.46	0.92	3531.00	1.28	1.60

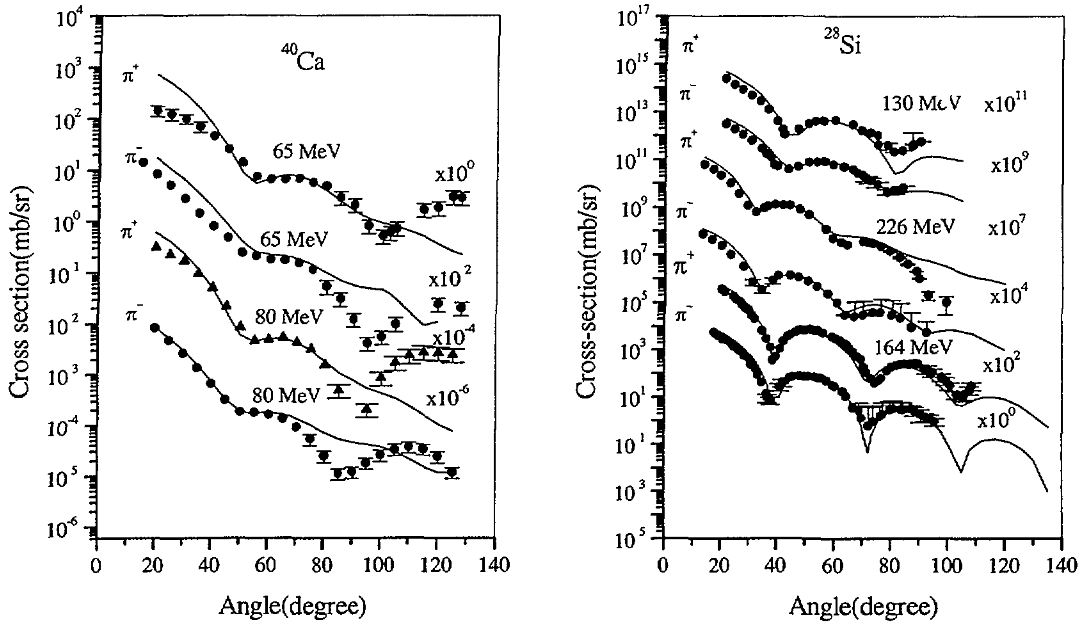

Fig 1. SAM fit to the elastic scattering of pions from different nuclei.

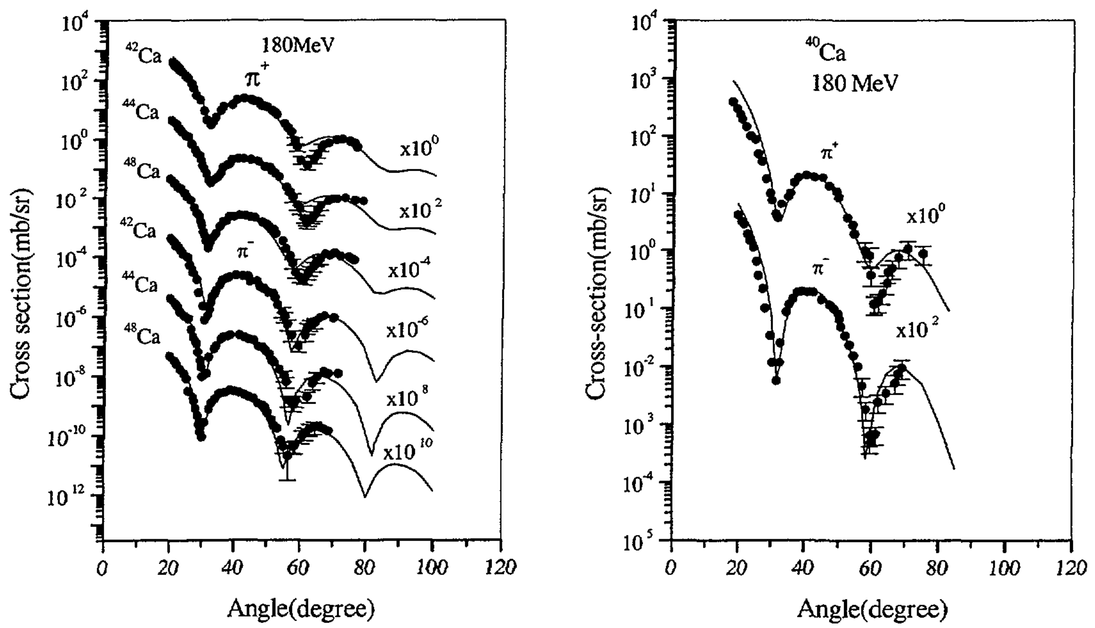

Fig 2. SAM fit to the elastic scattering of pions from different nuclei.

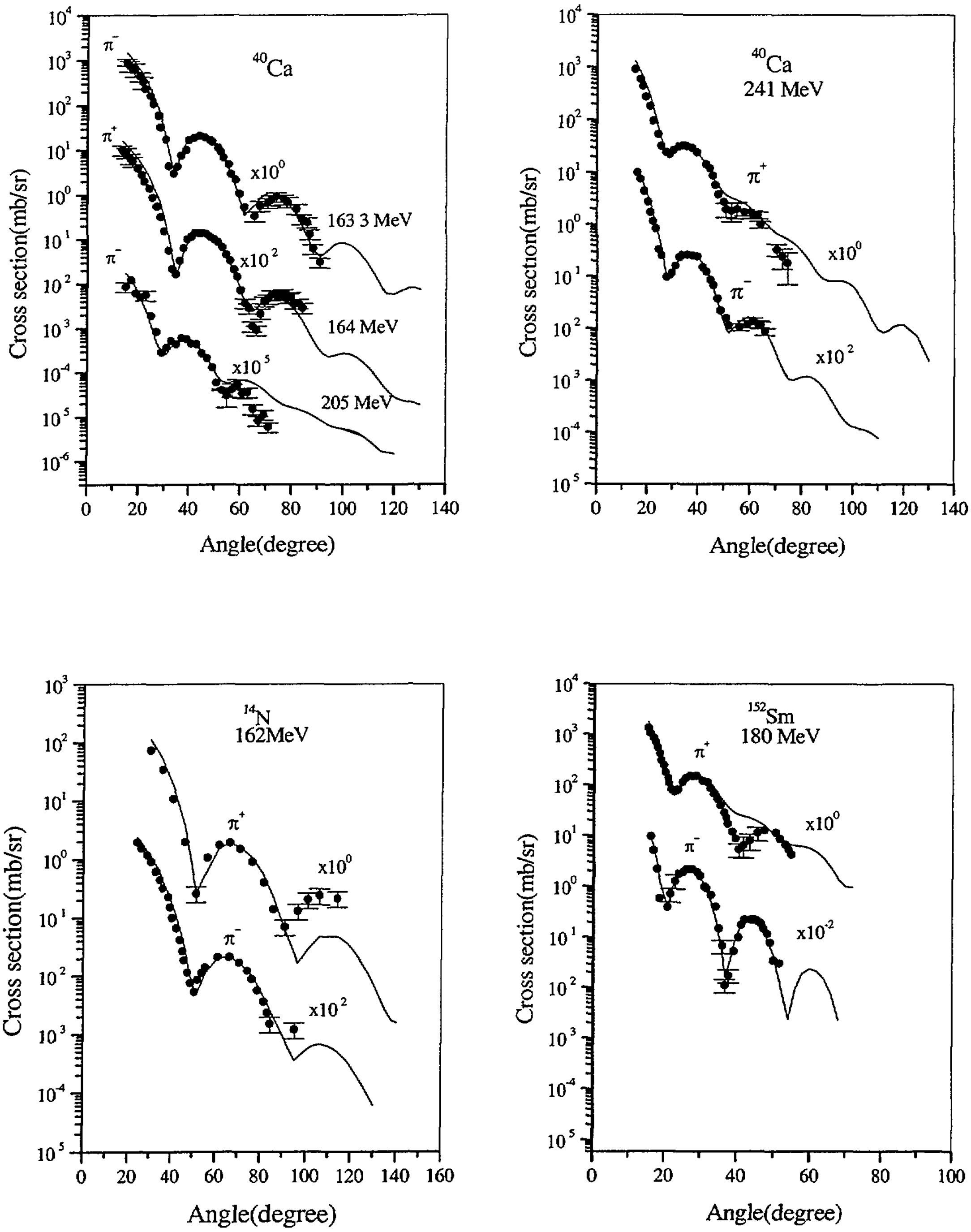

Fig 3. SAM fit to the elastic scattering of pions from different nuclei.

The surface diffuseness d remains roughly the same for the whole range of projectile energies and the target nuclei covered in this study. The surface diffuseness d remains approximately constant at the value given by,

$$d = 0.82 \pm 0.10 \text{ fm, for } \pi^+$$
$$= 0.76 \pm 0.07 \text{ fm, for } \pi^-$$

The agreement between two r_0 values is extremely good over a wide range of nuclear mass covered in the present study and the value of d in the two cases (π^+ and π^-) agree with each other quite well. These substantiate the confidence on the SAM parameters obtained.

The value of σ_r increases almost smoothly as both target mass and energy increase. The value of $\sigma_r / \pi R^2$ remains fairly constant (= 1.32) throughout the analyses, as it should be and which is more meaningful than the σ_r itself.

Mass dependence of σ_r is looked into, i.e. a plot of variation of $(\sigma_r / \pi)^{1/2}$ versus $A^{1/3}$ is accomplished. Results are shown in fig.4 together with the linear least squares fits to the SAM values at energy E_π=180 MeV covering a number of nuclei. The extracted values of σ_r at 180 MeV for π^- are consistently larger than those for π^+ (fig.4) and the difference between π^- and π^+ $(\sigma_r(\pi^-) - \sigma_r(\pi^+))$ absorption cross section increases systematically with an increase in mass number. These aspects are in conformity with strong absorption situation and in agreement with known (3,3) dominance at around 200 MeV.

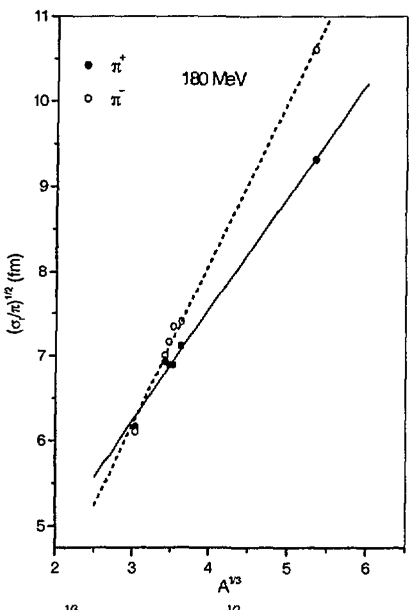

Fig4 $A^{1/3}$ dependence of $(\sigma_r/\pi)^{1/2}$ The solid and broken lines represent the least squares linear fits for π^+ and π^- respectively.

3.2 Inelastic scattering

The inelastic scattering angular distributions [12,20] of pions with the excitation of the lowest $2^+$and 3^- collective states in various nuclei are analyzed using the SAM parameters obtained from the corresponding elastic scattering (tables I and II). Some typical fits are illustrated in figs. 5-8. The deformation parameters estimated from these analyses are summarized in tables III-IV. The relevant parameters summarized by Raman et al. [13] and Spear [11] respectively for the lowest 2^+ and 3^- collective states are also included in the tables for comparison.

The inelastic scattering of charged pions leading to the lowest 2^+ and 3^- collective states are theoretically calculated using eqns. (13) and (14) with the SAM parameters fixed from the corresponding elastic scattering analyses. A reasonably good reproduction of the experimental data in most of the cases is possible, indicating thereby a dominant collective mode of excitation of the levels, as well as reliability of the SAM analyses.

Table III. Deformation parameters for the lowest 2^+ state.

Nucleus	E_x (MeV)	E_π (MeV)	Deformation Parameter, β_2 (a)	(b)	(c)	(d)
^{42}Ca	1.52	180	0.20	0.21	0.247	
^{44}Ca	1.16	180	0.21	0.22	0.253	
^{48}Ca	3.83	180	0.07	0.108	0.101	
^{152}Sm	0.122	180	0.285	0.31	0.305	0.287

(a) Present work from the inelastic scattering of π^+

(b) Present work from the inelastic scattering of π^-

(c) Adopted values: ref. [13]

(d) DWIA calculation : ref. [12]

Table IV. Deformation parameters for the lowest 3^- states.

Nucleus	E_x (MeV)	E_π (MeV)	Deformation Parameter, β_3 (a)	(b)	(c)	(d)
^{42}Ca	3.44	180	0.39	0.32	0.26	
^{44}Ca	3.31	180	0.32	0.25	0.23	
^{48}Ca	4.51	180	0.33	0.27	0.25	
^{152}Sm	1.041	180	0.10	0.15	0.12	0.12

(a) Present work from the inelastic scattering of π^+
(b) Present work from the inelastic scattering of π^-
(c) Adopted values: ref. [11]
(d) Previous work : DWIA calculation; ref. [12]

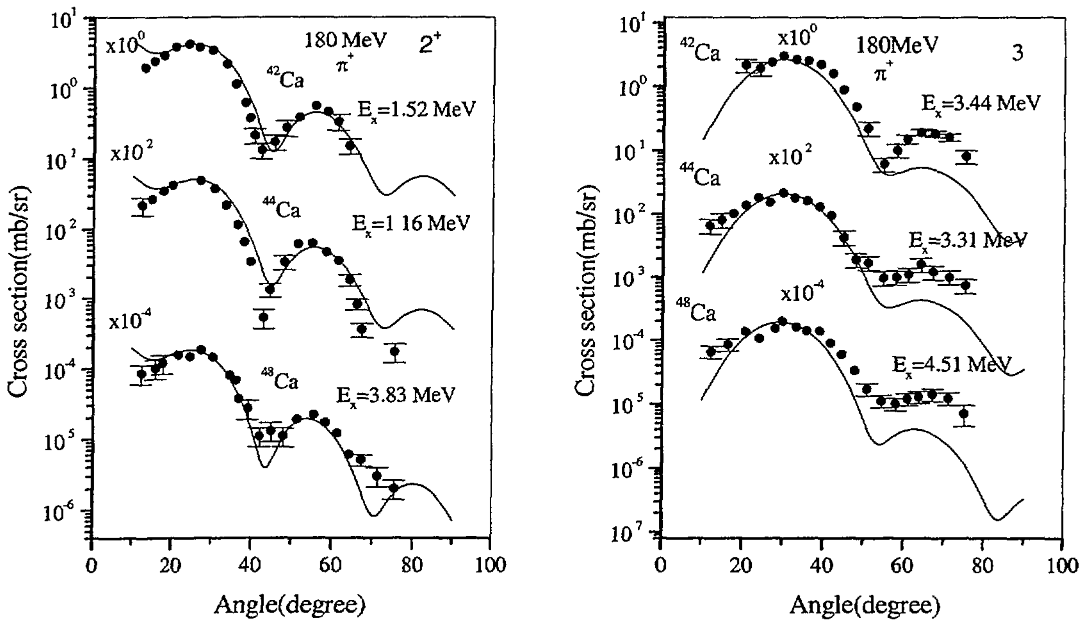

Fig.5. SAM fit to the inelastic scattering of pions leading to the lowest 2^+ and 3^- state.

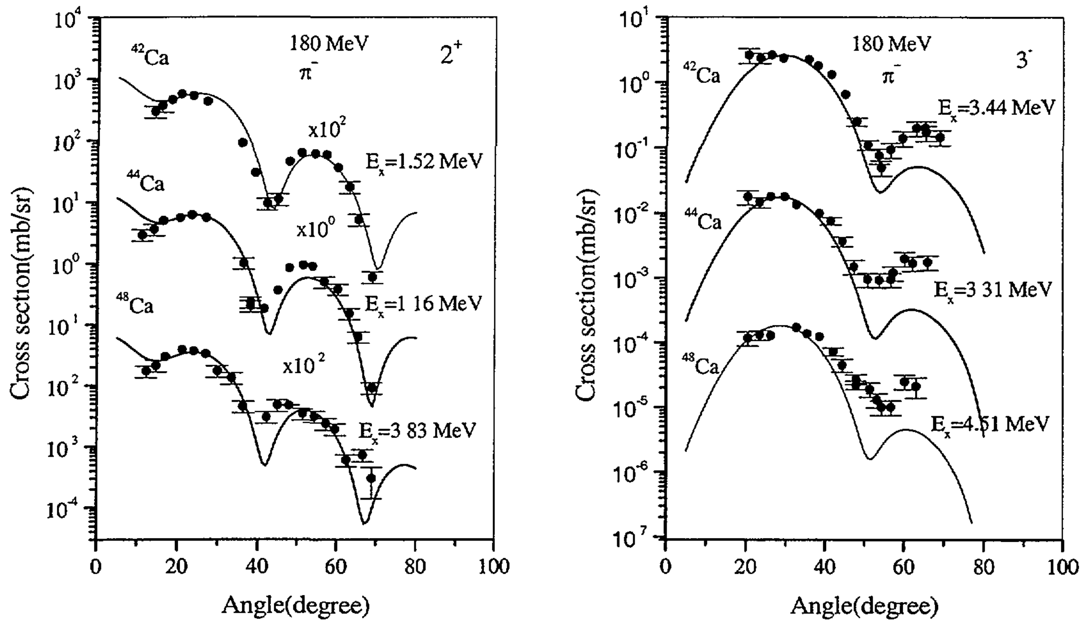

Fig.6. SAM fit to the inelastic scattering of pions leading to the lowest 2^+ and 3^- states.

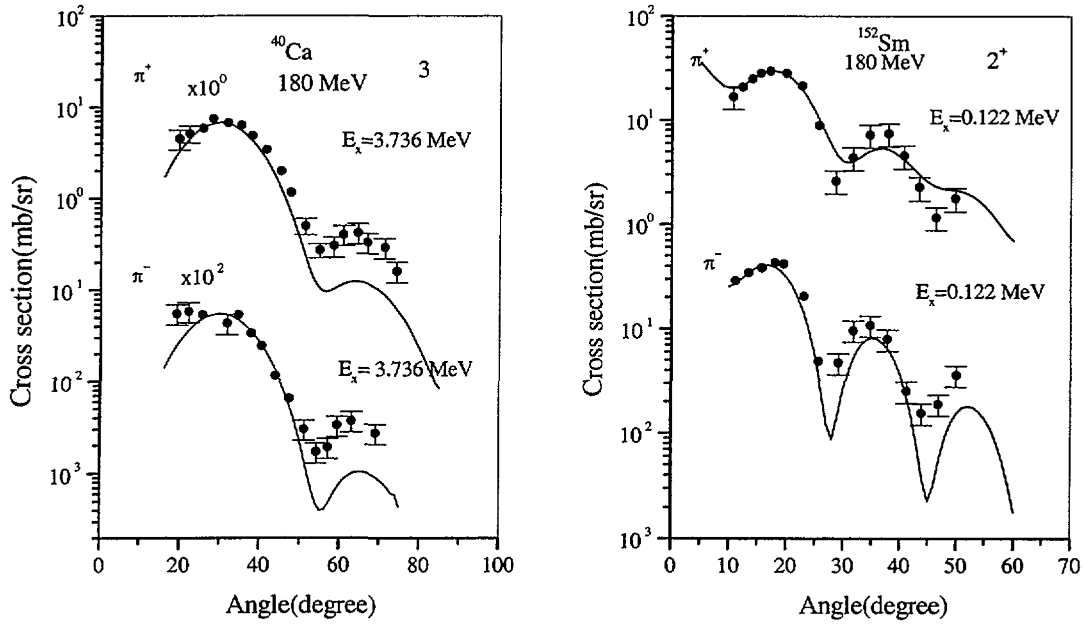

Fig.7. SAM fit to the inelastic scattering of pions leading to the lowest 2^+ and 3^- states.

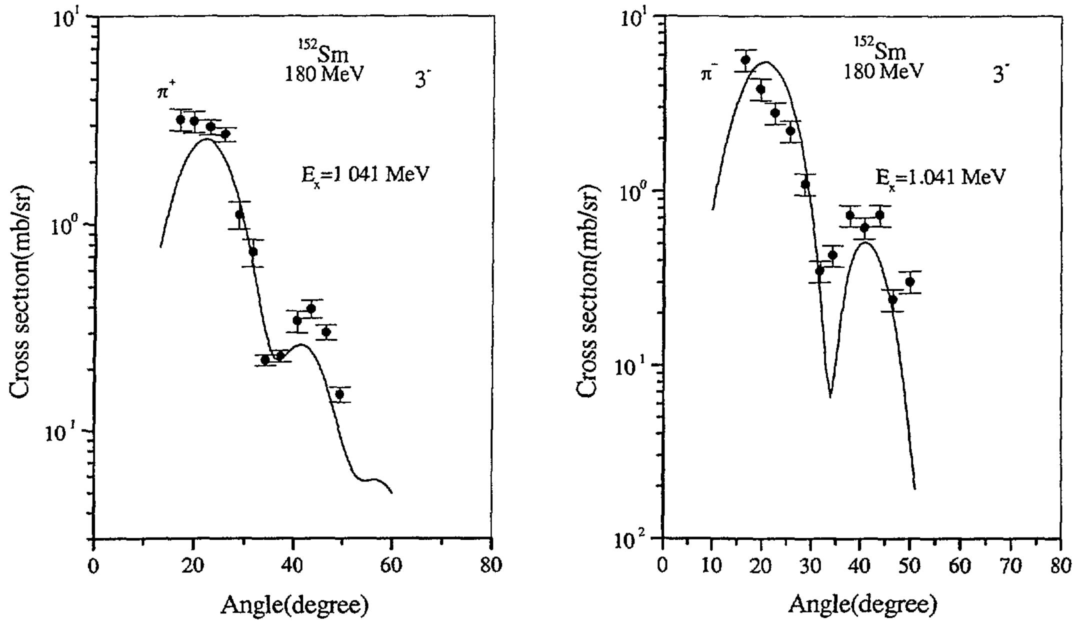

Fig 8. SAM fit to the inelastic scattering of pions leading to the lowest 2^+ and 3^- states.

Normalization of the SAM cross sections to the inelastic scattering data for a particular level of the same nucleus is done in exactly the same way for both π^+ and π^- by fitting over a wide angular range as possible and the normalization is done at the first maximum of the angular distributions of both π^+ and π^-, where the cross-sections at the subsequent peak or peaks are not given in magnitude or position.

The predicted angular distributions of 2^+ collective states for all the cases are very well reproduced. A reasonably good fit is also obtained in most of the cases for the 3^- collective states. The calculated deformation parameters β_2 and β_3 are compared with the values obtained from the DWIA analyses and with other values obtained by other workers for 2^+ and 3^- states. Our obtained values are usually close to DWIA analyses. A close scrutiny of the values of the parameters shows that the present values are fairly close to other values of the other workers. It is observed that there is hardly any difference between the values of $\beta_2(\pi^+)$ and $\beta_2(\pi^-)$ and between $\beta_3(\pi^+)$ and $\beta_3(\pi^-)$ values.

The matrix elements $G_p(\pi)$ and $G_n(\pi)$ have been extracted by Peterson [9] (and partly worked out in references therein) from the DWIA analyses of the inelastic scattering of pions at energies around the Δ-resonance based on the Kisslinger potential. These results being relevant to the present work are included in tables V(a) and V(b). The ratio $(G_n(\pi)/N)\,/(G_p(\pi)/Z)$ for most of the levels studied in the present work is approximately unity or so, in good agreement with previous results obtained from the DWIA analyses [9] within errors (tables VI(a) and VI(b)), suggesting thereby their isoscalar mode of excitation.

Table V. (a) Proton and neutron deformation parameters and transition matrix elements extracted from the inelastic scattering of 180 MeV pions leading to the lowest 2^+ state.

Nucleus	β_p	β_n	$G_p(\pi)$		$G_n(\pi)$		$(G_n/N)/(G_p/Z)$	
	(a)	(a)	(a)	(b)	(a)	(b)	(a)	(b)
^{42}Ca	0.205	0.2052	2.59	2.87	2.85	3.38	1.00	1.07
^{44}Ca	0.226	0.206	2.85	3.04	3.12	3.98	0.912	1.09
^{48}Ca	0.0612	0.11	0.80	1.56	2.00	3.71	1.78	1.70
^{152}Sm	0.334	0.272	13.06	11.9	15.44	18.9	0.82	1.094

(b)

Nucleus	M_p		M_n		$G_0(\pi)$		$G_0(\alpha)$
	(a)	(b)	(a)	(b)	(a)	(b)	(b)
^{42}Ca	17.03	18.9	18.73	22.2	5.44	6.25	3.53
^{44}Ca	19.37	20.7	21.18	27.7	5.97	7.02	4.77
^{48}Ca	5.56	11.2	13.98	26.7	2.8	5.27	3.79
^{152}Sm	202.76	184	239.68	202	28.5	30.8	7.67

(a) From SAM analysis (Present work)
(b) From DWIA analysis (Peterson [9] and references therein)

Table VI. (a) Proton and neutron deformation parameters and transition matrix elements extracted from the inelastic scattering of 180 MeV pions leading to the lowest 3⁻ state.

Nucleus	β_p	β_n	$G_p(\pi)$		$G_n(\pi)$		$(G_n/N)/(G_p/Z)$	
	(a)	(a)	(a)	(b)	(a)	(b)	(a)	(b)
^{42}Ca	0.446	0.272	5.71	4.41	3.83	3.95	0.61	0.81
^{44}Ca	0.391	0.197	5.00	3.44	3.02	3.21	0.5	0.78
^{48}Ca	0.432	0.206	5.53	3.13	3.69	3.57	0.5	0.81
^{152}Sm	0.092	0.148	3.65	4.31	8.52	5.44	1.6	0.87

(b)

Nucleus	M_p		M_n		$G_0(\pi)$		$G_0(\alpha)$
	(a)	(b)	(a)	(b)	(a)	(b)	(b)
^{42}Ca	124	119	103.68	107	9.54	8.36	5.08
^{44}Ca	114.94	98	85.82	98	8.02	6.65	4.80
^{48}Ca	117.08	96.9	114.2	111	9.22	6.7	5.59
^{152}Sm	357.67	434	835.2	575	12.17	9.75	

(a) From SAM analysis (Present work)
(b) From DWIA analysis (Peterson [9] and references therein)

4. Conclusions

A good account of the elastic scattering of pions at energies between 65 to 241 MeV i.e. below and above the Delta resonance from several target nuclei between ^{14}N to ^{152}Sm is given by the three parameters of the strong absorption model of Frahn and Venter. Mass number dependence of the reaction cross-section obtained in the present work is found to be consistent with previous studies using different models. The SAM with the same set of parameters T, Δ and μ as obtained from the elastic scattering analysis without any adjustment, is successful enough in describing simultaneously inelastic scattering phenomena. The estimated deformation parameters β_2 and β_3 are in excellent agreement with previous works. There is hardly any difference between $\beta_2(\pi^+)$ and $\beta_2(\pi^-)$ and that between $\beta_3(\pi^+)$ and $\beta_3(\pi^-)$. The proton and neutron deformation parameters are extracted from $\beta_L(\pi^+)$ and $\beta_L(\pi^-)$. Then, following Peterson [9] but using a different model, the proton and neutron matrix elements are obtained wherefrom it is possible to determine the relative importance of protons and neutrons in the excitation of various levels in nuclei. It is to be noted that the same set of elastic scattering parameters gives a fairly good account of the inelastic scattering of pions populating to the lowest 2^+ and 3^- collective states for most cases.

This no doubt speaks of the reliability of the SAM parameters. The crucial test of the parameters extracted from elastic scattering lies to the extent of its ability in describing the non-elastic data [21]. The SAM is thus a useful model in a situation dominated by the strong absorption by the nuclear surface and the uniqueness of the model parameters is its advantage. But the major drawback is that it does not give any insight into the fundamental interactions in nuclei-like NN and in the present context the πN and Δ-hole interactions. These are all summarily replaced by a 'black disc' with a diffuse surface. The present model takes advantage of the selectivity around the Delta resonance without provoking the strong energy dependence of the pion-nucleon amplitude. But nonetheless all the observed features in the pion-nucleus scatterings are reasonably well accounted for by the present model.

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