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DEM–NT–00–07

Rotations of Nuclei with Octupole Deformations

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We propose a collective Hamiltonian which incorporates interactions capable of generating rotations in nuclei with the simultaneous presence of octupole and quadrupole deformations. It is demonstrated that the model formalism can reproduce the staggering effects observed in nuclear octupole bands. On this basis we propose that the interactions involved should provide a relevant tool to study collective phenomena in nuclei and other quantum mechanical systems with reflection asymmetric correlations.

Lecture presented at the *NATO Advanced Study Institute on Nuclei Far from Stability and Astrophysics* (Predeal, Romania, 2000), to appear in the Proceedings, ed. H. Rebel and D. N. Poenaru, in the *Romanian Journal of Physics*.

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The properties of nuclear systems with octupole deformations [1] are of current interest due to the increasing of evidence for the presence of octupole instability in different regions of the nuclear table [2]. Various parametrizations of the octupole degrees of freedom have opened a useful tool for understanding the role of the reflection asymmetric correlations and for analysis of the collective properties of such kinds of systems [2]. An important step in this direction is to elucidate the question: what are the collective nuclear interactions that correspond to the different octupole shapes and how do they determine the structure of the respective energy spectra?

In the present work we address the above problem by examining the interactions that generate collective rotations in a system with octupole deformation. Based on the octahedron point symmetry parametrization of the octupole shape [3], we propose a general collective Hamiltonian which incorporates the interactions responsible for the rotations associated with the different octupole deformations. It will be shown that after taking into account the quadrupole degrees of freedom and the appropriate higher order quadrupole-octupole interaction the model formalism is able to reproduce schematically some interesting effects of the fine rotational structure of nuclear octupole bands.

Our model formalism is based on the principal that the collective properties of a physical system in which octupole correlations take place should be influenced by the most general octupole field $V_3 = \sum_{\mu=-3}^{3} \alpha_{3\,\mu}^{fix} Y_{3\,\mu}^{*}$, (in the intrinsic, body-fixed frame), which can be written in the form [3]:

$$V_3 = \epsilon_0 A_2 + \sum_{i=1}^{3} \epsilon_1(i) F_1(i) + \sum_{i=1}^{3} \epsilon_2(i) F_2(i) \ , \tag{1}$$

where the quantities

$$A_2 = -\frac{i}{\sqrt{2}}(Y_{3\,2} - Y_{3\,-2}) = \frac{1}{r^3}\sqrt{\frac{105}{4\pi}} xyz \ , \tag{2}$$

$$F_1(1) = Y_{3\,0} = \frac{1}{r^3}\sqrt{\frac{7}{4\pi}} z(z^2 - \frac{3}{2}x^2 - \frac{3}{2}y^2) \ , \tag{3}$$

$$\begin{aligned} F_1(2) &= -\frac{1}{4}\sqrt{5}(Y_{3\,3} - Y_{3\,-3}) + \frac{1}{4}\sqrt{3}(Y_{3\,1} - Y_{3\,-1}) \\ &= \frac{1}{r^3}\sqrt{\frac{7}{4\pi}} x(x^2 - \frac{3}{2}y^2 - \frac{3}{2}z^2) \ , \end{aligned} \tag{4}$$

$$\begin{aligned} F_1(3) &= -i\frac{1}{4}\sqrt{5}(Y_{3\,3} + Y_{3\,-3}) - i\frac{1}{4}\sqrt{3}(Y_{3\,1} + Y_{3\,-1}) \\ &= \frac{1}{r^3}\sqrt{\frac{7}{4\pi}} y(y^2 - \frac{3}{2}z^2 - \frac{3}{2}x^2) \ , \end{aligned} \tag{5}$$

$$F_2(1) = \frac{1}{\sqrt{2}}(Y_{3\,2} + Y_{3\,-2}) = \frac{1}{r^3}\sqrt{\frac{105}{16\pi}} z(x^2 - y^2) \ , \tag{6}$$

$$F_2(2) = \frac{1}{4}\sqrt{3}(Y_{3\,3} - Y_{3\,-3}) + \frac{1}{4}\sqrt{5}(Y_{3\,1} - Y_{3\,-1}) = \frac{1}{r^3}\sqrt{\frac{105}{16\pi}} x(y^2 - z^2) \ , \tag{7}$$

$$F_2(3) = -i\frac{1}{4}\sqrt{3}(Y_{3\,3} + Y_{3\,-3}) + i\frac{1}{4}\sqrt{5}(Y_{3\,1} + Y_{3\,-1}) = \frac{1}{r^3}\sqrt{\frac{105}{16\pi}} y(z^2 - x^2), \tag{8}$$

(with $r^2 = x^2 + y^2 + z^2$) belong to the irreducible representations (irreps) of the octahedron group (O). A_2 is one-dimensional, while F_1 and F_2 are three-dimensional irreps. The seven real parameters ϵ_0 and $\epsilon_r(i)$ ($r = 1,2$; $i = 1,2,3$) determine the amplitudes of the octupole deformation.

Our proposition is that the collective Hamiltonian which incorporates the shape characteristics of the octupole field (1) can be constructed on the basis of the above octahedron irreps. For this purpose we introduce operator forms of the quantities A_2, $F_1(i)$ and $F_2(i)$ ($i = 1,2,3$) in which the cubic terms of the Cartesian variables x, y and z in Eqs (2)–(8) are replaced by appropriately symmetrized combinations of cubic terms of the respective angular momentum operators $\hat{I}_x$, $\hat{I}_y$, $\hat{I}_z$ (with $\hat{I}^2 = \hat{I}_x^2 + \hat{I}_y^2 + \hat{I}_z^2$). The following Hamiltonian is then obtained:

$$\hat{H}_{oct} = \hat{H}_{A_2} + \sum_{r=1}^{2}\sum_{i=1}^{3} \hat{H}_{F_r(i)} \ , \qquad (9)$$

with

$$\hat{H}_{A_2} = a_2\frac{1}{4}[(\hat{I}_x\hat{I}_y + \hat{I}_y\hat{I}_x)\hat{I}_z + \hat{I}_z(\hat{I}_x\hat{I}_y + \hat{I}_y\hat{I}_x)] \ , \qquad (10)$$

$$\hat{H}_{F_1(1)} = \frac{1}{2}f_{11}\hat{I}_z(5\hat{I}_z^2 - 3\hat{I}^2) \ , \qquad (11)$$

$$\hat{H}_{F_1(2)} = \frac{1}{2}f_{12}(5\hat{I}_x^3 - 3\hat{I}_x\hat{I}^2) \ , \qquad (12)$$

$$\hat{H}_{F_1(3)} = \frac{1}{2}f_{13}(5\hat{I}_y^3 - 3\hat{I}_y\hat{I}^2) \ , \qquad (13)$$

$$\hat{H}_{F_2(1)} = f_{21}\frac{1}{2}[\hat{I}_z(\hat{I}_x^2 - \hat{I}_y^2) + (\hat{I}_x^2 - \hat{I}_y^2)\hat{I}_z] \ , \qquad (14)$$

$$\hat{H}_{F_2(2)} = f_{22}(\hat{I}_x\hat{I}^2 - \hat{I}_x^3 - \hat{I}_x\hat{I}_z^2 - \hat{I}_z^2\hat{I}_x) \ , \qquad (15)$$

$$\hat{H}_{F_2(3)} = f_{23}(\hat{I}_y\hat{I}_z^2 + \hat{I}_z^2\hat{I}_y + \hat{I}_y^3 - \hat{I}_y\hat{I}^2) \ , \qquad (16)$$

where a_2 and f_{ri} ($r = 1,2$; $i = 1,2,3$) are the Hamiltonian parameters.

The general collective Hamiltonian of a system with octupole correlations should contain also the standard (axial) quadrupole rotation part (a simultaneous presence of octupole and quadrupole degrees of freedom is assumed)

$$\hat{H}_{rot} = A\hat{I}^2 + A'\hat{I}_z^2 \ , \qquad (17)$$

where A and A' are the inertial parameters. In addition, we introduce the following higher order diagonal quadrupole-octupole interaction term (corresponding to the product $Y_{20} \cdot Y_{30}$):

$$\hat{H}_{qoc} = f_{qoc}\frac{1}{I^2}(15\hat{I}_z^5 - 14\hat{I}_z^3\hat{I}^2 + 3\hat{I}_z\hat{I}^4) \ . \qquad (18)$$

Finally, the Hamiltonian of the system can be written as

$$\hat{H} = \hat{H}_{bh} + \hat{H}_{rot} + \hat{H}_{oct} + \hat{H}_{qoc} \ . \tag{19}$$

Here $\hat{H}_{bh} = \hat{H}_0 + f_k \hat{I}_z$ is a pure phenomenological part which provides the bandhead energy $E_{bh} = E_0 + f_k K$ (E_0 and f_k are free parameters).

The physical relevance of the Hamiltonian (19) depends on the possibility to determine in a unique way the third angular momentum projection K. We suggest that for any given angular momentum I the quantum number K should be determined so as to minimize the respective collective energy. The resulting energy spectrum represents the yrast sequence of energy levels for our model Hamiltonian.

As a first step in testing our Hamiltonian we consider its diagonal part

$$\hat{H}^d = \hat{H}_{bh} + \hat{H}_{rot} + \hat{H}^d_{oct} + \hat{H}_{qoc} \ , \tag{20}$$

were the operator $\hat{H}^d_{oct} \equiv \hat{H}_{F_1(1)}$ represents the diagonal part of the pure octupole Hamiltonian $\hat{H}_{oct}$, Eq. (9). The following diagonal matrix element is then obtained:

$$\begin{aligned} E_K(I) &= E_0 + f_k K + AI(I+1) + A'K^2 + f_{11}\left(\frac{5}{2}K^3 - \frac{3}{2}KI(I+1)\right) \\ &+ f_{qoc}\frac{1}{I^2}\left(15K^5 - 14K^3I(I+1) + 3KI^2(I+1)^2\right) \ . \end{aligned} \tag{21}$$

The respective yrast sequence $E(I)$ is determined after minimizing Eq. (21) as a function of integer K in the range $-I \leq K \leq I$. The obtained energy spectrum depends on six model parameters: E_0 essentially responsible for the bandhead energy; f_k which provides minimal energy for $K = K_{bh} = I_{bh}$; A and A' are the quadrupole inertial parameters which should generally correspond to the known quadrupole shapes (axes ratios) of nuclei; f_{11} and f_{qoc} are the parameters of the diagonal octupole (11) and quadrupole-octupole (18) interactions respectively.

Table 1: The "yrast" energy levels, $E(I)$ (in KeV), and the respective K-values obtained by Eq. (21) for the parameter set $E_0 = 500$keV, $f_k = -7.5$keV, $A = 12$keV, $A' = 6.6$keV, $f_{11} = 0.56$keV, $f_{qoc} = 0.085$keV.

I	$E(I)$	K	I	$E(I)$	K	I	$E(I)$	K
1	522.772	1	13	2335.81	5	25	5453.12	11
2	568.327	1	14	2576.57	6	26	5694.49	12
3	637.095	1	15	2827.57	6	27	5935.5	12
4	728.71	1	16	3082.36	7	28	6157.5	13
5	840.857	2	17	3344.94	7	29	6378.29	13
6	971.155	2	18	3608.18	8	30	6575.37	14
7	1123.22	2	19	3877.05	8	31	6770.62	14
8	1288.09	3	20	4143.16	9	32	6937.23	15
9	1472.71	3	21	4413.03	9	33	7101.62	15
10	1668.56	4	22	4676.45	10	34	7232.21	16
11	1880.56	4	23	4942.01	10	35	7360.44	16
12	2101.68	5	24	5197.18	11	36	7449.45	17

We applied several sample sets of the above parameters and obtained the corresponding schematic energy spectra. One of them is given in Table 1. It is seen that the "yrast" values of the quantum number K gradually increase with the increase of the angular momentum I. We remark that they correspond to the local minima of Eq. (21) as a function of K. Such a behavior of the spectrum corresponds to a wobbling motion and could also be interpreted as a multiband-crossing phenomenon.

In addition we see that the K- values of the odd and the even sequence of levels are in groups, couples which implies the presence of an odd–even staggering effect. Indeed, the presence of such an effect is demonstrated in Fig. 1 (a)–(e), where the quantity

$$Stg(I) = 6\Delta E(I) - 4\Delta E(I-1) - 4\Delta E(I+1) + \Delta E(I+2) + \Delta E(I-2)\ , \quad (22)$$

with $\Delta E(I) = E(I+1) - E(I)$, is plotted as a function of angular momentum I for several different sets of model parameters. (The quantity $Stg(I)$ is the discrete approximation of the fourth derivative of the function $\Delta E(I)$, i.e. the fifth derivative of the energy $E(I)$. Its physical relevance has been discussed extensively in Refs [4, 5].)

Fig. 1(a) illustrates a long $\Delta I = 1$ staggering pattern with several irregularities, which looks similar to the "beats" observed in the octupole bands of some light actinides such as ^{220}Ra, ^{224}Ra and ^{226}Ra [4]. In Fig. 1(b) the increased values of f_{11} and f_{qoc} provide a wide angular momentum region (up to $I \sim 40$) with a regular staggering pattern. A further increase of f_{qoc} results in a staggering pattern with different amplitudes, as shown in Fig. 1(c). Further, staggering pattern with many "beats" is obtained, as shown in Fig 1(d). An example with almost constant staggering amplitude is shown in Fig. 1(e). It resembles the form of the odd–even staggering predicted in the SU(3) limit of various algebraic models (see Ref. [4] for details and relevant references). It also resembles the odd–even staggering seen in some octupole bands of light actinides, such as ^{222}Rn [4].

Now we can discuss the general Hamiltonian structure (19) including the various non-diagonal terms (10), (12)–(16) which provide a K-bandmixing interaction. In Fig. 1(f) a staggering pattern in the presence of K-bandmixing is illustrated. We see that the mixing leads to a decrease in the staggering amplitude with the increase of angular momentum, so that the staggering pattern is strongly suppressed. This pattern resembles the experimental situation in ^{228}Th [4] (odd–even staggering with amplitude decreasing as a function of I).

In such a way, we find that the axial symmetric (diagonal) term $\hat{H}_{F_1(1)}$ is the only pure octupole degree of freedom which provides a staggering behavior of the quantity (22). Thus, our analysis suggests that the $\Delta I = 1$ staggering effect observed in systems with octupole deformations should be considered as the manifestation of the axial symmetric "pear-like" shape.

The staggering patterns illustrated in Fig. 1 reproduce the form of almost all known $\Delta I = 1$ staggering patterns in nuclei. So, we suppose that the model parameters could be adjusted so as to reproduce quantitatively the staggering

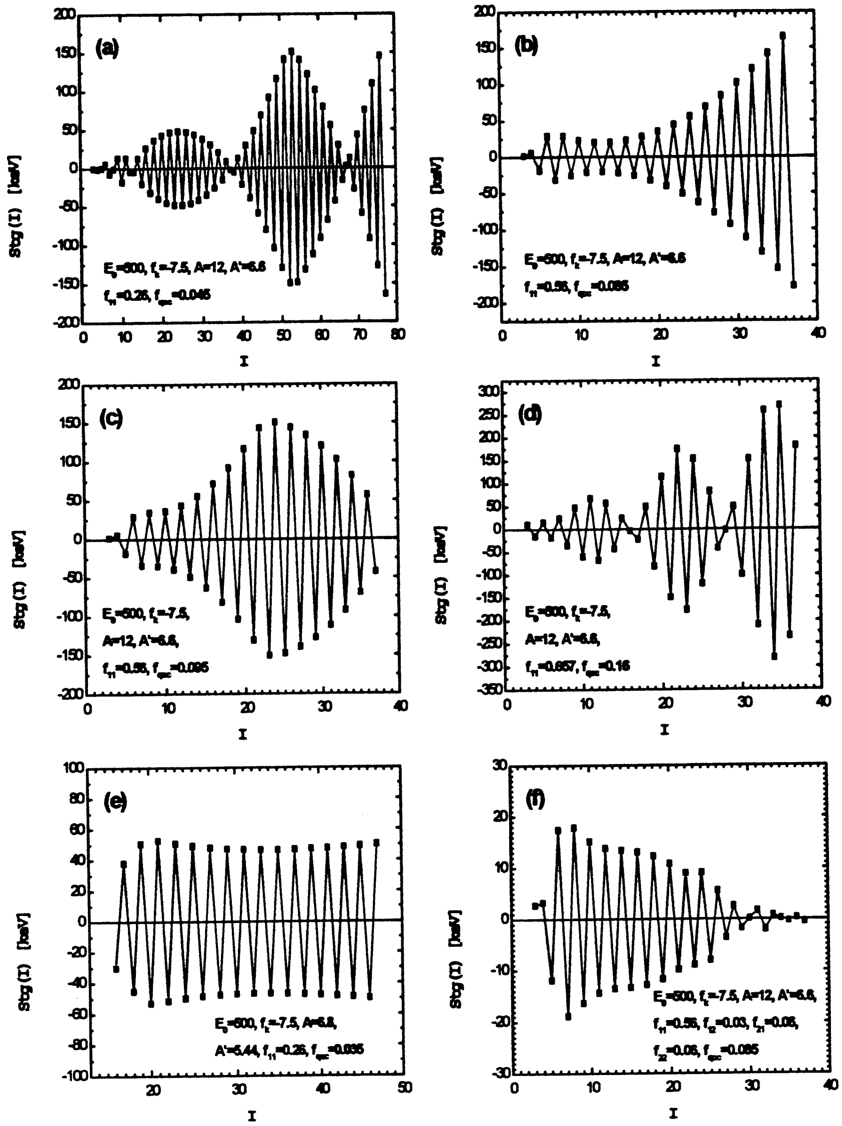

Figure 1: $\Delta I = 1$ staggering patterns [Eq. (22)] obtained: (a) – (e) by the diagonal Hamiltonian (20) for several different sets of model parameters; (f) by adding three non-diagonal terms $\hat{H}_{F_1(2)}$ [Eq. (12)], $\hat{H}_{F_2(1)}$ [Eq. (14)] and $\hat{H}_{F_2(2)}$ [Eq. (15)] to the diagonal Hamiltonian (20).

effects in all nuclear octupole bands as well as in some rotational negative parity bands built on octupole vibrations.

In closing, we note that the collective interactions considered in this work suggest the presence of various fine rotational band structures in nuclear systems with collective octupole correlations. In particular, they produce various staggering patterns, which appear as the result of a delicate interplay between the terms of a pure octupole interaction and those of a high order quadrupole–octupole interaction. The analysis carried out illustrates the dominant role of the axial symmetric "pear-like" shape for the presence of a $\Delta I = 1$ staggering effect. The obtained multi K- band crossing structures can be thought of as a wobbling collective motion of the system. Finally, the interactions used in this work should provide a natural handle for the study of collective phenomena in nuclei and in other quantum mechanical systems with complex shape correlations.

This work has been supported by the Bulgarian National Fund for Scientific Research under contract no MU–F–02/98.

References

[1] A. Bohr and B.R. Mottelson, *Nuclear Stucture*, vol. II (New York: Benjamin, 1975).

[2] P.A. Butler and W. Nazarewicz, Rev. Mod. Phys., **68**(1996)349.

[3] I. Hamamoto, X. Zhang and H. Xie, Phys. Lett **B257**(1991)1.

[4] D. Bonatsos, C. Daskaloyannis, S.B. Drenska, N. Karoussos, N. Minkov, P. P. Raychev and R.P. Roussev, Phys. Rev. **C62**(2000)024301.

[5] N. Minkov, S. Drenska, P. Raychev, R. Roussev and D. Bonatsos, Phys. Rev. **C61**(2000)064301.