Ac

Scalıng property of cosmıc ray leptons and protons

LBNL 54274

T F Hoang

Lawrence Berkeley National Laboratory Berkeley, Ca 94720
and
University of California, Space Science Laboratory, Berkeley, Ca 94720

Abstract

The scaling property, characteristic of cosmic ray acceleration, is investigated with the Kolmogorov distribution for chaotic processes The width parameter L is found to be about the same for the spectra of ν, μ, and p with $\overline{\mathrm{L}} = 0\ 249 \pm 0\ 010$, just like their spectral index $\overline{\gamma} = 2\ 773 \pm 0\ 019$ of the power law These scaling properties suggest that the spectrum may be represented by a one-dimensional Ulam map, its Liapunov exponent determined by $\overline{\gamma}$ indicates 4 jumps of shock waves to achieve the acceleration of these particles

An investigation of the spectrum of cosmic ray particles is imperative for understanding the complex process of their acceleration As the neutrinos are mostly from decays of muons, the width of their spectra is expected to be different from that of muons However, from recent precise measurements of ν_e spectrum by the AMANDA Collaboration [1] and ν_μ spectrum by the Super-K Collaboration [2], it is found that their width is about the same as those of muons at high altitudes by the MASS Collaboration [3] and the CAPRICE Collaboration [4], as well as muons at sea-level by the Kiel-Durham Collaboration [5,6] Furthermore, these widths are comparable to that of protons at high altitudes by the AMS Collaboration [7] On the other hand, the spectral index of all these spectra turns out to be practically the same These intrinsic properties of scaling, essentially different from the case of particles produced by high energy accelerators, are to be accounted for by any analytical model to describe the cosmic ray spectrum

As regards the momentum spectrum of cosmic ray particles, partly because of the chaotic nature of their acceleration process, and partly because of the parabolic shape of their spectrum in the log-plots, therefore for their description, it is appropriate to use phenomenologically the Kolmogorov distribution [8], together with the power law predicted by the shock wave acceleration [9] We will see that the spectral index represents actually the Lyapunov exponent [10] Consequently, it determines the number of shock jumps [11] for the acceleration of cosmic ray particles We recall that it is rather straightforward to derive the Kolmogorov distribution from the basic equation of energy gain of the original Fermi model [12]

We use the phase space

$$\zeta = \mathrm{Log\ P}, \tag{1}$$

as kinematic variable and write

$$\frac{\mathrm{dn}}{\mathrm{dP}} = \mathrm{N}e^{-(\zeta+\zeta^*)^2/2L} \qquad \mathrm{P} < \mathrm{P}_\times, \tag{2}$$

where ζ^* is the shift of the spectrum maximum, L is the width parameter and N is the normalization coefficient, whereas

$$\frac{\mathrm{dn}}{\mathrm{dP}} = \frac{\mathrm{C}}{\mathrm{P}^\gamma} \qquad \mathrm{P} > \mathrm{P}_\times, \tag{3}$$

γ being the spectral index and C the normalization coefficient As the second derivative $d^2n/d\mathrm{P}^2$ of the lognormal distribution (2) is negative definite, therefore the tangent slope starts from zero at the maximum, then decreases monotonically to $-\infty$ as $\mathrm{P} \to \infty$ It follows that the two distributions (2) and (3) may be joined tangentially at $\mathrm{P}_\times$ according to

$$\mathrm{Log\ P}_\times = 2\ 303\gamma\ \mathrm{L} - \zeta^*, \tag{4}$$

so that the power law (3) is to correct the tail of the lognormal distribution (2) and represents actually the asymptotic behavior of the spectrum

We use these distributions to analyze the spectrum of cosmic ray particles Consider first the ν_e spectrum of the AMANDA Collaboration [1] and the ν_μ spectrum of the Super-K Collaboration [2], as shown in Fig 1 The dotted and solid curves represent the least-squares fits with (2) and (3), respectively The parameters of lognormal fits are summarized in Table I, together with the average momentum $< P >$ (in GeV/c) computed according to the fit Note that their average momentum differs by $\sim$ 200, due to the large difference in the position of their maximum at antilog($-\zeta^*$) = 37 07 GeV/c for ν's of AMANDA compared to 0 109 GeV/c in the case of Super-K

As regards the power law fit with Eq (3), it is not applicable to the Super-K spectrum, as its momentum range is too restricted For the AMANDA data, we find (in solid line)

$$\gamma_\nu = 2\ 775 \pm 0\ 253, \ \ C_\nu = (4\ 28 \pm 1\ 48)10^6$$

A comparison with the data indicates that the fits are very satisfactory indeed, as shown in Fig 2 the test by the moment analysis of both spectra The points corresponding to the first 5 moments (triangles and nablas for the Super-K and the AMANDA data) are all close to the bisector If we tentatively assume

$$(\mathrm{Mmt})_{fit} = \mathrm{c}[(\mathrm{Mmt})_{exp}]^\alpha, \tag{5}$$

the validity of the phenomenological distribution (2) requires both parameters α and c to be consistent with 1 We find

$$\alpha = 0\ 996 \pm 0\ 005, \ \ c = 0\ 849 \pm 0\ 0172 \qquad \text{for AMANDA,}$$

$$\alpha = 1\ 063 \pm 0\ 004, \ \ c = 1\ 190 \pm 0\ 020 \qquad \text{for Super-K}$$

It is interesting to note that the width parameters L of these two distributions so different in their shape are about the same within $\sim$ 1 2 standard deviations In fact, L = 0 218 may fit the AMANDA spectrum as well with $\zeta^* = -1\ 586 \pm 0\ 009$ comparable to the free-parameter fit listed in Table I This implies the scaling property, namely one of the two spectra may be superposed onto another by sliding itself along its coordinate axes Furthermore, the spectral index γ is comparable to those of muons and protons (see below), as well as α-particles and heavy nuclei of cosmic rays [13]

Next, we consider negative muons at various high altitudes from 26 to 225g/cm^2 of the MASS Collaboration [3], and at mountain altitude 886 g/cm^2 residual atmosphere of the CAPRICE Collaboration [4], then at sea-level 1036 g/cm^2 from the Kiel Collaboration [5] and the Durham Collaboration [6] The fits according to (2) and (3) are shown in Fig 3(b) by the dotted and the solid curves, respectively The parameters of the lognormal fits and the estimates of $< P >$ are listed in Table I Note that the estimates of $< P >_\mu$ for the CAPRICE experiment [4] at 24 - 255 g/cm^2 are less than those of other μ experiments because of restricted momentum range

As for the power law fits (in solid lines) we find

$$\gamma_\mu = 2\ 763 \pm 0\ 032, \ \ C_\mu = 363\ 0 \pm 25\ 4 \ \text{ for CAPRICE,}$$

and

$$\gamma_\nu = 2\ 982 \pm 0\ 146, \ \ \mathrm{C}_\mu = 2150 \pm 12 \ \text{ for Kiel-Durham}$$

Here, we find both parameters, ζ^* and L remain practically the same, independent of altitudes and especially, independent of the location and the time of these experiments This is due to negligible energy loss suffered by high energy muons As regards the spectral index of the muons, here again, we find it equal to that of protons within standard errors

Finally, let us turn to the protons at high altitude, at 3 8 g/cm^2 of the AMS Collaboration [7] shown in Fig 4, together with the curves of fits according to (2) and (3) The parameters of the lognormal fit are in Table I As for the power law fit, we find

$$\gamma_p = 2\ 768 \pm 0\ 249, \ \ \mathrm{C}_p = (6\ 136 \pm 0\ 102)10^3$$

Here again, we find the same spectral index as for the leptons, the mean value of these indices being

$$\overline{\gamma} = 2\ 773 \pm 0\ 019 \tag{6}$$

It is most remarkable that the width parameter L of these very different cosmic ray spectra of ν, μ, and p is practically the same, their weighted average being

$$\overline{\mathrm{L}} = 0\ 249 \pm 0\ 010 \simeq 0\ 250 \tag{7}$$

Therefore the dynamical properties of cosmic ray particles differ essentially from those of accelerator particles We recall that in the latter case, the decay of $\mu \rightarrow e + \nu + \overline{\nu}$ leads to $\mathrm{L}_\nu \neq \mathrm{L}_\mu$

This remarkable scaling property implies that as far as the computation of the momentum distribution is concerned, the phenomenological distributions (2) and (3) may be replaced by a one-dimensional logistic map of chaotic approach, namely

$$f(x) = 1 - \mu x^2 \tag{8}$$

by replacing

$$\zeta + \zeta^* \rightarrow x, \qquad \frac{1}{\mathrm{N}}\frac{\mathrm{dn}}{\mathrm{dP}} \rightarrow e^{f(x)} \tag{9}$$

The parameter μ, characteristic of the map, is related to the mean value $\overline{\mathrm{L}}$ of the spectrum width as follows

$$\mu = \frac{1}{2\overline{\mathrm{L}}} = 2\ 008 \pm 0\ 024 \simeq 2 \tag{10}$$

so that we are dealing with the Ulam map [14] Its fixed point defined by $\dot{x}^* = f(x^*)$ takes place at the end points of the map, namely $x^* = \pm 1$ Whereas its Liapunov exponent

$$\lambda = \ln 2, \tag{11}$$

may be either positive or negative in the case of dissipative process

Therefore we may regard the spectral index γ of the cosmic ray spectrum as a negative Liapunov exponent, the ratio of its magnitude to ln 2 of the Ulam map determines the number of cycles to accelerate the cosmic ray particles

$$n = \frac{|\gamma|}{\lambda} = 4\ 001 \pm 0\ 027 \simeq 4,$$

i e 4 jumps of chock waves to achieve the acceleration

Finally, we note, in passing, that there exist in the literature sophisticated simulation models to predict the cosmic ray spectrum by inspiring the Feynman-Yang scaling for particles from high energy accelerators, characteristic of the forward narrow peak As the scaling property is different for cosmic ray particles, therefore, bias may be inherent in models using such an analogy As in the case of predictions for the neutrino spectrum of the Super-K experiment [2], namely $< P >_\nu = 0\ 871 \pm 0\ 106$ GeV/c, $0\ 725 \pm 0\ 196$ GeV/c and $0\ 650 \pm 0\ 026$ GeV/c according to the Bartol model [15], the Fluka model [16] and the HKKM model [17], respectively, compared to the experimental value $0\ 885 \pm 0\ 067$ GeV/c The underestimation of these predictions are tested by the moment analysis according to (5), the parameter c is found to be less than 1 foı all these models [15,16,17]

On the other hand, for the cosmic ray muons at sea-level, the predicted $< P >_\mu = 4\ 276 \pm 0\ 165$ GeV/c and $4\ 804 \pm 0\ 338$ GeV/c by the Fluka and the HKKM model [16,17] are rather overestimated, compared to the experimental value $\overline{< P >}_\mu = 3\ 675 \pm 0\ 133$ GeV/c, still worse is the Bartol prediction [18] $< P >_\mu = 29\ 50 \pm 1\ 04$ GeV/c These overestimated $< P >_\mu$ are indicated by the slopes $\alpha >> 1$ (see Ref [15,16,17]) as is found by the plots of moment analysis according to (5)

Acknowledgements

The author wishes to thank H M Steiner, D Groom, R G Stokstad, and G Smoot for information on cosmic ray experiments, J Engelage and P N Price for valuable help and discussions Thanks to H J Crawford for the kind hospitality at LBNL and the Tsi Jung Memorial Fund for the support The work is done through the U S Department of Energy Contract No FG03-09 ER4057

References and Footnotes

[1] AMANDA Collaboration, R Porreta, 25 International Cosmic Ray Conference, vol 7, p 9, Durbam, (1997)

[2] Super-K Collaboration, R Becker Szendy et al , Phys Rev Lett 69, 1010 (1992) The data here used are from Review of Particle Properties, Phys Rev D 66, 010001-185 (2002)

[3] MASS Collaboration, R Bellotti, et al, Phys Rev D 60, 032002 (1999)

[4] CAPRICE Collaboration, J Kremer et al, Phys Rev Lett , 83, 4241 (1999) and M Boezio et al, Astrophys J, 518, 457 (1999) and Astropart Phys 19, 583 (2003)

[5] Kiel Collaboration, O C Allhoffer et al , Phys Lett B 36,4(1971)

[6] Durham Collaboration C A Ayre et al , Phys G, 1, 584(1975)

[7] AMS Collaboration, J Alcarez et al , Phys Lett B 400, 27 (2000)

[8] A M Kolmogorov, Doll, Akad Sci USSR 119, 861 (1985)

[9] See, v g B T Draine and C F McKee, Ann Rev Astrophys 31, 273 (1993)

[10] M A Liapunov, Probleme General de la Stabilite du Mouvement, Annales de la Facdeds Sciences de Seconde Serie, vol 9 (1907), Reprinted by Princeton University (1947)

[11] E E Fermi, Physical 75, 1169 (1949)

[12] A R Bell, Mon Notes RoyAstr Soc 18147 (1978)

[13] J R Horemdel, Astropart Phys 19, 193 (2003)

[14] See, v g , C Beck and S Schlozl, Thermodynamics of Chaotic Systems, Cambridge University Press (1995), p 29

[15] V G Barr, T K Gaisser, and T Stanev, Phys Rev D 39, 3532 (1989) For the neutrinos, leaving aside two dispersed points near the end of the spectrum at P = 2 5 GeV/c and P > 3 GeV/c, we find for the lognormal fit to their prediction $\zeta^* = 0\ 782 \pm 0\ 033, \mathrm{L} = 0\ 174 \pm 0\ 028$ and $\mathrm{N} = 2\ 346 \pm 0\ 443$ and for the moments analysis $\alpha = 0\ 979 \pm 0\ 028$ and $c = 0\ 833 \pm 0\ 288$ For muons $\zeta^* = -0\ 285 \pm 0\ 005$, $\mathrm{L} = 0\ 336 \pm 0\ 005$, $\mathrm{N} = 3317 \pm 19$, and $\alpha \simeq 1\ 505$ and $c \simeq 6\ 06$, both much greater than 1

[16] G Battistoni et al , Astropart Phys 12, 365 (2000) and ar Xiev hep/020703v2 20 Jan (2003) The parameters of the lognormal fit to their prediction for neutrinos are $\zeta^* = 1\ 695 \pm 0\ 045$, L $= 0\ 326 \pm 0\ 009$ and N $= 2\ 947 \pm 1\ 026$ and for the moments analysis $\alpha = 0\ 979 \pm 0\ 028$ and $c = 0\ 833 \pm 0\ 050$ For muons $\zeta^* = 0\ 308 \pm 0\ 011$, L $= 0\ 274 \pm 0\ 009$, N $= 4096 \pm 261$, and $\alpha \simeq 1\ 225$ and c $\simeq 0\ 764$, both differ from 1

[17] M Honda T Kajita, K Kasohara and S Midorikawa, Phys Rev D 64, 05011 (2001) According to their prediction for ν, $\zeta^* = 1\ 765 \pm 0\ 126$, L $= 0\ 314 \pm 0\ 022$ and N $= 3\ 296 \pm 0\ 143$ and $\alpha = 0\ 802 \pm 0\ 063$ and $c = 0\ 684 \pm 0\ 257$ For muons $\zeta^* = 0\ 410 \pm 0\ 010$, L $= 0\ 304 \pm 0\ 015$, N $= 4809 \pm 31$, and $\alpha \simeq 1\ 272$ and c $\simeq 1\ 117$, both differ from 1

[18] The muon spectrum predicted by the Bartol model V Agramal, T Gaisser, P Lipari and T Stanov, Phys Rev D 53, 1314 (1996), is very broad The width parameter of their predicted spectrum L $= 0\ 336$ is much larger than that 0 257 measured by the Kiel-Durham experiment Whereas the position of the momentum of the maximum determined by ζ^* is at 1 93 GeV/c, compared to 0 446 GeV/c of the experimental spectrum in Fig 3(b) This displaced maximum leads to a much larger $< P >_\mu = 29\ 5$ GeV/c compared to 3 668 GeV/c measured experimentally as listed in Table I

Table I- Parameters of lognormal distributions Eq (2) for ν_e of the AMANDA Collaboration [1], ν_μ of the Super-K Collaboration [2], μ^- of the MASS Collaboration [3], the CAPRICE Collaboration [4], the Kiel-Durham Collaborations [5,6] and protons of the AMS Collaboration [7] The normalization coefficient N is in events/GeV/c for the neutrinos, its unit for other particles see figures Average momentum < P > computed according to the fit

Part	g/cm^2	ζ^*	L	N	< P > GeV/c
ν_e	5000	$-1\ 569 \pm 0\ 013$	$0\ 190 \pm 0\ 016$	$3\ 88 \pm 0\ 12$	$169\ 5 \pm 22\ 3$
ν_μ	6000	$0\ 962 \pm 0\ 097$	$0\ 218 \pm 0\ 023$	$108\ 6 \pm 22\ 3$	$0\ 885 \pm 0\ 067$
μ^-	25-47	$0\ 674 \pm 0\ 086$	$0\ 253 \pm 0\ 020$	$(1\ 83 \pm 0\ 07)\times10^{-4}$	$1\ 804 \pm 0\ 022$
μ^-	48-83	$0\ 700 \pm 0\ 156$	$0\ 252 \pm 0\ 038$	$(1\ 12 \pm 0\ 13)\times10^{-3}$	$1\ 731 \pm 0\ 020$
μ^-	83-106	$0\ 510 \pm 0\ 079$	$0\ 213 \pm 0\ 018$	$(1\ 12 \pm 0\ 30)\times10^{-2}$	$1\ 859 \pm 0\ 015$
μ^-	106-164	$0\ 560 \pm 0\ 107$	$0\ 231 \pm 0\ 025$	$(1\ 42 \pm 0\ 130\times10^{-1}$	$1\ 888 \pm 0\ 012$
μ^-	164-255	$0\ 511 \pm 0\ 062$	$0\ 265 \pm 0\ 037$	$1\ 09 \pm 0\ 02$	$1\ 806 \pm 0\ 181$
μ^-	886	$0\ 360 \pm 0\ 010$	$0\ 271 \pm 0\ 005$	$(1\ 46 \pm 0\ 02)\times10^{-3}$	$3\ 682 \pm 0\ 035$
μ^-	1036	$0\ 351 \pm 0\ 010$	$0\ 257 \pm 0\ 010$	$(3\ 86 \pm 0\ 26)\times10^{-3}$	$3\ 668 \pm 0\ 170$
p	3 8	$0\ 346 \pm 0\ 028$	$0\ 230 \pm 0\ 006$	$530\ 3 \pm 3\ 1$	$3\ 368 \pm 0\ 074$

Figure Captions

[1] Log-plots of the momentum spectrum for cosmic ray ν_e of the AMANDA Collaboration (in circles) and ν_μ of the Super-K Collaboration [2] (in triangles) The dashed curves are least-squares fits with Eq (2), the parameters are in Table I The solid line represents the power law (3) fit with $\gamma_{nu} = 2.775 \pm 0.250$

[2] Plots of the first 5 moments for momentum spectra of ν_e, ν_μ, μ computed according to the fits Eq (2) vs those of data Moments test requires all the points lie on the bisector as shown by the solid line, see text

[3] Muon momentum at various altitudes (in g/cm^2) of the MASS Collaboration [3], the CAPRICE Collaboration [4], the Kiel Collaboration [5] and the Durham Collaboration [6]. The curves are lognormal fits, the parameters are listed in Table I The straight lines are power law fits with $\gamma_\mu = 2.763 \pm 0.032$ and 2.982 ± 0.146 for the CAPRICE and the Kiel-Durham curves

[4] Spectrum of high altitude protons at 3 g/cm^2 of the AMS Collaboration [7] The dotted curve represents the lognormal fit, the parameters are in Table I The straight line represents the power law fit with $\gamma_p = 2.768 \pm 0.249$

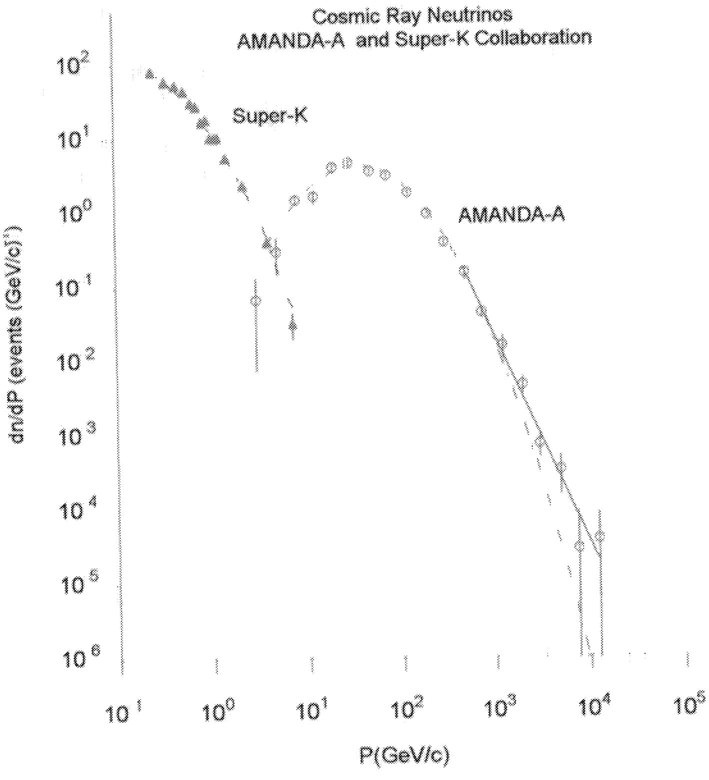

Cosmic Ray Neutrinos
AMANDA-A and Super-K Collaboration
Super-K
AMANDA-A
dn/dP (events (GeV/c)^{1})
10^2
10^1
10^0
10 1
10 2
10 3
10 4
10 5
10 6
10 1
10^0
10^1
10^2
10^3
10^4
10^5
P(GeV/c)

Fig 1

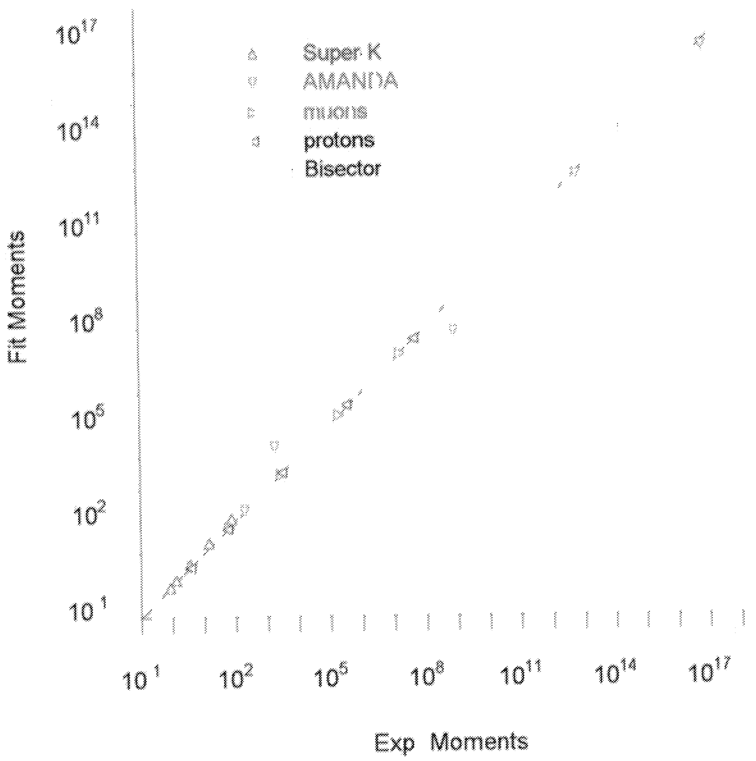
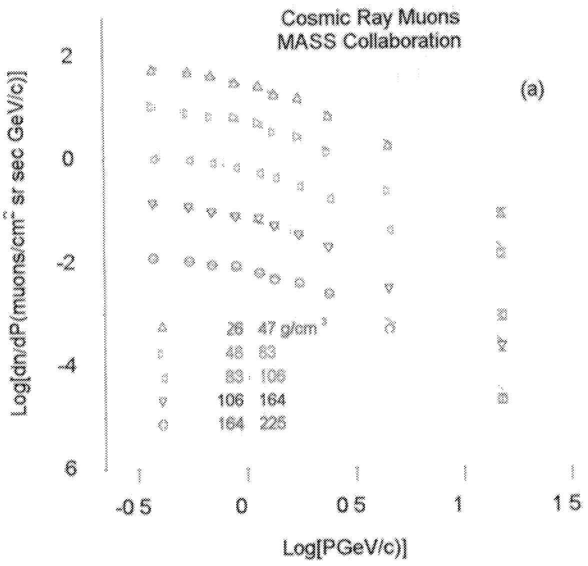

Fit Moments
Exp Moments
Super K
AMANDA
muons
protons
Bisector
10^{17}
10^{14}
10^{11}
10^{8}
10^{5}
10^{2}
10^{1}

Fig 2


Cosmic Ray Muons
MASS Collaboration
(a)
Log[dn/dP(muons/cm² sr sec GeV/c)]
26 47 g/cm³
48 83
83 106
106 164
164 225
Log[PGeV/c)]

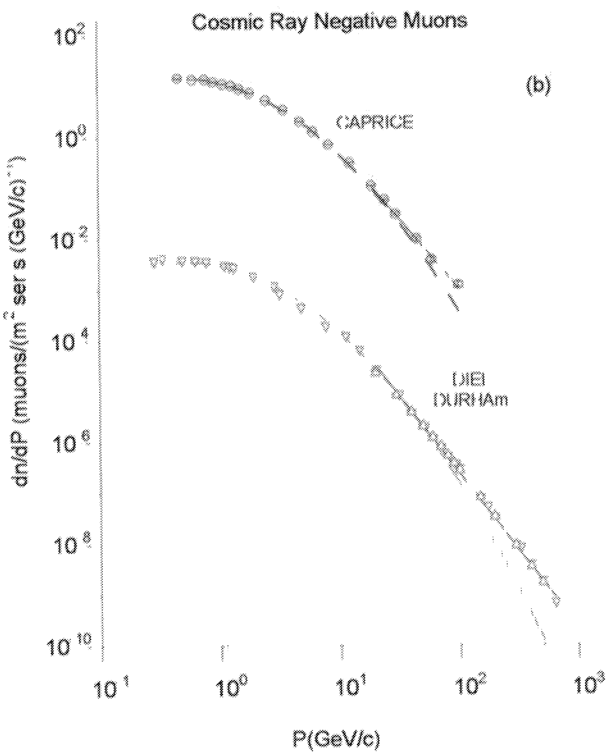

Cosmic Ray Negative Muons
(b)
CAPRICE
DURHAm
dn/dP (muons/(m² ser s (GeV/c)⁻¹)
P(GeV/c)

Fig 3

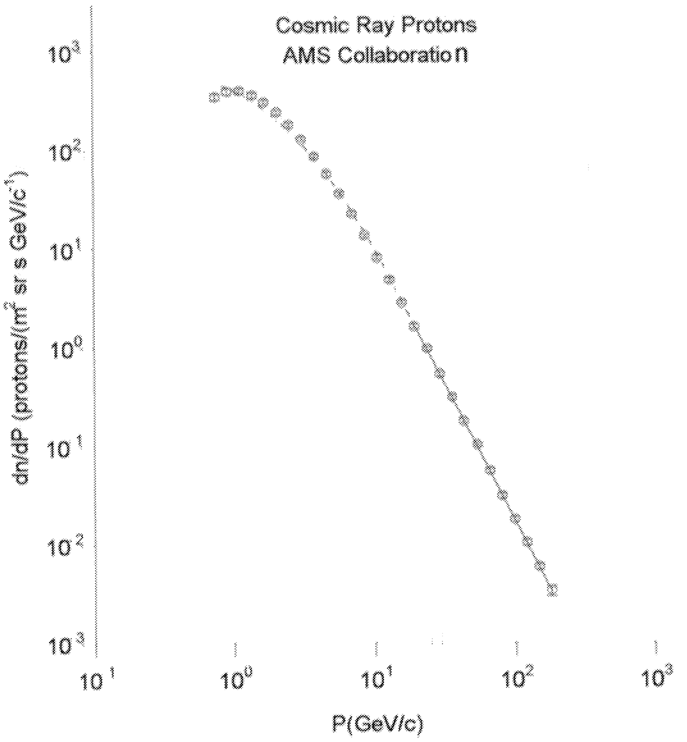

Cosmic Ray Protons
AMS Collaboration
dn/dP (protons/(m^2 sr s GeV/c^{-1}))
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-1}
10^0
10^1
10^2
10^3
P(GeV/c)

Fig 4