CERN - European Laboratory for Particle Physics

CLIC Note 273
26 June 1995

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Interaction Region for CLIC:

Apertures for the Disrupted Beam

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EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

CERN SL-Note 95-57 (AP)
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Abstract

We study the clearance of CLIC beams at 250 GeV passing through the apertures of the last quadrupole doublets after being disrupted by the collision with each other. Two versions of the interaction region geometry had been proposed: co-linear axes of the last doublets, and axes inclined in the horizontal plane so that the incoming beam remains at the quadrupole centers. However, due to the large emittance blow-up in the strong magnetic field for off-axis beams, the first version cannot reach the required small spot size. The computer program IRCLIC has been developed which calculates and plots the trajectories of both beams through the IR. Angular disruption and the increased energy spread of the outgoing particles due to collisions at the interaction point are taken into account. Also the kicks due to parasitic crossings for trains of up to four bunches is included but was found to be small. The minimum apertures for the quadrupoles in the final doublets have been determined.

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1 Introduction

In order to increase the CLIC luminosity it has been envisaged that each of its beams should consist of a train of several bunches. Because of the rather close bunch spacing needed for efficient operation of the linacs, parasitic crossing will then occur inside the free space around the interaction point (IP) and would limit the bunch currents to very low values. This can be avoided by introducing a crossing angle α between the two beams. The further the beams are separated at the position of the parasitic crossings, the weaker become their interaction with the bunches of the opposite beam. However, a large crossing angle decreases the bunch overlap and hence luminosity is substantially reduced when it exceeds the bunch diagonal angle. For horizontal crossing, $\alpha_{diag} = \sigma_x/\sigma_z$, which is about 1.25 mrad for the present 500 GeV c.o.m CLIC parameters[1].

To avoid the loss of luminosity, it has been proposed to rotate the bunches[2] such that they will overlap completely just at collision (" crabbing"). However, in spite of frequent discussion of this idea for linear colliders as well as for B-factories, it has never been tried or proved experimentally. An estimate of the tolerances on the stability of the rf phase in the crabbing cavities for a previous version of somewhat shorter bunches for CLIC seemed to be extremely tight. Besides, there is apprehension [3] for other reasons, such as wake fields created by additional crabbing cavities, and creation of undesired vertical dispersion due to strongly inclined trajectories in the strong solenoidal field desired around the IR for most experiments.

Another possibility is to use a crossing angle so small that the luminosity loss is negligible and that "crabbing" is not required. The crossing angle should be chosen smaller than the diagonal angle. The transverse deflection of the incoming beams due to the close distance at the positions of parasitic crossings has been calculated and its effects were found to be tolerable[4]. In this report, we calculate the required apertures for letting the outgoing beams pass without interception, based on approximate formulae for the beam disruption. Two realistic doublet solutions are shown, with equal or unequal apertures in the two quadrupoles. Based on these considerations, a small crossing angle with no crabbing is preferable for CLIC.

The transverse particle distribution in the bunches is assumed to be Gaussian, with spot sizes determined by the focusing in the last doublet. The aspect ratio of a bunch σ_x/σ_y should be chosen as large as possible in order to increase the luminosity while keeping disruption and beamstrahlung low. For the present CLIC parameters it is over 30. Under such circumstances, crossing in the horizontal plane is preferable. [5] The schematic layout of such a beam crossing with 4 bunches is illustrated in Fig.1.

The interaction of strong, dense bunches at the IP is required for high luminosity, but is accompanied by some unavoidable, and mostly undesirable, effects. The strong electromagnetic (EM) fields of each bunch distort the trajectories and the particles energies in the opposing bunch [6]. In first approximation, these effects can be described as focusing (enhancing the luminosity), disruption (enlarging the beam emittance after the collision), average energy loss, and increase of the energy spread of the outgoing beam. Several others effects, like beamstrahlung and pair production, may be important but have not yet been included completely.

To avoid large backgrounds in the physical apparatus, as well as damage to the collider equipment, the last quadrupoles should be designed in such a way as to ensure free passage of the enlarged outgoing beams. This can be done by making the quadrupole apertures large enough. However, to reach the required focussing strength with a given maximum pole-tip field strength, they also have to become longer.

We present here the results of a study of the disrupted beam clearance through the apertures of the last doublet quadrupoles, taking into account the energy spread in the outgoing beam and the parasitic crossing for the train of four bunches in each beam. Two versions of the Interaction Region (IR) geometries have been considered:

a) the axes of the last doublets coincide with each other, and

b) the axes of the last doublets are crossing at the small angle α in the horizontal plane.

The first scheme has the advantage of requiring smaller apertures for the free passage of the disrupted beams. An example is shown in Fig.2 for quadrupole apertures of 12 mm. However, since the incoming beams pass through the quadrupoles off-center, enhanced synchrotron radiation of the electrons in the stronger magnetic field would increase the achievable spot size at the IP [7]. It turns out that it is not possible to obtain the desired small spot sizes for CLIC under these conditions, and the version has therefore been put aside. The second scheme avoids this limitation but requires somewhat larger apertures. The smaller crossing angles also reduce the effects of a solenoidal field, which therefore was not included in the calculations.

2 Results

The relevant input parameters for CLIC 500 GeV c.o.m. energy are presented in Table 1. Several configurations of the Interaction Region (IR), have been computed with the help of the program FFADA [8] and are presented in Table 2.

The horizontal deflection angle $\Delta\theta_x$ (change of the transverse momentum relative to the longitudinal one) due to the interaction of bunches at a parasitic crossing was evaluated by the approximate formula (see Apendix, Eq. 31):

$$\Delta\theta_x = -\frac{2Nr_e}{\gamma\Delta x} , \qquad (1)$$

where N is the number of electrons per bunch, $r_e \equiv e^2/4\pi\epsilon_0 mc^2$ is the classical radius of electron, γ is its Lorentz factor and Δx is the horizontal distance between bunch centers at the place of a parasitic collision.

Under the assumption of negligible emittance and head-on collision, the maximum disruption angle is given [6] :

$$\Theta_x^{disr} = 0.765 D_x \frac{\sigma_x}{\sigma_z} \, , \tag{2}$$

For the present parameters of CLIC 500 GeV c.o.m., the horizontal disruption parameter is $D_x = 0.286$. The maximum disruption angle then becomes about 270 nm, more than ten times larger than the natural divergence $\sigma_{x'} = \sqrt{\epsilon_x/\beta_x^*}$ of about 25 nm.

The effect of disruption of a bunch by the beam-beam interaction at the IP was included as an emittance increase:

$$\epsilon_x^{disr} = \sqrt{\sigma_x^2\left(\sigma_{x'}^2 + (\Theta_x^{disr})^2\right)} \, . \tag{3}$$

In the absence of a full theory of beam disruption with finite emittance and crossing angles, we have thus included the disruption angle by quadratic addition. This may not be the best estimate, since the disrupted angular distribution is far from Gaussian, with two sharp peaks at the maximum angle. A linear addition of the divergence and disruption angle might be more realistic, but the difference is anyhow rather small.

Fig.3 illustrates the results of the calculations of the particle trajectories through the IR with quadrupole axes inclined by half the crossing angle, such that the incoming beam will remain on the magnetic axis. The outgoing beam then will be deflected more, and therefore the second quadrupole of the doublet needs to have a wider aperture (20 mm) than the first one (12mm). In the figure three trajectories are plotted: one for the center of the bunch, and two which show the maximum excursions.

The effect of the energy loss and spread $\Delta\mathcal{E}$ due to the beam-beam interaction was included approximately by scaling the quadrupole strengths and the kicks by a factor $1-\Delta\mathcal{E}/\mathcal{E}$. For CLIC, an rms energy spread of $\Delta\mathcal{E}/\mathcal{E} = 6.0\%$ was obtained with the program ABEL [9], but only the top 2 % of the energy distribution are actually of interest for most experiments. The average energy loss was found to be about $\delta_E = 3.8\%$ with the same program. The effect of an energy reduction by 6 % on the beam trajectories was found to be hardly visible on the plot and can thus be ignored.

3 Conclusions

We have studied the required quadrupole apertures for a small crossing-angle interaction region (0.52 mrad full crossing angle) in CLIC at 500 GeV c.o.m.

(250 GeV per beam), For hybrid quadrupoles with pole-tip fields of 1.4 T, the final doublet needs to consist of a first, longer (2.7m) quadrupole with 12 mm (full) aperture, and a second, somewhat shorter one (2.1 m) of 20 mm (full) aperture to pass the outgoing beams which were enlarged by disruption at the IP.

Naturally, larger apertures are preferable, and would be possible - in particular if super-conducting quadrupoles with stronger fields are used. A provisional design of a final doublet with SC-LHC quads was made some time ago[10], but has not been completed so far.

Compared to the effect of the widening of the beam by disruption, the parasitic collisions have only a very small effect on the required apertures for the chosen crossing angle of 0.52 mrad and a bunch spacing of 20 cm. Also an energy loss of 6.0% by disruption and beamstrahlung influence the trajectories only very little.

The present results have been derived assuming a Gaussian distribution for the beams, and analytic formulae for the disruption angle which have limited validity. Calculations of the beam trajectories using more realistic distributions need to be done by multi-particle tracking including the beam-beam collision.

Acknowledgements

We profited from the program FFADA by O. Napoly, which was used to design several versions of the CLIC interaction region. One of us (S.K.) is grateful to the CLIC study group for the invitation to work at CERN.

Table 1: **Final Focus System Parameters**

Quantity	*Symbol*	*Value*	*Units*
Energy	$\mathcal{E}$	$2.5 \cdot 10^{11}$	eV
Horiz. Emittance	ϵ_x	$6.0 \cdot 10^{-12}$	m
Vert. Emittance	ϵ_y	$3.0 \cdot 10^{-13}$	m
Horiz. β function at IP	$\beta_x^\star$	$10 \cdot 10^{-3}$	m
Vert. β function at IP	$\beta_y^\star$	$1.8 \cdot 10^{-3}$	m
Bunch length	σ_z	$0.2 \cdot 10^{-3}$	m
Bunch population	N	$0.8 \cdot 10^{10}$	–
Number of Bunches	N_B	4	–
Distance between Bunches	d	0.20	m
Last drift	L_1	1.25	m
Drift between quads	L_2	0.35	m
Full crossing angle	α	$0.52 \cdot 10^{-3}$	rad

Table 2: **Parameters of Last Doublet**

Configuration		Value				Units
		CLIC508	CLIC512	CLIC514	CLIC512/20	
Q_1						
Length	L_d	2.145	2.843	3.154	2.745	m
Strength	K_d	−0.420	−0.280	−0.240	−0.280	m^{-2}
Aperture	D_d	8.00	12.00	14.00	12.00	mm
Q_2						
Length	L_f	1.096	1.381	1.506	2.119	m
Strength	K_f	0.420	0.280	0.240	0.168	m^{-2}
Aperture	D_f	8.00	12.00	14.00	20.00	mm

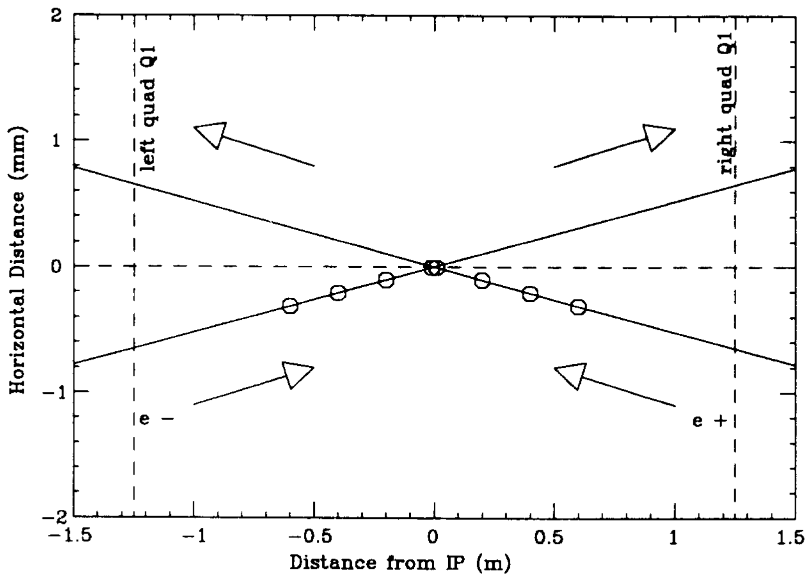

Figure 1: Geometry of Collision

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Appendix: A Kick at a Parasitic Crossing

We want to evaluate the kick which a positron of an incoming bunch experiences when it encounters the outgoing electron bunch at a parasitic crossing. The geometry of the collision has been shown schematically in Fig.1. It is assumed that bunches collide with a total crossing angle α. For the time being we use a coordinate system x, y, z which is rotated with respect to the accelerator reference frame around the vertical axis y by an angle $\alpha/2$. In this system the bunch with the charge $q = -Ne$ moves along the z-axis with velocity $\mathbf{v}$. We chose the initial time so that its center passes through the interaction point (IP) at time $t = 0$.

We begin with a case of a point charge of the electron bunch. The electric $\mathbf{E}$ and magnetic $\mathbf{B}$ fields of a point charge q are described by the following formulae: [11]

$$\mathbf{E}(x,y,z,t) = \frac{q}{4\pi\epsilon_o\gamma^2}\frac{\mathbf{R}(t)}{R_\star^3(t)} , \quad \mathbf{B}(x,y,z) = \frac{1}{c^2}\mathbf{v}\times\mathbf{E} . \tag{4}$$

Here γ is the Lorentz factor, $\mathbf{R}$ is a radius-vector from the charge q at the time t to the observation point (x, y, z).

The quantity $R_\star$ is defined by:

$$R_\star^2 = (z - vt)^2 + \frac{x^2 + y^2}{\gamma^2} . \tag{5}$$

The kick $\mathbf{\Delta}\theta$ which a test particle experienced from the charge q is:

$$\mathbf{\Delta}\theta = \frac{mc^4}{\gamma v_e}\int_{-\infty}^{\infty} dt\mathbf{F}(t) , \tag{6}$$

where $\mathbf{F}$ is the Lorentz force acting on the test particle with the rest mass m and velocity $\mathbf{v}_e$:

$$\mathbf{F}(t) = e(\mathbf{E} + \frac{1}{c^2}\mathbf{v}_e\times\mathbf{v}\times\mathbf{E}) . \tag{7}$$

Using for $\mathbf{E}$ expression Eq. 4 we obtain:

$$\mathbf{\Delta}\theta = A\int_{-\infty}^{\infty}\frac{dt}{R_\star^{3/2}}\Big(\mathbf{R}(1 - \frac{\mathbf{v}_e\cdot\mathbf{v}}{c^2}) + \mathbf{v}\frac{(\mathbf{v}_e\cdot\mathbf{R})}{c^2}\Big) . \tag{8}$$

The constant A is

$$A = -\frac{Nr_e c^2}{\gamma^3 v_e} . \tag{9}$$

Since we are interested in the fields at the position of a test charge e the observation point is actually its position at any time t: $\mathbf{R}_e = (x_e(t), y_e(t), z_e(t))$. Then

$$\mathbf{R} = \mathbf{R}_e - \mathbf{R}_q , \tag{10}$$

where $\mathbf{R}_q = (0, 0, vt)$.

The test particle moves in almost opposite direction along the line which is defined by Euler angles α and ψ. Its velocity v_e is assumed to be also constant. It will eventually go through the IP but at a later time. Their closest parasitic encounter happens at the distance $d/2$ in the accelerator frame, where d is the distance between the two next bunches. Then the collision time is $t_{col} = dv/2\cos(\alpha/2)$ and the position of the test particle at this time is defined by:

$$x_{col} = \frac{d \sin\alpha \cos\psi}{2\cos\alpha/2} , \quad y_{col} = \frac{d\sin\psi}{2\cos\alpha/2} , \quad z_{col} = \frac{d\cos\alpha\cos\psi}{2\cos\alpha/2} . \tag{11}$$

Under these conditions the particle trajectory is described by:

$$z_e(t) = a + bt , \quad x_e(t) = f + gt , \quad y_e(t) = h + kt , \tag{12}$$

where

$$a = \frac{d\cos\alpha\cos\psi}{2\cos(\alpha/2)}(1 + \frac{v_e}{v}) , \quad b = -v_e \cos\alpha\cos\psi , \tag{13}$$

$$f = \frac{d\sin\alpha\cos\psi}{2\cos(\alpha/2)}(1 + \frac{v_e}{v}) , \quad g = -v_e \sin\alpha\cos\psi , \tag{14}$$

$$h = \frac{d\sin\psi}{2\cos(\alpha/2)}(1 + \frac{v_e}{v}) , \quad k = -v_e \sin\psi . \tag{15}$$

We need also to calculate two vector products:

$$\mathbf{v}_e \cdot \mathbf{R} = -v_e[(a + bt - vt)\cos\alpha\cos\psi + (f + dt)\sin\alpha\cos\psi + (h + kt)\sin\psi] , \tag{16}$$

and

$$\mathbf{v}_e \cdot \mathbf{v} = -vv_e \cos\alpha\cos\psi . \tag{17}$$

Combining all the previous expressions, three projections of the kick are expressed by the same function of two arguments:

$$\Delta\theta_{x,y,z} = AI(p, q) , \tag{18}$$

where the function I is

$$I(p, q) = \int_{-\infty}^{\infty} \frac{dt(p + qt)}{R_*^{3/2}} . \tag{19}$$

In terms of this function

$$\Delta\theta_z = AI(\alpha, \beta) , \tag{20}$$

with

$$\alpha = a - \frac{vv_e}{c^2} f \sin\alpha\cos\psi , \quad \beta = b - v - \frac{vv_e}{c^2} g \sin\alpha\cos\psi , \tag{21}$$

$$\Delta\theta_x = AI(\sigma,\tau) \ , \tag{22}$$

with

$$\sigma = f(1+\frac{vv_e}{c^2}\cos\alpha\cos\psi) \ , \quad \tau = g(1+\frac{vv_e}{c^2}\cos\alpha\cos\psi) \ , \tag{23}$$

$$\Delta\theta_y = AI(\xi,\chi) \ , \tag{24}$$

with

$$\xi = h(1+\frac{vv_e}{c^2}\cos\alpha\cos\psi) \ , \quad \chi = k(1+\frac{vv_e}{c^2}\cos\alpha\cos\psi) \ , \tag{25}$$

The function $I(p,q)$ can be expressed in terms of tabulated integrals. After some algebra the final result is:

$$I(p,q) = \frac{2(pP-qQ)}{(SP-Q^2)\sqrt{P}} \ , \tag{26}$$

where

$$P = (b-v)^2 + \frac{g^2+k^2}{\gamma^2} \ , \quad Q = a(b-v) + \frac{fg+hk}{\gamma^2} \ , \quad S = a^2 + \frac{f^2+g^2}{\gamma^2} \ . \tag{27}$$

It is easy to verify that

$$SP - Q^2 = \frac{[(ag-f(b-v)]^2 + [(ak-h(b-v)]^2}{\gamma^2} \ , \tag{28}$$

so I is proportional to γ^2.

Now we can come back to the accelerator reference frame. The y kick does not change by the rotation. In terms of the calculated quantities the other two components in the accelerator reference frame are:

$$\Delta\theta_{z,acc} = \Delta\theta_z\cos(\alpha/2) + \Delta\theta_x\sin(\alpha/2) \ , \tag{29}$$

$$\Delta\theta_{x,acc} = -\Delta\theta_z\sin(\alpha/2) + \Delta\theta_x\cos(\alpha/2) \ . \tag{30}$$

Combining the appropriate expressions it is straightforward to write the formulae for the kicks. The result is rather cumbersome for the general case and there is little sense to write it down. But for an ultra-relativistic case $v \approx c, v_e \approx c$ terms of the order γ^{-2} can be neglected. When in addition the angles α and ψ are small, $\cos\alpha \approx 1, \cos\psi \approx 1, \sin\alpha \approx \alpha, \sin\psi \approx \psi$, the expressions become simple:

$$\Delta\theta_{x,acc} = -\frac{2Nr_e}{\gamma\Delta x(1+\psi^2/\alpha^2)} \ , \quad \Delta\theta_{y,acc} = -\frac{2Nr_e(\psi/\alpha)}{\gamma\Delta x(1+\psi^2/\alpha^2)} \ , \tag{31}$$

In these formulae we have introduced the distance between the bunches in horizontal plane at the parasitic collision point $\Delta x = d\alpha/2$. The minus sign indicates that the bunches are attracted to each other. When the crossing is purely horizontal, $\psi = 0$ and we get the equation for the kick used in the text.

There is a small asymmetry of the collision with respect to the sign of z. That explains a small residual kick in the z direction:

$$\Delta\theta_{z,acc} = \frac{4Nr_e}{d\gamma} \; . \tag{32}$$

The calculation of the electric and magnetic parts of the Lorentz force showed that each of them contributes half of the full force in all three components. That concludes the calculation for the point electron bunch.

Would the effective sizes of the bunches be all much smaller then the distance between them at the parasitic collision point, no further calculations were required. In CLIC with small angle crossing that is not the case, at least for the longitudinal size of the bunch. For example, the bunch length is $\sigma_z \approx 200\mu$m, the distance between bunches $d = 0.2$ m, and when $\alpha/2 \approx 0.5 \cdot 10^{-3}$, $\Delta x \approx 50\mu$m.

In this situation the kicks can be evaluated from the results obtained by averaging over a normalized longitudinal bunch distribution $\rho(s)$. Eq.27 is still valid when the parameter a is replaced by $a - s$. The kicks then are obtained by integrating over s:

$$\Delta\theta = A \int ds\rho(s) I(p,q) \; . \tag{33}$$

Assuming, e.g., a waterbag model for the longitudinal distribution:

$$\rho(s) = \frac{2\sqrt{\sigma_z^2 - s^2}}{\pi\sigma_z} \tag{34}$$

we find for small angles in the ultra-relativistic approximation:

$$\Delta\theta_x = -\frac{2A\alpha\gamma^2}{c(\alpha^2+\psi^2)} J(d) \; , \Delta\theta_y = -\frac{2A\psi\gamma^2}{c(\alpha^2+\psi^2)} J(d) \; , \tag{35}$$

where

$$J(d) = \frac{2}{\pi\sigma_z} \int_{-1}^{1} dx \frac{\sqrt{1-x^2}}{x + d/\sigma_z} . \tag{36}$$

For $d \gg \sigma_z \quad J \approx 1/d$ and the kicks are given by the same formulae Eqs.31 and 32.

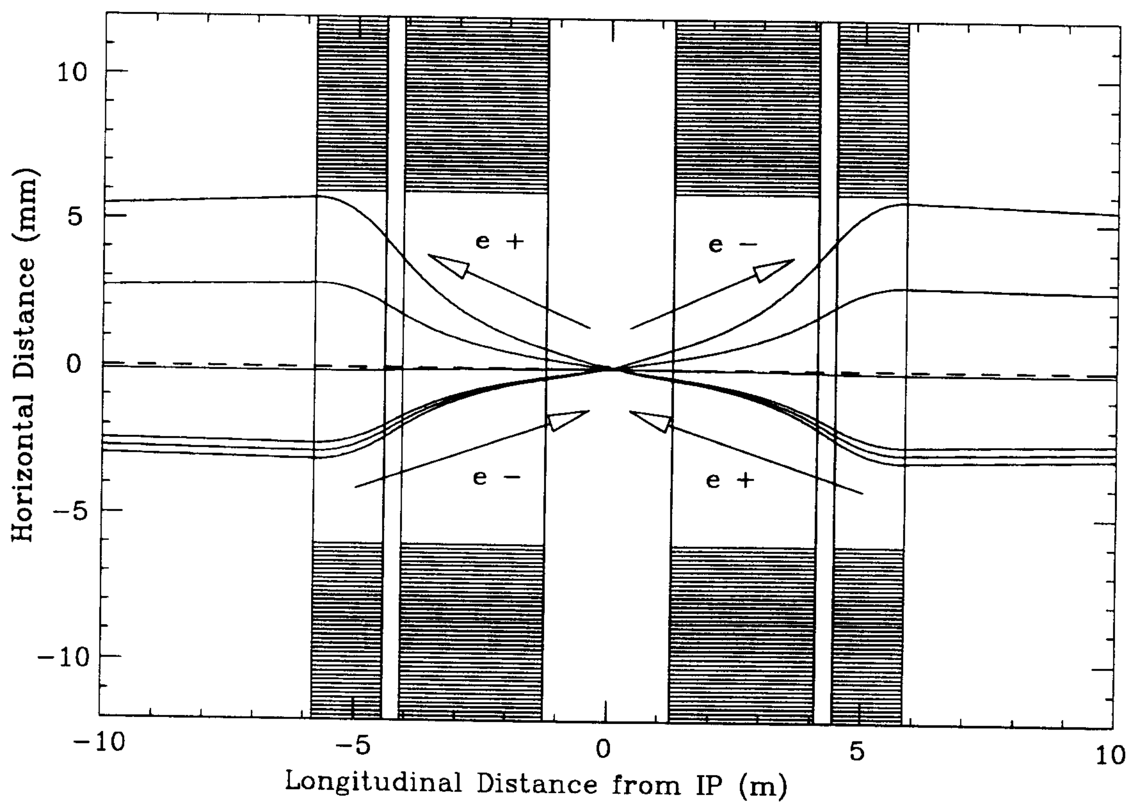

Figure 2: Trajectories for FFS CLIC 512 (12 mm aperture coaxial quads) with full crossing angle $\alpha_x = -.52$ mrad, $dE/E_o = -6\%$, Disruption and parasitic kicks ON

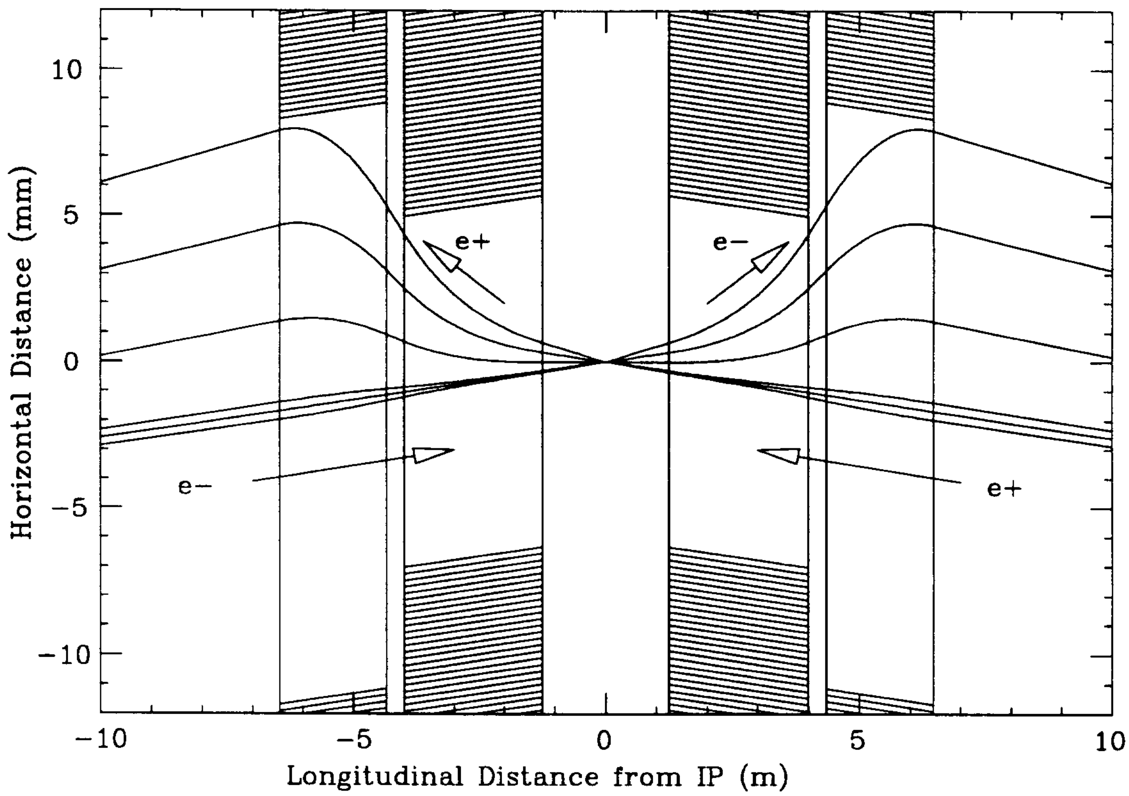

Figure 3: Trajectories for FFS CLIC 512/20 (12 and 20 mm aperture quadrupoles parallel to incoming beam) with full crossing angle $\alpha_x = -.52$ mrad, $dE/E_o = -6\%$, Disruption and parasitic kicks ON