# Phenomenology of Doubly Special Relativity


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## ABSTRACT

Investigations of the possibility that some novel "quantum" properties of spacetime might induce a Planck-scale modification of the energy/momentum dispersion relation focused at first on scenarios with Planck-scale violations of Lorentz symmetry, with an associated reduced $n$-parameter ($n < 6$) rotation-boost symmetry group. More recently several studies have considered the possibility of a "doubly special relativity", in which the modification of the dispersion relation emerges from a framework with both the Planck scale and the speed-of-light scale as characteristic scales of a 6-parameter group of rotation-boost symmetry transformations (a deformation of the Lorentz transformations). For the schemes with broken Lorentz symmetry at the Planck scale there is a large literature on the derivation of experimental limits. We provide here a corresponding analysis for the doubly-special-relativity framework. We find that the analyses of photon stability, synchrotron radiation, and threshold conditions for particle production in collision processes, the three contexts which are considered as most promising for constraining the broken-Lorentz-symmetry scenario, cannot provide significant constraints on doubly-special-relativity parameter space. However, certain types of analyses of gamma-ray bursts are sensitive to the symmetry deformation. A key element of our study is an observation that removes a possible sign ambiguity for the doubly-special-relativity framework. This result also allows us to characterize more sharply the differences between the doubly-special-relativity framework and the framework of $\kappa$-Poincaré Hopf algebras, two frameworks which are often confused with each other in the literature.

# 1 Introduction

Because of its central role in modern physics, Lorentz symmetry has been investigated in great detail. The analysis of test theories that could be used as a measure of our level of experimental verification of Lorentz symmetry, and could describe violations of Lorentz symmetry, was already rather mature in the 1940s [1]. The level of interest in this subject has grown gradually over the last three decades (see, *e.g.*, Refs. [2, 3, 4, 5, 6]), and in particular in these past four or five years a large number of studies (see, *e.g.*, Refs. [7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19] and references therein) has explored the possibility that Lorentz symmetry might be broken by Planck-scale physics ("quantum gravity").

It is plausible that the unification of general relativity and quantum mechanics could involve some sort of spacetime quantization, such as spacetime noncommutativity or spacetime discreteness, and this can indeed justify some interest in the fate of Lorentz symmetry at the Planck scale. If spacetime is fundamentally discrete or noncommutative then this should in particular apply to the spacetimes that low-energy probes perceive as Minkowski (flat classical and continuous), and the recent quantum-gravity literature provides support for the expectation that, in the flat-spacetime limit, spacetime discreteness [8, 12, 20] and spacetime noncommutativity [21, 22, 23, 24] are likely to require some departures from Lorentz symmetry.

At first interest in Planck-scale departures from Lorentz symmetry focused [7, 8, 9, 10, 11, 12, 13, 14, 16, 15] on some mechanisms for breaking Lorentz symmetry through quantum-gravity effects. These scenarios are characterized by "symmetry loss": only a (possibly empty) subset of the 6-parameter family of transformations that constitute the Lorentz group would survive as symmetries in the Planck regime. As proposed in Ref. [25], and verified in a variety of schemes in Refs. [25, 26, 27, 28], it is also possible that Planck scale effects would induce departures from ordinary Lorentz symmetry which do not however involve a loss of symmetry. In this so-called "doubly special relativity" ("DSR") scenario one contemplates the possibility that by gaining access to data involving particles of higher energies we might discover that in Nature the laws of transformation between inertial observers are characterized by an invariant Planckian energy/momentum/length scale, just like nearly a century ago, as data on higher velocities became available it was established that the speed-of-light scale is a characteristic of the laws of transformation between inertial observers. In Galilei-Newton relativity there is a 6-parameter family of rotation/boost symmetry transformations between inertial observers, with laws of transformation that are scale independent (in the sense that they do not involve any characteristic scale). In special relativity one still has a 6-parameter family of rotation/boost (Lorentz) symmetry transformations, but the laws of transformation are characterized by an invariant velocity scale, the speed-of-light scale. In doubly-special relativity once again one has a 6-parameter family of rotation/boost (Lorentz) symmetry transformations, but the laws of transformation are characterized by two invariant scales, the speed-of-light scale and the Planck scale. The doubly-special-relativity rotation/boost transformations are a Planck-scale "deformation" of the special-relativity transformations, which in turn are a $c$-scale "deformation" of the Galilei transformations. In the low-energy limit a DSR scheme turns into special relativity, just like in the low-velocity limit special relativity turns into Galilei-Newton relativity.

We focus here on this possibility that at the Planck scale Lorentz symmetry be deformed (rather than broken). Specifically we focus on certain types of observations which could allow to test some of the predictions of the most popular DSR schemes. For the case of Planck-scale-broken Lorentz symmetry there is already a rather wide literature on experimental tests [7, 8, 9, 10, 11, 13, 14, 15, 29, 30, 31, 32]. For the DSR case, with Planck-scale-deformed Lorentz symmetry, a corresponding analysis is still missing, and we intend to fill this gap here.

In the next section we start by introducing the most studied model with Planck-scale-broken Lorentz symmetry and the most studied model with Planck-scale-deformed Lorentz symmetry, and we show that these two models can be very naturally compared since they predict the same leading-order modification of the dispersion relation[a]

$$0 \simeq E^2 - \vec{p}^2 - m^2 - \eta \frac{E}{E_p} \vec{p}^2 \ , \tag{1}$$

[a]While sometimes, especially when commenting on the logical structure of the DSR framework, it is convenient for us to indicate explicitly the speed-of-light scale $c$, in most equations we adopt conventions such that $c = \hbar = 1$.

where $\eta$ is a dimensionless coefficient, $E_p$ is the Planck energy scale ($E_p \simeq 10^{28} eV$), and we are considering the possibility that such a dispersion relation would describe the low-energy limit $E \ll E_p$ (we only included a term linear in $E_p^{-1}$).

A key objective of the type of phenomenology that is here of interest is the one of setting limits on the parameter $\eta$, which cannot be much smaller than 1 if the conventional quantum-gravity intuition is to be realized (for $\eta \ll 1$ the scale the characterizes the onset of the new effects, $E_p/\eta$, would be much bigger than the Planck scale). In Section 3 we observe that even without resorting to data one can investigate from a theory perspective the issue of the "sign of $\eta$". In particular, we argue that in the scenario with Lorentz symmetry broken at the Planck scale it is natural to expect $\eta < 0$, although one cannot completely rule out the case of positive $\eta$. We also show that in the DSR scenario, with deformed Lorentz symmetry, $\eta$ must be positive ($\eta > 0$) in order to have a genuine 6-parameter symmetry group of rotation/boost transformations, as required by the DSR principles. The "sign of $\eta$" had been considered [25, 27] as a key ambiguity for the DSR framework, and we show that this potential ambiguity can be completely removed. This proves to be a key asset for the phenomenological analysis that follows, and also allows us to characterize more sharply the differences between the doubly-special-relativity framework and the framework of $\kappa$-Poincaré Hopf algebras, two frameworks which are often confused with each other in the literature.

In Section 4 we briefly review the analysis of photon stability in the scenario with Lorentz symmetry broken at the Planck scale, and we provide a corresponding analysis for the DSR scenario. Although the two scenarios we consider adopt the same leading-order modification of the dispersion relation, in the broken-symmetry scenario, for $\eta > 0$, one finds that high-energy photons can decay into an electron positron pair, whereas in the DSR scenario the process $\gamma \to e^+e^-$ is forbidden. Some observations in astrophysics allow to establish that the photon is stable enough to exclude the possibility $\eta > 0$ for the case with broken Lorentz symmetry (therefore in the broken-symmetry case $\eta > 0$, which was already disfavored conceptually on the basis of the points raised in Section 3, is also ruled out by data).

In Section 5 we consider synchrotron radiation. We review a preliminary analysis [30] which has been used to argue that, for $\eta < 0$, a scheme in which Lorentz symmetry is broken at the Planck scale should affect significantly the analysis of certain astrophysical contexts involving synchrotron radiation. On the basis of our result of Section 3, showing that $\eta < 0$ is not admissible for the DSR scenario, we find that instead synchrotron radiation is not significantly affected in DSR.

In Section 6 we briefly review the analysis of "threshold anomalies" in the case of broken Lorentz symmetry, and we also analyze (on the basis of the corresponding results of Ref. [25]) the possibility of threshold anomalies in DSR. One speaks of a threshold anomaly [15, 33] when the Planck-scale effects induce a significant modification of the threshold conditions for particle production in collision processes. In particular, there has been strong interest [11, 13, 15, 34, 35] in the possibility that Planck-scale effects might modify the estimate of the GZK [36] threshold, *i.e.* the estimate of the minimum value of energy needed for a cosmic-ray proton to interact with a CMBR photon, leading to photopion production. In the case in which Lorentz symmetry is broken at the Planck scale one indeed finds [11, 13, 15, 34] that, for $\eta < 0$, the GZK threshold is significantly modified, and forthcoming cosmic-ray observatories could test this effect. Also in the corresponding DSR analysis there is a modification of the threshold condition, but it is extremely small, negligible even for ultra-high-energy cosmic rays.

In Section 7 we consider certain types of time-of-travel analyses for astrophysical signals with rich time-versus-energy structure. In presence of a deformation of the dispersion relation one expects a (small, Planck-scale suppressed) energy-dependence of the speed of photons. In the case of a burst of photons of different energies all emitted at the same (within a certain accuracy) time from a point far away from a Earth observatory, one then expects a correlation between time of arrival on Earth and energy. The analysis in this context proceeds basically in the same way independently of whether Lorentz symmetry is broken or deformed. However, in Sections 3 and 4 we established that the broken-symmetry scenario requires negative $\eta$ whereas the DSR scenario requires positive $\eta$. This "sign difference" opens the way for studies able to discriminate between the broken-symmetry and the DSR scenarios using the next generation of gamma-ray observatories (such as the GLAST space telescope, which is already planning [37] this type of time-of-arrival studies motivated by Planck-scale physics).

In the last section (Section 8) we summarize our key results and comment on the outlook of this research programme.

# 2 Broken versus deformed Lorentz symmetry

## 2.1 A scenario for Planck-scale-broken Lorentz symmetry

There is growing interest [7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19] in the possibility that some novel quantum properties of spacetime may have important implications for the analysis of Lorentz transformations. Some approaches to the quantum-gravity problem attribute to the Planck scale $E_p$ the status of an intrinsic characteristic of space-time structure. For example $E_p$ can have a role in spacetime discretization or in the commutation relations between spacetime observables. In discretized versions or non-commutative versions of Minkowski space-time it is not uncommon to find departures from ordinary Lorentz symmetry (see, *e.g.*, the pedagogical discussions in Ref.[20] for the case of discretization and in Refs.[21, 22, 23, 38] for the case of non-commutativity). The action of ordinary (classical) boosts on discretization length scales (or non-commutativity length scales) can be such that different inertial observers would attribute different values to these lengths scales, just as one would expect from the mechanism of FitzGerald-Lorentz contraction.

Models based on an approximate Lorentz symmetry, with Planck-scale-dependent departures from exact Lorentz symmetry, have been recently considered in most quantum-gravity research lines, including models based on "spacetime foam" pictures [7, 39], "loop quantum gravity" models [8], certain "string theory" scenarios [23, 40], and "noncommutative geometry [23, 25, 26].

The most studied model of Planck-scale departures from Lorentz symmetry is the one which evolved primarily through the studies reported in Refs. [7, 11, 13, 15]. This is a kinematic in which the Planck-scale $E_p$ enters the energy/momentum dispersion relation as in (1), which we report here for convenience

$$0 = f(E, \vec{p}^2, m; E_p) \simeq E^2 - \vec{p}^2 - m^2 - \eta \frac{E}{E_p} \vec{p}^2 \ , \tag{2}$$

while the laws of energy-momentum conservation remain unaffected by the Planck scale, and therefore for example in a process $a + b \to c + d$

$$E_a + E_b = E_c + E_d$$

$$\vec{p}_a + \vec{p}_b = \vec{p}_c + \vec{p}_d \ . \tag{3}$$

In different quantum-gravity models with Planck-scale departures from Lorentz symmetry the exact dispersion relation (the exact form of the function $f(E, \vec{p}^2, m; E_p)$) will in general be different, but it has been argued that for a variety of quantum-gravity models one should find that for $E \ll E_p$ (small energies) $f \simeq E^2 - \vec{p}^2 - m^2 - \eta E \vec{p}^2 / E_p$, as in (2). The fact that in this popular scenario one combines the modified dispersion relation (2) with the unomdified law of energy-momentum conservation (3) is an explicit indication of the assumed breaking of Lorentz symmetry. There is in fact no 6-parameter family of rotation/boost-like symmetry transformation which are compatible with both requirements (2) and (3). (We elaborate more on this point in the next subsection).

The fact that the literature has focused primarily on this scenario is mostly due to its simplicity, which makes it a natural first step in a phenomenology of Planck-scale departures from Lorentz symmetry. However, this scenario is also more or less directly connected with various quantum-gravity proposals. In Loop Quantum Gravity preliminary results [8, 41] provide support for the dispersion relation (2). Proposals based on noncommutative spacetimes inevitably lead to departures from ordinary Lorentz symmetry and there are various examples [22], in which one finds (1). In String Theory it appears that the modification of the dispersion relation is not automatic but emerges in presence of certain natural background fields [23, 40], and for some background configurations a dispersion relation of the type (1) is encountered [40]. The role that phenomenological analyses of this scenario (1), (3) could have in the overall development of quantum-gravity research has been stressed in most recent reviews by experts of the field (see, *e.g.*, Refs. [42, 43, 44]).

At an intuive level this scenario can be seen in analogy with the emergence of deformed dispersion relations in the description of collective modes (such as phonons) in certain materials, whose propagation satisfies a relativistic dispersion relation only up to corrections governed by the scale of atomic structure of the material[b]. Intuitively one can indeed attempt to introduce in quantum

[b]Similar considerations apply to the decription of the propagation of light in water.

gravity the concept of "spacetime foam", and spacetime foam might affect particle propagation in a way that to some extent can be viewed in analogy with the way that the presence of other media affects particle propagation.

## 2.2 A corresponding doubly-special-relativity scenario

The fact that the quantum-gravity scenario discussed in the preceding subsection "breaks" Lorentz symmetry, *i.e.* requires a preferred class of inertial observers, is not necessarily a disappointment. As mentioned, certain quantum-gravity ideas, notably some perspectives on the spacetime-foam picture, can provide motivation for exploring this possibility. Moreover, in the study of particle physics the concept of spontaneous breaking of a symmetry has proven very useful, and it is conceivable that quantum gravity would host a similar mechanism for the spontaneous breaking of Lorentz symmetry at the Planck scale. For example, in string theory, the most popular approach to the quantum-gravity problem, mechanisms for the spontaneous breaking of Lorentz symmetry have been investigated (see, *e.g.*, Ref. [5, 24] and references therein).

However, the existence of a preferred class of inertial observers is anyway not simple conceptually. At present we can only make wild guesses about this preferred class of observers. Many studies attempt to identify this preferred class of inertial observers with a natural class of observers for the CMBR, but this conjecture (although it cannot be excluded a priori) appears to lack any justification, since it would be rather surprising to find a connection between the physics responsible for the CMBR and the Planck-scale realm.

Even if one is not troubled by the possibility of a preferred class of inertial observers, it is natural to wonder [25] whether there are other alternatives in addition to the cases in which Lorentz symmetry is exactly preserved at the Planck scale and the case in which Lorentz symmetry is broken, with associated emergence of a preferred class of inertial observers. The so-called "doubly-special relativity" ("DSR") theories were proposed [25] as a scenario in which **(i)** the Planck energy $E_p$ takes the role of relativistic invariant, in the same sense that the speed-of-light scale "$c$" is a relativistic invariant, and **(ii)** ordinary Lorentz symmetry is not exactly preserved at the Planck scale, but there is no special class of inertial observers. As mentioned, this DSR proposal can be described, using a terminology which is popular in the mathematics community, as a "deformation" of Lorentz symmetry, without any actual "loss of symmetry". This means that the Lorentz laws of transformation between inertial observers are replaced by different laws of transformation between inertial observers (in the DSR case this involves the introduction of a second observer-independent scale $E_p$) but without any loss of equivalence between inertial observers: all inertial observers remain equivalent (there is no preferred class of inertial observers), but the laws of transformation between inertial observers are modified with respect to the original Lorentz transformation laws.

It is emerging that the DSR framework is needed in order to understand some aspects of certain noncommutative spacetimes [25, 26], and possibly also of the "loop quantum gravity" approach [45]. Here, since we are focusing on phenomenological issues, we prefer to stress that the DSR hypothesis can be viewed as a logical continuation of the path that already connects Galilei-Newton relativity and special relativity. In Galilei-Newton relativity there is no observer-independent scale characteristic of the rotation/boost transformations, and in fact the dispersion relation is written as $E = p^2/(2m)$ (whose structure fulfills the requirements of dimensional analysis without the need for dimensionful coefficients). As experimental evidence in favour of Maxwell equations started to grow, the fact that those equations involve a special velocity scale appeared to require (assuming the Galilei symmetry group should remain unaffected) the introduction of a preferred class of inertial observers (the "ether"). However, in the end we discovered that the Maxwell theory does not require a preferred class of inertial observers, but rather it was wrong to assume that Galileian relativity should apply in all regimes. In the high-velocity regime it must be replaced by special relativity. Special relativity introduces the first observer-independent scale, the velocity scale $c$, and its dispersion relation takes the form $E^2 = c^2p^2 + c^4m^2$. As interest in dispersion relations of the type $0 = E^2 - c^2\vec{p}^2 - c^4m^2 + f(E, \vec{p}^2, m; c, E_p)$ is starting to grow within the quantum-gravity community (also because of the analysis of noncommutative spacetimes and loop quantum gravity) the fact that these dispersion relations involve a special energy scale, $E_p$, was leading to the assumption that a preferred class of inertial observers might have to be introduced. The DSR proposal essentially raises the possibility that the assumption of a preferred class of inertial observers might once again be incorrect, and once again we might need to deform the laws of transformation between inertial observers (as already done in going from Galilei-Newton relativity to special relativity). A dispersion relation of the type $0 = E^2 - c^2\vec{p}^2 - c^4m^2 + f(E, \vec{p}^2; E_p)$ can in fact hold for all inertial observers [25, 26].

In particular, the dispersion relation[c]

$$0 = \frac{2}{\lambda^2}\left[\cosh(\lambda E) - \cosh(\lambda m)\right] - \vec{p}^2 e^{\lambda E} \simeq E^2 - \vec{p}^2 - m^2 - \lambda E \vec{p}^2 \tag{4}$$

can be valid in all inertial frames, at the cost of a $\lambda$-dependent deformation of the boost transformations [25, 26]. And one easily notices that for $\lambda \equiv \eta/E_p$ the dispersion relation (4) is consistent with the dispersion relation (1) considered in the popular broken-Lorentz-symmetry scenario discussed in the previous subsection.

The fact that the same dispersion relation can be considered both in a broken-Lorentz-symmetry scenario and in a deformed-Lorentz-symmetry scenario raises an interesting challenge from a phenomenological perspective: how can we distinguish experimentally between the two scenarios? We will show that, although (as established previously [7, 8, 9, 10, 17, 18]) the modification of the dispersion relation is the most important ingredient of the strategies for testing Planck-scale departures from ordinary Lorentz symmetry, it is possible to distinguish experimentally between a DSR scheme and a broken-Lorentz-symmetry scheme which adopt the same Planck-scale modification of the dispersion relation. Of course, one must exploit the fact other aspects of kinematics (in addition to the dispersion relation) must also be Planck-scale modified in a DSR scheme, for consistency with the request of equivalence among inertial frames (whereas the same is not true for the broken-Lorentz-symmetry scenario).

We will stress that, in particular, the unmodified law of energy-momentum conservation, which is assumed in the broken-Lorentz-symmetry scenario of the previous subsection, is not compatible with the DSR requirements. An easy way to show this incompatibility is based on the form of the dependence of energy-momentum on the rapidity parameter $\xi$ (the coefficient of a boost generator $N$ in the exponentiation $e^{\xi N}$ that implements a finite boost transformation). For a DSR scenario with dispersion relation (4) one finds [25, 26] that the rapidity/energy-momentum relation is such that

$$\cosh(\xi) = \frac{e^{\lambda E} - \cosh(\lambda m)}{\sinh(\lambda m)}\ , \quad \sinh(\xi) = \frac{\lambda p e^{\lambda E}}{\sinh(\lambda m)}\ , \tag{5}$$

where $\xi$ here is the amount of rapidity needed to take a particle from its rest frame ($E = m$, $p = 0$) to a frame in which its energy is $E$ (and its momentum is $p(E)$, which is fixed in terms of $E$ by the dispersion relation and the direction of the boost). Of course, in the $\lambda \to 0$ limit these relations reproduce the corresponding special relativistic relations

$$\cosh(\xi) = \frac{E}{m}\ , \quad \sinh(\xi) = \frac{p}{m}\ . \tag{6}$$

And, as needed for the DSR requirement, one can easily verify that the $E(\xi)$ and $p(\xi)$ implicitly defined by (5) satisfy the dispersion relation (4) for every value of $\xi$ (the $\lambda$-modified dispersion relation (4) holds in every frame connected by the relevant boost). However, one can also easily verify that it is not possible to have $E_a(\xi) + E_b(\xi) = E_c(\xi) + E_d(\xi)$, $\vec{p}_a(\xi) + \vec{p}_b(\xi) = \vec{p}_c(\xi) + \vec{p}_d(\xi)$ (in a process $a + b \to c + d$) for every $\xi$. If one was to enforce the unmodified law of energy-momentum conservation in a framework with boost transformations of type (5) then this unmodified law of energy-momentum conservation could only hold in one inertial frame (it would be violated in other frames reacheable by boosting).

This incompatibility (within a DSR framework) between modified dispersion relation and unmodified energy-momentum conservation was already noticed in the first papers on DSR [25], leading to the realization that in order to have the equivalence of all inertial frames the presence of a modification of the dispersion relation required a corresponding modification of the law of energy-momentum conservation. For the specific DSR scheme we are considering (with dispersion relation (4) and boost

[c]This type of dispersion relation was considered in some of the first papers on the DSR proposal [25, 26], and remains the focus of much of DSR research. As mentioned, the leading-order form of this dispersion relation had been previously considered for the broken-Lorentz-symmetry scenario of Ref. [7], and the full exact form of this dispersion relation had been considered even earlier in the Hopf-algebra literature [21, 22].

transformations codified in (5)) this needed modification[d] of the law of energy-momentum conservation has been worked out [25, 26]. The exact form of this law is a rather messy combination of exponentials, but fortunately in the following sections we will only the need the leading-$\lambda$-order form of the law of energy-momentum conservation, which is [25, 26]

$$E_a + E_b - \lambda p_a p_b - E_c - E_d + \lambda p_c p_d = 0 \ , \tag{7}$$

$$p_a + p_b - \lambda(E_a p_b + E_b p_a) - p_c - p_d + \lambda(E_c p_d + E_d p_c) = 0 \ . \tag{8}$$

This law of energy-momentum conservation is indeed compatible with the DSR boost transformations; in fact, using (5) one can easily verify that (to leading order in $\lambda$) if energy-momentum is conserved in one inertial frame then it is automatically conserved also in all other inertial frames.

This observation already provides us a valuable key for phenomenological analyses. In the popular broken-Lorentz-symmetry scenario of the previous subsection one has the modified dispersion relation (4) with unmodified law of energy-momentum conservation. A DSR scheme can adopt the same modified dispersion relation (4) but then for consistency with the required equivalence of inertial frames it must also impose a modified law of energy-momentum conservation (7)-(8).

In the phenomenology discussed in the following Section 4, 5, 6 and 7 this previous result on the law of energy-momentum conservation in DSR will play a key role. In addition we will also use a new result which we obtain in the next Section 3: we will show that in the DSR framework one must necessarily assume $\lambda E_p > 0$ (*i.e.* the dimensionful parameter $\lambda$ that appears in (4) and (5) must necessarily be positive). This result proves very useful since the possibility that, in the broken-Lorentz-symmetry scenario of the previous subsection the parameter $\eta$ be positive (and we remind the reader that $\eta \sim \lambda E_p$) is already excluded experimentally (see Section 4).

## 2.3 Other schemes for Planck-scale departures from Lorentz symmetry

There are clearly three possibilities for the fate of Lorentz symmetry at the Planck scale: **(I)** Lorentz symmetry, with its dispersion relation and its other key features, remains unmodified even at the Planck scale **(II)** Lorentz symmetry is broken by Planck-scale effects, with associated emergence of a preferred class of inertial observers, and **(III)** Lorentz symmetry is deformed, in the DSR sense, preserving the equivalence of inertial observers.

The scenario discussed in Subsection 2.1 is the most studied broken-Lorentz-symmetry scenario, while the scenario discussed in Subsection 2.2 is the most studied DSR (deformed-Lorentz-symmetry) scenario. But there are other ways to break Lorentz symmetry at the Planck scale which have been considered in the literature, and there are other DSR scenarios which have been considered in the literature. In particular, both in work on broken Lorentz symmetry and in work on DSR, there has been some interest in the possibility that the dispersion relation be modified more softly, *i.e.* that the term linear in $\lambda$ ($\eta/E_p$) might be absent, leading to a dispersion relation which at low energies takes the form

$$0 \simeq E^2 - \vec{p}^2 - m^2 + \eta_2 \frac{E^2}{E_p^2} \vec{p}^2 \ . \tag{9}$$

[d]Since we stressed the analogy between the transition Galilei relativity → special relativity and the conjectured transition special relativity → doubly special relativity, it is perhaps worth mentioning that this DSR modification of energy-momentum conservation must be viewed in analogy with the special-relativistic modification of the law of composition of velocities. In Galilei relativity there is no special velocity scale, and inevitably the law of composition of velocities must take the form $V_0 + V$ (where, in particular, $V$ could be the velocity of a particle in a given inertial frame $O$ and $V_0 + V$ could be the velocity of that same particle in another inertial frame $O'$, if $V_0$ is the relative velocity for the frame $O$ and $O'$). In special relativity there is a special velocity scale $c$ and the law of composition of velocities becomes nonlinear and $c$-dependent (it must saturate at the maximum velocity $c$). In turn in special relativity there is no special energy-momentum scale, and therefore the law of composition of energy-momentum (in particular in obtaining the total momentum of a two-particle system) must be linear, $P_\mu + P'_\mu$. But then in DSR there is special energy-momentum scale, $1/\lambda$, and the the law of composition of energy-momentum becomes nonlinear and $\lambda$-dependent.

Although we are choosing to focus on the most studied scenarios, the ones of Subsections 2.1 and 2.2, in the following Sections we shall sometimes briefly comment of the possibility (9).

Especially in the DSR framework there is a line of analysis that can lead to considering [46] a scenario with a very mild modification of the dispersion relation at energies low enough that $E \leq (m^j E_p^{n-j})^{1/n}$ (where $n$,$j$ are some integers, while $m$ and $E_p$ denote again the mass of the particle and the Planck scale respectively) but a rather significant modification of the dispersion relation for energies such that $(m^j E_p^{n-j})^{1/n} < E < E_p$. In this case also the relation between rapidity and energy (in the sense of (5)) would take a form such that it appears to be nearly exactly special relativistic for $E \leq (m^j E_p^{n-j})^{1/n}$ but is significantly modified for $(m^j E_p^{n-j})^{1/n} < E < E_p$. An example of this type of rapidity-energy relation was considered in Ref. [46]:

$$\cosh(\xi) = (E/m)(2\pi)^{-E^2 \tanh[m^2 E_p^4/E^6]/(mE_p+E^2)} \ . \tag{10}$$

We will only briefly comment on the phenomenological implications of this possibility in Section 6 (in association with the GZK threshold for ultra-high-energy cosmic rays).

Besides possible alternative forms of the Planck-scale-dependent terms in the dispersion relation, there has been also some interest in the literature in "nonuniversal" dispersion relations. The scenarios on which we focus (the ones of Subsections 2.1 and 2.2) are implicitly "universal", in the sense that the relation between energy and momentum depends (once $c$ and $E_p$ are fixed) only on the mass of the particle, and not on other properties of the particle, such as spin and electromagnetic charge. Especially from a broken-Lorentz-symmetry perspective, some authors (see, *e.g.*, Refs. [30, 32]) have considered the possibility of a "nonuniversal" Planck-scale modification of the dispersion relation, *i.e.* a dispersion relation of type (1) (or (9)) in which however the coefficient $\eta$ (or $\eta_2$) is different for different particles. For example, the value of $\eta$ for photons could be different [16, 30, 32] from the value of $\eta$ for electrons. We will comment briefly on this possible "nonuniversality" in the sections that report our phenomenological analysis.

# 3 Removing the sign ambiguity

Of course, the key objective of this phenomenology is to establish whether or not $\eta \neq 0$ in the broken-Lorentz-symmetry scenario (or $\lambda \neq 0$ in the DSR scenario). At the next level of priority clearly it is important to establish "the sign of the modification", *i.e.* to establish whether $\eta \leq 0$ or $\eta \geq 0$ (and similarly for $\lambda E_p \leq 0$ or $\lambda E_p \geq 0$).

As announced earlier, one of the key results of the analysis we are reporting is that $\eta$ (the parameter of the broken-Lorentz-symmetry scenario) must be smaller or equal to 0, while $\eta_{DSR} \equiv \lambda E_p$ (the parameter of the DSR scenario) must be greater or equal to 0.

The condition $\eta \leq 0$ is favoured conceptually, as argued in the next Subsection (3.1), and one then finds, as we show in Section 4, that $\eta > 0$ is excluded by data.

As we show in Subsection 3.2, the condition $\eta_{DSR} \equiv \lambda E_p \geq 0$ is obtained already at the level of the mathematical consistency of the DSR framework. For $\eta_{DSR} < 0$ it is not possible to enforce the dispersion relation (4) in all inertial frames.

## 3.1 A natural sign choice for broken Lorentz symmetry

As mentioned, at least at an intuive level, the idea of Planck-scale broken Lorentz symmetry is usually viewed in analogy with the emergence of modified dispersion relations in the description of the propagation of particles in certain material media (as in the case of the propagation of light in water or in certain crystals). One can indeed attempt to introduce in quantum gravity the concept of "spacetime foam" and spacetime foam might affect particle propagation in a way that to some extent can be viewed in analogy with the way that the presence of material media affects particle propagation.

The sign of $\eta$ in the dispersion relation will fix (making the natural assumption that the relation $v = dE/dp$ still holds) whether or not $c$ is still the maximum velocity. In the analogous situation of dispersion induced by a material medium one naturally finds that $c$ is still the maximum speed. The presence of the medium does not change the fact that the theoretical framework at the fundamental level constrains speeds to be lower than $c$. In a broken-Lorentz-symmetry scenario Lorentz symmetry still plays a role at the fundamental level of the theory, but the presence of some medium allows to select a preferred frame and makes room of the possibility that the speed of photons be different from $c$. But this speed still cannot exceed $c$, in light of the fundamental constraint imposed by the Lorentz symmetry of the fundamental theory (before the backgorund is introduced.

This reasoning (however limited, since it is simply based on an analogy) suggests that in the scenario with broken Lorentz symmetry at the Planck scale the sign of $\eta$ should negative. This theoretical expectation is confirmed by data, in the sense that, as we will discuss in Section 4, consistency with certain observations in astrophysics requires that $\eta < 0$ in the broken-Lorentz-symmetry scenario of Subsection 2.1.

### 3.2 The necessary sign choice for doubly special relativity

Just like in the case of broken Lorentz symmetry, for a doubly special relativity framework the sign choice which specifies whether or not the relativistic invariant $c$ sets the maximum value of speed is one of the most important features. In particular, in the DSR scenario discussed in Subsection 2.2 (with the dispersion relation (4) and the dependence of energy-momentum on rapidity governed by (5)) using $v = dE/dp$ one finds that for negative $\lambda$ the invariant $c$ is the maximum velocity, which is achieved by massless particles in the low-energy ($E \rightarrow 0$) limit (and the speed of massless particles decreases with energy). For positive $\lambda$ the invariant $c$ is still the speed of massless particles in the low-energy limit, but the speed of massless particles increases with energy.

While in the broken-Lorentz-symmetry case it is puzzling to find speeds higher than $c$ (for the reason discussed in the previous subsection), in a doubly-special-relativity scenario there is no in-principle obstruction for speeds higher than $c$. In fact, in doubly special relativity $c$ is defined operatively as the speed of massless particles in the low-energy ($E \rightarrow 0$) limit, and there is no *a priori* reason for assuming a description of $c$ as maximum speed.

So far the choice between positive and negative $\lambda$ has been treated [25, 26, 27] as a free choice allowed by the DSR framework, but we intend to show here that actually only for positive $\lambda$ one obtains a scenario which is genuinely consistent with the DSR requirements.

This point comes from a simple analysis of the DSR laws of transformation between inertial observers. A first indication comes already from the structure of the equations (5). These equations can be easily derived by imposing that for every value of rapidity the DSR energy/momentum dispersion relation would be satisfied. However, we observe that for negative $\lambda$ the equations (5) correspond to a satisfactory behaviour only for relatively small rapidity, and for a critical <u>finite</u> value of rapidity a divergence of energy in encountered.

In an effort to find the root of this problem, we observe that the differential equations that govern the dependence of energy-momentum on rapidity are of a type that does not necessarily lead to the existence of global solutions $E(\xi)$, $p(\xi)$, since the structure of the equations does not fulfill the standard Cauchy requirements for the existence of global solutions $E(\xi)$, $p(\xi)$. It is sufficient for us to discuss this issue considering a single boost (along a given direction). In this case one finds [25, 26] that in the DSR framework the dependence of energy-momentum on rapidity is governed by the differential equations

$$p'(\xi) = \frac{\lambda}{2} p^2(\xi) + \frac{1 - e^{-2\lambda E(\xi)}}{2\lambda} , \tag{11}$$

$$E'(\xi) = p(\xi) , \tag{12}$$

where we used the standard notation $f'(\xi) \equiv df/d\xi$ (for any function $f$).

For the context we are considering the Cauchy requirements can be compactly stated introducing the two-component function $Y(\xi) \equiv \{Y_1(\xi), Y_2(\xi)\} \equiv \{E(\xi), p(\xi)\}$, and using the notation $Y'_l = F_l(Y)$ ($l \in \{1, 2\}$) to denote compactly our system of equations (11)-(12):

- (i) F must be continuous;
- (ii) for every $M \in \Re$ and for every $X \in \Re^2$ and $Z \in \Re^2$, such that $|X| \leq M$,$|Z| \leq M$, there must exist an $L_M \in \Re$, such that $\mid F(X) - F(Z) \mid \leq L_M \mid X - Z \mid$;
- (iii) for every $X \in \Re^2$ there must exist $L_1 \in \Re$ and $L_2 \in \Re$ such that $\mid F(X) \mid \leq L_1 + L_2 \mid X \mid$.

(Of course, with $|W|$ we are denoting $\sqrt{W_a^2 + W_b^2}$ for every $W \equiv \{W_a, W_b\} \in \Re^2$.)

The requirements (i) and (ii) are easily verified, but a possible problem for (iii) originates from the nonlinear structure of our equation (11). The corresponding differential equations of ordinary special relativity ($\lambda \to 0$ limit) are linear and automatically verify the Cauchy "sublinearity requirement" (iii). Instead the nonlinearity of the DSR differential equations imposes a detailed analysis. Our system of equations (11)-(12) evidently satisfies the Cauchy requirements for existence and uniqueness of a local solution (in a neighborhood of a given value of $\xi$), but we are not *a priori* assured of the existence of a global solution.

A detailed analysis shows that for positive $\lambda$ there is no problem: the Cauchy "sublinearity requirement" (iii) is satisfied (in spite of the nonlinearity of the equations) and therefore the existence of global solutions is assured. But for negative $\lambda$ the Cauchy "sublinearity requirement" is not satisfied. The interested reader can straightforwardly (but somewhat tediously) verify that indeed for positive $\lambda$ one can find two real numbers $L_1$ and $L_2$ with he property required in (iii). Instead for negative $\lambda$ the requirement (iii) is not satisfied, for any pair of real numbers $L_1$ and $L_2$.

It is for us here sufficient to discuss a simplified proof, restricting our interest to the case relevant for on-shell particles, in which energy and momentum satisfy the dispersion relation (4). Imposing the dispersion relation one can of course reduce our system of two differential equations to a single differential equation:

$$E'(\xi) = \frac{1}{|\lambda|}\sqrt{1 - 2\cosh(\lambda m) e^{-\lambda E(\xi)} + e^{-2\lambda E(\xi)}} \ . \tag{13}$$

Here the Cauchy "sublinearity requirement" asks us to find a pair of real numbers $L_1$ and $L_2$ such that $E'(\xi) \leq L_1 + L_2 E(\xi)$ for every $E(\xi)$. Indeed for positive $\lambda$ (where the exponentials in (13) are of the type $e^{-|\lambda| E}$, and $e^{-|\lambda| E} \leq 1$) one can find such pairs of real numbers. For example, the choice $L_1 = 1/\lambda$ and $L_2 = 0$ is acceptable. Instead in the case of negative $\lambda$ one finds that for any given pair of real numbers $L_1$ and $L_2$ there is always a value of $E$ such that, according to (13), $E'(\xi) > L_1 + L_2 E(\xi)$. This is due to the fact that for negative $\lambda$ the exponentials in (13) are of the type $e^{|\lambda| E}$, and diverge exponentially for large $E$.

This leads us, as anticipated, to the conclusion that, in the framework[e] discussed in Subsection 2.2, for negative $\lambda$ one does not genuinely obtain a DSR scenario, since the absence of global solutions for our system of differential equations would lead to the paradox that a particle with well-defined energy-momentum for certain inertial observers, would not have a well-defined momentum for some other observers. For positive $\lambda$ there is no such problem and all the DSR requirements are satisfied.

### 3.3 An illustrative example of $\kappa$-Poincaré algebra which is not admissible in DSR

While the objectives of the analysis we are reporting are primarily phenomenological, it seems appropriate to devote this subsection to the discussion of some conceptual (rather than phenomenological) implications of the result obtained in the previous subsection.

As clarified earlier, the proposal [25] of doubly special relativity is the idea that the quantum-gravity problem might lead us to the introduction of a second relativistic invariant, a small-length/large-energy scale, possibly given by the Planck scale. And the DSR (rotation-)boost transformations should be characterized by two scales, $\lambda$ ($1/\lambda$) and $c$, in the same sense that $c$ already characterizes the (rotation-)boost transformations of special relativity. It is interesting to ask which types of mathematical formalisms could play a role in the construction of the physics idea of a doubly special relativity. Einstein's special relativity can rely on the mathematics of Lorentz and Poincaré. For

[e] In this subsection we provided the solution for the "sign ambiguity" of the DSR scenario of Subsection 2.2. We are finding that an analogous result also holds in DSR scenarios that adopt a dispersion relation that is different from the one of Subsection 2.2. In particular, the dispersion relation $[m^2/(1-\tilde{\lambda} m)^2] = [E^2 - p^2/(1-\tilde{\lambda} m)^2]$ is adopted in a DSR scenario considered in Ref. [27], and the issue of the choice of sign of $\tilde{\lambda}$ was indeed raised. Also in that case our line of analysis allows to fix the sign of the deformation parameter (the careful reader will easily find that a genuine DSR scenario is obtained for positive $\tilde{\lambda}$, while the possibility of a negative $\tilde{\lambda}$ is not acceptable).

doubly special relativity such a clear conclusion has not yet been reached, but it appears that to some extent the mathematics of "$\kappa$-Poincaré Hopf algebras" [22, 47] can play a role in the description of some aspects of a DSR theory, at least in the cases (as the one we are here considering) in which the second relativistic invariant appears in a deformation of the dispersion relation.

This observation has led to some confusion: sometimes the concept of a DSR physical theory and the concept of $\kappa$-Poincaré Hopf algebra are treated interchangeably, as if they were the same concept. This would anyway be inappropriate in the same sense that the physical proposal of Einstein's special relativity cannot be strictly identified with the mathematics of Lorentz and Poincaré. And it is even more inappropriate since in the DSR/$\kappa$-Poincaré context one can easily verify that there some aspects of $\kappa$-Poincaré mathematics that are not compatible with the DSR physical principles. Some of these incompatibilities have already been emphasized in the literature. In particular, it is well known [26, 22] that the laws of conservation of energy-momentum for multiparticle processes adopted in the $\kappa$-Poincaré literature (see Ref. [48] and references therein) are incompatible [49] with the DSR requirements. The result obtained in the previous subsection, which shows how the DSR requirements impose that we only consider the positive-$\lambda$ case, illustrates even more clearly that, even restricting our attention to the simple one-particle sector, the requirements for the mathematical consistency of a $\kappa$-Poincaré Hopf algebra are in general not sufficient to obtain structures which are compatible with the DSR requirements.

Let us see this by considering the $\kappa$-Poincaré Hopf algebra that comes closest to playing a role in the DSR scenario of Subsection 2.2. This is the $\kappa$-Poincaré Hopf algebra with commutation relations $\kappa \sim 1/\lambda$[f]

$$
\begin{aligned}
[P_\mu, P_\nu] &= 0 \\
[M_j, M_k] &= i\varepsilon_{jkl} M_l \quad [N_j, M_k] = i\varepsilon_{jkl} N_l \quad [N_j, N_k] = -i\varepsilon_{jkl} M_l \\
[M_j, P_0] &= 0 \quad [M_j, P_k] = i\epsilon_{jkl} P_l \\
[N_j, P_0] &= iP_j \\
[N_j, P_k] &= i\left[\left(\kappa\frac{1-e^{2P_0/\kappa}}{2} + \frac{\vec{P}^2}{2\kappa}\right)\delta_{jk} - \frac{P_j P_k}{\kappa}\right] .
\end{aligned}
\tag{14}
$$

While a Lie algebra is fully specified by the commutation relations, a Hopf algebra also involves a "co-algebra sector", which encodes the rules for the action of generators on tensor products of one-particle states:

$$
\begin{aligned}
\Delta(P_0) &= P_0 \otimes 1 + 1 \otimes P_0 \quad \Delta(P_i) = P_i \otimes 1 + e^{-P_0/\kappa} \otimes P_j \\
\Delta(M_i) &= M_i \otimes 1 + 1 \otimes M_i \\
\Delta(N_j) &= N_j \otimes 1 + e^{-P_0/\kappa} \otimes N_j + \frac{1}{\kappa}\epsilon_{jkl} P_k \otimes M_l
\end{aligned}
\tag{15}
$$

The connection between these algebraic relations and the DSR scenario of Subsection 2.2 is most easily seen by using the fact that a Casimir of (14) is

$$
2\kappa^2 \cosh(P_0/\kappa) - \vec{P}^2 e^{P_0/\kappa} . \tag{16}
$$

The correspondence with the DSR dispersion relation (4) is seen upon $\kappa \to 1/\lambda$, $P_0 \to E$, $\vec{P} \to \vec{p}$.

However, the Hopf-algebra requirements do not specify in any way the sign of $\kappa$ whereas instead the DSR requirements impose, as we showed in the previous subsection, $\lambda \geq 0$. Both for $\kappa > 0$ and for $\kappa < 0$ the relations (14),(15) satisfy the Hopf algebra criteria [21, 22]. This "sign issue" is

[f] Here the careful reader will recognize the structure of the Majid-Ruegg bicrossproduct basis [21].

actually not surprising: the pathology for $\lambda < 0$ which we encountered in the previous subsection emerged at the level of analysis of finite boost transformations, while none of the Hopf algebra requirements pertains to (or can be related with) the concept of a finite symmetry transformation[g]. In other "$\kappa$-Poincaré bases" the Lorentz sector of the commutation relations is actually modified, with commutators among Lorentz (boost/rotation) generators that depend also on the translation generators. While a deformed (nonlinear) action of boosts is a natural possibility for a DSR scenario, it appears [25, 26, 27] that in order to have a consistent DSR scheme the deformation must be such that the commutators among rotation/boost generators still close the Lorentz algebra. For the cases in which the "$\kappa$-Poincaré basis" involves a modification of the Lorentz-sector commutators it appears [25, 26, 27] that there is no hope of ending up with a DSR scenario. The result of the previous subsection shows that even in some cases (at least in the $\kappa < 0$ case of the basis we are considering) in which the Lorentz-sector commutators are not modified a "$\kappa$-Poincaré basis" leads to an inconsistency with the DSR requirements.

Of course, the reverse is also true: some structures which are acceptable from the DSR perspective are not consistent with the $\kappa$-Poincaré. A well known example of this possibility is the fact that, as stressed in Ref. [48], the energy-momentum-conservation law (7)-(8), which is perfectly ok from a DSR perspective, is not compatible with the $\kappa$-Poincaré mathematics (in particular, the law (7)-(8) is symmetric under exchange of the incoming or outgoing particles, whereas $\kappa$-Poincaré mathematics requires [48] that this law should not be symmetric[h].

In summary, while indeed certain specific technical aspects of some DSR proposals and some $\kappa$-Poincaré proposals are closely related (most notably a possible modification of the dispersion relation), the conditions for a DSR structure are in general clearly different from the conditions for a $\kappa$-Poincaré structure. The DSR idea concerns a modification of the laws of transformation that connect arbitrarily different inertial observers, in such a way that the large-velocity scale $c$ and the small-length scale $\lambda$ (large energy-momentum scale $1/\lambda$) both acquire the status of relativistic invariants that characterize these transformations. The $\kappa$-Poincaré idea concerns the possibility of finding a Hopf algebra which involves a scale $\kappa$ and in the $\kappa \to \infty$ limit reduces to the Poincaré Lie algebra. As seen through the example of the law of energy-momentum conservation (7)-(8), some structures that are perfectly acceptable from the DSR perspective may not satisfy some key $\kappa$-Poincaré requirements. And analogously, as seen in the simplest way in the case of the "sign of $\kappa$" issue, certain possibilities which are fully compatible with the $\kappa$-Poincaré requirements are inadmissable from a DSR perspective.

# 4 Photon stability

Although from a conceptual perspective they are extremely significant, Planck-scale departures from Lorentz symmetry lead to negligible small new effects in ordinary situations, when the energies of the particles involved are much smaller than the Planck scale. As it will also emerge from the phenomenological analysis reported in this paper, in order to reach an observable large level the relevant Planck-scale effects must be "amplified" [10] by the presence, in the data being considered, of a large (cosmological) distance and/or the Planck-scale suppression must be softened by the presence of some particles of ultra-high energies, as sometimes happens in the context of certain observations in astrophysics. We will assume in our presentation that the reader is familiar with the analyses (see, *e.g.*, Refs. [10, 17, 18] and references therein) which showed that in controlled on-Earth laboratory setups the new effects introduced by Planck-scale departures from Lorentz symmetry are always negligibly small. However, it is also emerging that in certain contexts of interest in astrophysics one can test the different scenarios for the fate of Lorentz symmetry at the Planck scale. The key opportunities come from observations which provide indirect evidence of the stability of the photon,

[g]Even in the well-understood subject of Lie-algebra symmetries the algebra relations (the commutation relations) are not directly related with the description of finite symmetry transformations (they are however used directly in the description of infinitesimal symmetry transformations).

[h]Note however that, while in the relevant $\kappa$-Poincaré literature [48] it is firmly argued that the coproduct should not be symmetric, there are some works on Hopf algebras which, at least in the case of 1+1-dimensional systems, considered the possibility of a symmetrized coproduct [50].

observations which involve synchrotron radiation, observations which are sentivite to the precise form of the threshold conditions for particle production in collision processes, and observations that are sensitive to a possible wavelength dependence of the speed of photons. We shall consider all of these four opportunities, starting in this section with photon stability.

### 4.1 Brief review of the broken-Lorentz-symmetry result

It has been recently realized [16, 30, 51, 52, 53] that when Lorentz symmetry is broken at the Planck scale there can be significant implications for certain decay processes. One of the decay processes whose analysis leads to a striking result is photon decay into an electron-positron pair: $\gamma \to e^+e^-$. Let us analyze this process using the dispersion relation (1) and unmodified energy-momentum conservation, *i.e.* in the context of the broken-Lorentz-symmetry scenario of Subsection 2.1. One easily finds a relation between the energy $E_\gamma$ of the incoming photon, the opening angle $\theta$ between the outgoing electron-positron pair, and the energy $E_+$ of the outgoing positron (of course the energy of the outgoing electron is simply given by $E_\gamma - E_+$). For the region of phase space with $m_e \ll E_\gamma \ll E_p$ one easily finds

$$\cos(\theta) \simeq \frac{E_+(E_\gamma - E_+) + m_e^2 - \eta E_\gamma E_+(E_\gamma - E_+)/E_p}{E_+(E_\gamma - E_+)} , \tag{17}$$

where $m_e$ is the electron mass.

The fact that for $\eta = 0$ Eq. (17) would require $cos(\theta) > 1$ reflects the fact that if Lorentz symmetry is preserved the process $\gamma \to e^+e^-$ is kinematically forbidden. For $\eta < 0$ the process is still forbidden, but for negative positive $\eta$ high-energy photons can decay into an electron-positron pair. In fact, for $E_\gamma \gg (m_e^2 E_p/|\eta|)^{1/3}$ one finds that $\cos(\theta) < 1$, *i.e.* there is a physical phase space available for the decay.

The energy scale $(m_e^2 E_p)^{1/3} \sim 10^{13} eV$ is not very high. The fact that certain observations in astrophysics allow us to establish [16, 52, 30] that photon of energies up to $\sim 10^{14} eV$ are not unstable implies that the case of positive $\eta$ is severely constrained. Since the positive-$\eta$ case, for this broken-Lorentz-symmetry scenario, is already disfavoured conceptually (see Subsection 3.1), we shall, without further hesitation, assume that $\eta \leq 0$ in the broken-Lorentz-symmetry scenario of Subsection 2.1.

### 4.2 No photon instability in DSR

The fact that one finds that a certain particle decay can occur only at energies higher than a certain minimum decay energy ($E_\gamma \gg (m_e^2 E_p/|\eta|)^{1/3}$) is of course a manifestation of the break down of Lorentz symmetry. A given photon will have high energy for some inertial observers and low energy for other inertial observers. And of course it is not possible[i] to assume that the decay would be allowed according to some observers and disalloed according to others. So clearly such a picture requires a preferred frame: the energy of the particle should be measured in the preferred frame and the decay is allowed if the energy of the particle in the preferred frame exceeds a certain given value.

For the particle-decay picture of the previous subsection the existence of a preferred class of inertial observers is therefore a prerequisite. And even without any calculations we can conclude that there is no such mechanism in a consistent DSR scenario, where the presence of preferred frames is excluded by construction.

For completeness we can verify explicitly that the process $\gamma \to e^+e^-$ is not allowed in the DSR scenario of Subsection 2.2. We must simply combine the dispersion relation (4) (which coincides with the dispersion relation (1) of the broken-symmetry scenario upon the identification $\eta = \lambda E_p$)

[i] We are of course assuming the "objectivity of particle-production processes" [25]. If according to one observer the "in state" (a time "$-\infty$") is a photon and the "out state" (a time "$+\infty$") is composed of an electron-positron pair, then all other observers must agree.

with the DSR-deformed energy-momentum conservation law (see (7)-(8)) which in this case takes the form

$$E_\gamma \simeq E_+ + E_- - \lambda \vec{p}_+ \cdot \vec{p}_- \ , \quad \vec{p}_\gamma \simeq \vec{p}_+ + \vec{p}_- - \lambda E_+ \vec{p}_- - \lambda E_- \vec{p}_+ \ . \tag{18}$$

From this, considering again the region of phase space with $m_e \ll E_\gamma \ll E_p \sim 1/\lambda$, one easily finds that the relation between $E_\gamma$, the opening angle $\theta$, and $E_+$ must take the form

$$\cos(\theta) \simeq \frac{2E_+(E_\gamma - E_+) + 2m_e^2 + \lambda E_\gamma E_+(E_\gamma - E_+)}{2E_+(E_\gamma - E_+) + \lambda E_\gamma E_+(E_\gamma - E_+)} \ , \tag{19}$$

Evidently in this DSR case, no matter how large is $E_\gamma$, one inevitably finds[j] that the process would require $\cos(\theta) > 1$, *i.e.* there is no physical phase space available for the process $\gamma \to e^+ e^-$.

We conclude that the fact that certain observations in astrophysics allow us to establish that photons of energies up to $\sim 10^{14} eV$ are not unstable does not introduce any constraints on the parameters of a DSR scenario. It led to an important constraint for the broken-Lorentz-symmetry scenario of Subsection 2.1, but it is completely unconsequential for the DSR scenario of Subsection 2.2, even though the two scenarios adopt the same modification of the dispersion relation.

# 5 Synchrotron radiation

## 5.1 Brief review of the broken-Lorentz-symmetry analysis

As observed recently in Ref. [30] (and observed already earlier in Ref. [54]) in the mechanism that leads to the production of synchrotron radiation a key role is played by the special-relativistic velocity law $v = dE/dp \simeq 1 - m^2/(2E^2)$. Making the natural assumption that the relation $v = dE/dp$ still holds at the Planck scale the modified dispersion relation (1) replaces the special-relativistic velocity law, with

$$v \simeq 1 - \frac{m^2}{2E^2} + \eta \frac{E}{E_p} \ . \tag{20}$$

Assuming that all other aspects of the analysis of synchrotron radiation remain unmodified at the Planck scale (an assumption which of course must be subject to careful further scrutiny), one is led [30] to the conclusion that, if $\eta < 0$, the energy/wavelength dependence of the Planck-scale term in (20) can severely affect the value of the cutoff energy for synchrotron radiation [55]. In fact, for $\eta < 0$, an electron on, say, a circular trajectory (which therefore could emit synchrotron radiation) cannot have a speed that exceeds the maximum value

$$v_e^{max} \simeq 1 - \frac{3}{2}\left(|\eta| \frac{m_e}{E_p}\right)^{2/3} \ , \tag{21}$$

whereas in special relativity $v_e^{max} = 1$ (although values of $v_e$ that are close to 1 require a very large electron energy).

This may be used to argue that for negative $\eta$ the cutoff energy for synchrotron radiation should be lower than it appears to be suggested by certain observations of the Crab nebula [30]. In the near future, when the relevant observations of the Crab nebula will be more precise and better understood and the assumptions that are used in the analysis of Ref. [30] will be more carefully examined, could allow to set a very stringent limit ($|\eta| \ll 1$) on the negative values of $\eta$ for the broken-Lorentz symmetry scenario of Subsection 2.1. Therefore, since positive values of $\eta$ for the broken-Lorentz symmetry scenario are already ruled out (see Subsections 3.1 and 4.1), these planned studies of the Crab nebula and of synchrotron radiation have the possibility of ruling out completely[k] the scenario of Subsection 2.1.

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[j] We notice that this result formally applies both to the positive-$\lambda$ and the negative-$\lambda$ cases. But of course, since (as shown in Subsection 3.2) the consistency of the DSR scheme requires $\lambda > 0$, the case of true interest is positive-$\lambda$.

[k] As mentioned, it is however possible to contemplate "nonuniversal" modifications of the dispersion relation, in

## 5.2 Implications of DSR for synchrotron radiation are negligible

Assuming again that the relation $v = dE/dp$ still holds at the Planck scale[1] from the DSR modified dispersion relation (4) one obtains the velocity law

$$v \simeq 1 - \frac{m^2}{2E^2} + \lambda E \ . \tag{22}$$

For $\lambda < 0$ one would also in this case find that for a particle of mass $m$ there is an $m$-dependent (and smaller than 1) maximum value of the velocity. However, we have seen in Subsection 3.2 that negative values of $\lambda$ are not compatible with the logical structure of the DSR framework, since they require the existence of a preferred class of inertial frames. And for positive $\lambda$, since there is no maximum value of the speed of electrons, the formula (22) does not suggest any modification of the cutoff energy for synchrotron radiation.

It is perhaps useful to summarize our findings before the additional phenomenological analyses of the following Sections 6 and 7. Our first level of analysis, in Section 3, focused on the logical structure of the scenarios, and we found that in the DSR scenario of Subsection 2.2 it was necessary to impose $\lambda > 0$, while for the broken-Lorentz-symmetry scenario of Subsection 2.1 there was no absolute requirement on the sign of $\eta$, although $\eta \leq 0$ appeared to be preferable. We then, in Section 4, considered on photon-stability issues, and we found that some relevant data rule out $\eta > 0$ in broken-Lorentz-symmetry scenario of Subsection 2.1, while they have no significant implication for the analysis of the DSR scenario of Subsection 2.2. So at the end of Section 4 we had ruled out $\eta > 0$ and $\lambda < 0$, but negative values of $\eta$ and positive value of $\lambda$ were still completely unconstrained. Then, here in Section 5, we focused on synchrotron-radiation issues, and we found that some relevant data (when better understood) provide an opportunity to rule out the residual possibility for the broken-Lorentz-symmetry scenario of Subsection 2.1, *i.e.* the case of negative $\eta$, while the same synchrotron-radiation data are not significant for the positive-$\lambda$ DSR scenario of Subsection 2.2. In the remainder of this paper we will focus on positive values of $\lambda$ for the DSR scenario, and we will continue to consider negative values of $\eta$ for the broken-Lorentz-symmetry scenario. Indeed our results so far leave still completely uncostrained the positive values of $\lambda$. For $\eta$ we are in a situation in which positive values are clearly excluded (through the photon-stability analysis) but negative values are still allowed (pending a more careful examination of the synchrotron-radiation analysis of Ref. [30]).

# 6 Threshold anomalies

## 6.1 Brief review of the broken-Lorentz-symmetry result

Another opportunity to investigate Planck-scale departures from Lorentz symmetry is provided by certain types of energy thresholds for particle-production processes that are relevant in astrophysics.

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which case some of these points are significantly modified. In particular, it was argued in Ref. [32] that a specific picture of spacetime foam [56] suggests that $\eta = 0$ for the electron and $\eta \sim 1$ for photons. If $\eta = 0$ for electrons then there is no modification of the velocity law for electrons.

[1]We must stress however that in the DSR framework the validity of the relation $v = dE/dp$ might depend on the relevant spacetime picture. Because of the nature of the phenomenological analyses here of interest we could here focus on the energy-momentum sector but of course a full DSR theory must include a description of the spacetime sector. In particular, for one of the spacetimes which are being considered as candidate DSR-compatible spacetimes, the so-called $\kappa$-Minkowski noncommutative spacetime, the validity of the relation $v = dE/dp$ is still a subject of an active debate [57, 58, 59, 60, 61]. We restrict here our attention to the relation $v = dE/dp$ because we are not trying to examine a variety of DSR scenarios, but we are rather focusing on the DSR scenario of Subsection 2.2, and on an analysis of that DSR scenario which makes it as similar as possible to the broken-Lorentz-symmetry scenario of Subsection 2.1 (in order to expose some aspects of phenomenology which are genuinely sensitive to the difference between breaking and deforming Lorentz symmetry).

A simple way to see this is found in the analysis of collisions between a soft photon of energy $\epsilon$ and a high-energy photon of energy $E$ that creates an electron-positron pair: $\gamma\gamma \to e^+e^-$. For given soft-photon energy $\epsilon$, the process is allowed only if $E$ is greater than a certain threshold energy $E_{th}$ which depends on $\epsilon$ and $m_e^2$. In the broken-Lorentz-symmetry scenario of Subsection 2.1 this threshold energy can be evaluated combining the dispersion relation (1) with ordinary energy-momentum conservation. One easily obtains (assuming $\epsilon \ll m_e \ll E_{th} \ll E_p$)

$$E_{th}\epsilon + \eta \frac{E_{th}^3}{8E_p} = m_e^2 \ . \tag{23}$$

The special-relativistic result $E_{th} = m_e^2/\epsilon$ corresponds of course to the $\eta \to 0$ limit of (23). For $|\eta| \sim 1$ the Planck-scale correction can be safely neglected as long as $\epsilon/E_{th} > E_{th}/E_p$. But eventually, for sufficiently small values of $\epsilon$ and correspondingly large values of $E_{th}$, the Planck-scale correction cannot be ignored. This occurs for $\epsilon < (m_e^4/E_p)^{1/3}$ (when, correspondingly $E_{th} > (m_e^2 E_p)^{1/3}$). For $\epsilon \sim 0.01eV$ the modification of the threshold is already significant, and this is relevant for the observation of multi-$TeV$ photons from certain Blazars [14, 15]. Blazar photons travel to us from very far and they travel in an environment populated by soft photons, some with energies (roughly $\epsilon \sim 0.01eV$) suitable for acting as targets for the disappearance of the hard photon into an electron-positron pair. According to the standard ($\eta = 0$) threshold formula, Blazar photons of energies in the neighborhood of $10TeV$ should be already significantly absorbed before reaching our Earth telescopes. If $\eta$ is negative (the only case we are still considering, since positive $\eta$ is safely ruled out) and of order 1 one finds from (23) a significantly higher value of the threshold (varying $\eta$ between, say, 0.1 and 10 one finds that the threshold can be shifted up to $20TeV$ and higher). Therefore the availability of good-quality data on the observed spectrum of these Blazar photons, as we should surely have at least once the space telescope GLAST [37] starts operating in 2006, could be used to set stringent constraints on negative $\eta$.

A similar argument can be applied to cosmic rays. The fact that ultra-high-energy cosmic-ray protons travel toward the Earth in an environment populated by CMBR photons leads to the expectation of a GZK [36] cutoff on the observed spectrum. In the standard special-relativistic framework the GZK cutoff is around $5 10^{19}eV$, whereas in the broken-Lorentz-symmetry scenario with dispersion relation (1) one finds a modification of the threshold [11, 13, 15] analogous to (23). And again for negative $\eta$ of order 1 one finds a value of the threshold that is significantly higher than the GZK value (comfortably above $10^21eV$). Good-quality data on the observed spectrum of cosmic rays at energies of order $10^{20}$ or $10^{21}eV$ would therefore allow to test a key prediction of the negative-$\eta$ case. At present the situation could be described as encouraging, since some observations reported by AGASA [62], arguably the best cosmic-ray observatory presently taking data, provide encouragement for the idea of a cutoff scale significantly higher than the GZK prediction. But it is necessary to proceed with caution: there are other (non-Planck-scale-related) possible descriptions [31, 63] of the AGASA anomaly, and also the experimental situation requires further scrutiny (since the findings of the HIRES observatory [64, 65] are inconsistent with ones of AGASA). Important elements for this analysis should become available in the coming months, when the Pierre Augerre cosmic-ray observatory should announce its first data.

## 6.2 Threshold anomalies are typically small in DSR

While, as shown in the previous subsection, "threshold anomalies" can be observably large in the broken-Lorentz-symmetry scenario of Subsection 2.1, in DSR the threshold anomalies are typically negligibly small. In particular, in the DSR scenario of Subsection 2.2 one finds negligibly small threshold anomalies.

One can easily verify this by analyzing again the process $\gamma\gamma \to e^+e^-$. In the DSR scenario of Subsection 2.2 one should analyze the kinematics of this process using again the dispersion relation (4) (which is essentially the same as the dispersion relation (1)), but taking into account [25, 66] the DSR-deformed energy-momentum conservation law (see (7)-(8)) which in this case takes the form

$$E + \epsilon - \lambda \vec{P} \cdot \vec{p} \simeq E_+ + E_- - \lambda \vec{p}_+ \cdot \vec{p}_- \ , \quad \vec{P} + \vec{p} - \lambda E \vec{p} - \lambda \epsilon \vec{P} \simeq \vec{p}_+ + \vec{p}_- - \lambda E_+ \vec{p}_- - \lambda E_- \vec{p}_+ \tag{24}$$

where we denoted with $\vec{P}$ the momentum of the photon of energy $E$ and we denoted with $\vec{p}$ the momentum of the photon of energy $\epsilon$.

The presence of correction terms both in the dispersion relation and in the energy-momentum-conservation law (with coefficients fixed by the requirement of equivalence of inertial frames) leads to rather large cancellations in the threshold formula. Assuming again that $\epsilon \ll m_e \ll E_{th} \ll E_p$ one ends up finding

$$E_{th} \simeq \frac{m_e^2}{\epsilon} , \tag{25}$$

*i.e.* (for $\epsilon \ll m_e \ll E_{th} \ll E_p$) one ends up with the same result as in the special-relativistic case. If, rather than working within the approximations allowed by the hierarchy $\epsilon \ll m_e \ll E_{th} \ll E_p$, one considers the exact DSR threshold formula, one finds a result which is actually different from the special-relativistic one. There are "threshold anomalies" in the DSR scenario of Subsection 2.2. But they are very small, well below an observable level, when $\epsilon \ll m_e \ll E_{th} \ll E_p$, which corresponds to the only real opportunity for testing such threshold anomalies.

An analogous result holds for the cosmic-ray threshold. The calculation is somewhat more tedious but we find that the end result is again affected by large cancellations in the approximations that are natural when the energy of the cosmic ray is much higher that the proton mass and much smaller than the Planck scale (and also using the fact that the energies of CMBR photons are much smaller than the proton mass). For $\epsilon_{CMBR} \ll m_{proton} \ll E_{cosmic-ray} \ll E_p$ one finds only a minute difference (well below the level observable at cosmic-ray observatories) between the special-relativistic threshold and the DSR threshold.

We conclude that planned studies of "threshold anomalies" at gamma-ray and cosmic-ray observatories will not allow to discriminate between special relativity and the DSR scenario of Subsection 2.2.

We must stress here that there is no in-principle obstruction for observably large threshold anomalies in DSR. It just happens to be difficult to construct a DSR scenario with observably large threshold anomalies. We have verified (and the reader can easily verify) even considering the possibility of changing rather arbitrarily the dispersion relation, if done consistently with the DSR requirements (*i.e.* introducing a corresponding modification of the energy-momentum conservation law) one nearly inevitably finds a final result in which the threshold anomalies are negligibly small. But if one really desperately wants observably large threshold anomalies, even at the cost of a completely *ad hoc* choice of the DSR dispersion relation, a suitable DSR scenario can be found, and indeed such a scheme was proposed in Ref. [46]. This is the scenario which we already mentioned in discussing Eq. (10) (and the "creative" presence of tanh functions in Eq. (10) reflects the *ad hoc* choices that guided the construction of that scenario).

So, in summary, the threshold anomalies of the DSR scenario of Subsection 2.2 (with dispersion relation (4)) are negligibly small, and the threshold anomalies are also negligibly small in other DSR scenarios, but with some *ad hoc* choices in introducing the elements of a DSR scenario one can manage to produce sizeable threshold anomalies.

This status of threshold anomalies in DSR is rather different from the results of our analysis of photon stability in DSR. We have verified explicitly that in the most studied DSR scenario, the one we discussed in Subsection 2.2, photon decay into an electron-positron pair is not allowed and there are no sizeable threshold anomalies. But for photon decay we observed that one can exclude it in any, however structured, DSR scenario because it would conflict with the required equivalence of inertial observers, whereas the presence of sizeable threshold anomalies is not in conflict with any general requirement of the DSR framework (they simply turns out to be absent in the DSR scenario of Subsection 2.2, and we found that some rather *ad hoc* choices must be made in constructing a DSR scenario if one has the objective of introducing sizeable threshold anomalies).

# 7 Time-of-travel analyses

In Section 5 we already noticed that both in the broken-Lorentz-symmetry scenario of Subsection 2.1 and in the DSR scenario of Subsection 2.2 one finds a Planck-scale contribution to the energy dependence of the speed of particles. It is convenient to introduce a unified notation here for the relevant velocity laws (for $m < E \ll E_p \sim 1/\lambda$):

$$v \simeq 1 - \frac{m^2}{2E^2} + \eta_* \frac{E}{E_p} , \tag{26}$$

where $\eta_* = \eta \leq 0$ for the broken-Lorentz-symmetry scenario and $\eta_* = \lambda E_p \geq 0$ for the DSR scenario.

In Section 5 we were focusing on synchrotron radiation, where a significant effect might be expected if $\eta_* < 0$ (with $|\eta_*| \sim 1$), so the analysis turned out to be only relevant for the broken-Lorentz-symmetry scenario. There is of course another, more direct, way to investigate the possibility (26): whereas in ordinary special relativity two photons ($m = 0$) with different energies emitted simultaneously would reach simultaneously a far-away detector, those two photons should reach the detector at different times according to (26).

This type of effect emerging from an energy dependence of the speed of photons can be significant [7, 9] in the analysis of short-duration gamma-ray bursts that reach us from cosmological distances. For a gamma-ray burst it is not uncommon[m] to find a time travelled before reaching our Earth detectors of order $T \sim 10^{17}s$. Microbursts within a burst can have very short duration, as short as $10^{-3}s$ (or even $10^{-4}s$), and this means that the photons that compose such a microburst are all emitted at the same time, up to an uncertainty of $10^{-3}s$. Some of the photons in these bursts have energies that entend at least up to the $GeV$ range. For two photons with energy difference of order $\Delta E \sim 1GeV$ an $\eta_* \Delta E/E_p$ speed difference over a time of travel of $10^{17}s$ would lead to a difference in times of arrival of order $\Delta t \sim \eta_* T \Delta E/E_p \sim 10^{-2}s$, which is significant (the time-of-arrival differences would be larger than the time-of-emission differences within a single microburst).

Such a Planck-scale-induced time-of-arrival difference could be revealed [7, 9] upon comparison of the structure of the gamma-ray-burst signal in different energy channels. The next generation of gamma-ray telescopes, such as GLAST [37], will exploit this idea to search for energy dependence of the speed of photons.

An even higher sensitivity to a possible energy dependence of the speed of photons could be achieved by exploiting the fact that, according to current models [68], gamma-ray bursters should also emit a substantial amount of high-energy neutrinos. Some neutrino observatories should soon observe neutrinos with energies between $10^{14}$ and $10^{19}$ $eV$, and one could, for example, compare the times of arrival of these neutrinos emitted by gamma-ray bursters to the corresponding times of arrival of low-energy photons. But a robust estimate of the sensitivity achievable following this strategy will require an improved understanding of gamma-ray bursters, good enough establish whether there are typical at-the-source time delays (there could be a systematic tendency of gamma-ray bursters to emit high-energy neutrinos with, say, a certain delay with respect to microburst of photons).

In any case, this type of time-of-arrival analyses (both the ones that use exclusively photons and the ones that might be able to use also neutrinos) is clearly important both for the broken-Lorentz-symmetry scenario and for the DSR scenario, since in both scenarios one expects an energy-dependent speed of photons. Our analysis (particularly the results reported in Subsection 3.2) provides a new tool for these experimental studies. In fact, we can now establish that data in favour of $\eta_* < 0$ would not only signal a departure from Lorentz symmetry but would also suggest that there is a break down of Lorentz symmetry at the Planck scale (since $\eta_* < 0$ is not allowed in the DSR scenario). And *vice versa* data in favour of $\eta_* > 0$ would not only signal a departure from Lorentz symmetry but would also suggest that there is a DSR-deformation of Lorentz symmetry at the Planck scale (since the possibility of $\eta_* > 0$ in the broken-Lorentz-symmetry scenario is disfavoured by the phenomenological analysis of Section 4).

# 8 Closing remarks

Our analysis shows that the possibility of a Planck-scale deformation of Lorentz symmetry, in the sense of DSR, leads to a phenomenology which in many ways is significantly different from the more studied corresponding scenario with broken Lorentz symmetry. In some contexts in which the broken-Lorentz-symmetry scenario could lead to observably-large departures from the predicitions of the ordinary Lorentz-symmetry case, as in the observations relevant for photon stability, synchrotron radiation and anomalous thresholds, the DSR scenario is instead basically indistinguishable from

[m] Up to 1997 the distances from the gamma-ray bursters to the Earth were not established experimentally. By a suitable analysis of the gamma-ray-burst "afterglow" [67], it is now possible to establish the distance from the gamma-ray bursters to the Earth for a significant portion of all detected bursts. $10^{10}$ light years ($\sim 10^{17}s$) is not uncommon.

ordinary special relativity. These differences are due to the fact that in DSR one still has a 6-parameter rotation/boost symmetry group, and this introduces various limitations to the new effects that may arise. For example, as we discussed in Section 4 (on the basis of the points already raised in Ref. [25]), just like in special relativity also in DSR it is not possible for a particle which is stable at low energies to become unstable above some high-energy scale. The same features of DSR kinematics (energy-momentum conservation) which enforce this principle end up playing a role also in the analysis of the possibility of anomalous thresholds (Section 6). A sizeable modification of the threshold relations for particle-physics reactions is not *a priori* excluded by the DSR requirements, but since the analysis of particle decays and the analysis of reaction thresholds share many computational aspects it ends up being the case that the same structures that protect DSR from unadmissable decay properties also render typically small the modifications of the reaction thresholds.

A DSR scenario is instead not much different from a broken-Lorentz-symmetry scenario for what concerns the type of time-of arrival analyses discussed in Section 7.

This leads us to a rather sharp characterization of the possibility of departures from Lorentz symmetry in which the dispersion relation involves a leading-order term that goes linearly with the Planck scale, as the ones of Subsections 2.1 and 2.2. Ordinary special relativity of course predicts that no "anomaly" should be found in the observations considered in Sections 4, 5, 6 and 7. The DSR scenario of Subsection 2.2 predicts that there should be no anomalies for what concerns the observations considered in Sections 4, 5 and 6 but there should be a trace of the Planck-scale effects in the time-of-arrival analyses of Section 7. The broken-Lorentz-symmetry scenrio of Subsection 2.1 definitely predicts observably-large anomalies in all of the types of observations considered in Sections 4, 6 and 7 and possibly also the ones considered in Section 5 (pending further scrutiny of some of the assumtpions in the analysis of the implications of broken Lorentz symmetry for synchrotron radiation reported in Ref. [30]).

Clearly the investigation this possibility of departures from Lorentz symmetry in which the dispersion relation involves a leading-order term that goes linearly with the Planck scale is within the reach of the gamma-ray and cosmic-ray observatories that should start taking data in the next 4 or 5 years. On the basis of the analysis we reported here, one can be confident that with these observatories the possibility of such Planck-scale-linear modifications will be fully explored. We will either be able to rule it out completely on the basis of actual data or find definite evidence of such new effects. And if the data do provide support for the new Planck-scale effects their analysis, on the basis of the characterization we provided here, will also allow us to distinguish between the DSR scenario and the broken-Lorentz-symmetry scenario.

Of course, once the possibility of effects that go linearly with the Planck scale has been fully explored (and, as one tends to expect, possibly ruled out), the next item on the agenda of investigation of the fate of Lorentz symmetry at the Planck scale will concern the possibility of effects that go quadratically with the Planck scale. Since these quadratically-suppressed effects are obviously much smaller than the linear ones, their investigation with actual data is somewhat further in the future. The fact that instead data relevant for the linear case is soon becoming available motivated us to focus on that possibility, but we did mention, although somewhat parenthetically, that some opportunities for testing the quadratic case might not be too far in the future. From that perspective a key role should be played by cosmic-ray observations and the powerful tool that could be provided by high-energy-neutrino observatories [69].

On the technical side our result showing that $\lambda$ must be positive in the DSR scenario of Subsection 2.2 could have several applications, and already proved to be rather poweful (together with the presence of the 6-parameter group of rotation/boost symmetries) for our phenomenological analysis. This result for the sign of $\lambda$ should also contribute to a deeper understanding of the DSR conceptual structure, and in particular, as we stressed, should allow to resolve completely the frequent confusion between the DSR proposal and the mathematics of $\kappa$-Poincaré Hopf algebras.

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