Understanding of the freeze-out in ultra-relativistic heavy-ion collisions*

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Abstract

I discuss the quantities and effects important for the freeze-out and outline a formalism for the description of continuous decoupling of particles from the fireball. Then I present a calculation of the scattering rates of pions at various temperatures and argue that it is important to take continuous particle decoupling into account when modelling the collision dynamics.

Hadronic single-particle spectra and correlations carry information about the freeze-out state of the fireball which results from the collective expansion of the strongly interacting matter. Often, the freeze-out state is being modelled in framework of the Cooper-Frye [1] mechanism where all particles—regardless their identity and/or momentum—are emitted from a single sharp three-dimensional freeze-out hypersurface. This is the case for most hydrodynamic simulations. A question appears: does the assumption of a common sharp three-dimensional freeze-out still provide reliable approximation of the real process?

In case of a sharp freeze-out, the freeze-out hypersurface is usually characterised by some prescription. There were some attempts to identify a *universal freeze-out criterion* [2, 3], i.e., a condition which determines the freeze-out hypersurface for heavy-ion collisions of any size and at any ultrarelativistic energy. In case of the continuous gradual decoupling there is no hypersurface to be determined and the concept of a universal freeze-out criterion is not applicable. I will focus on gradual decoupling since the sharp freeze-out can be defined as its limiting case.

Let me focus on the mechanism of freeze-out and identify the important effects and quantities. Freeze-out occurs when scattering ceases. It has been suggested that the mean-free path is the relevant quantity to look at [3, 4]. Here, densities of the individual species are weighted with cross-sections for scattering on them. An example: density of nucleons is much more important for pion scattering rate than the density of pions, because the πN cross-section is bigger than the one for $\pi\pi$ scattering. The CERES collaboration argued that the "universal" pion mean-free path at freeze-out should be something *of the order of* 1 fm (maybe 2–3 fm) [3].

On the first sight, this is rather surprising, because this length is much shorter than the size of the system. So far, however, we did not mention the expansion and the decrease of the density due to it. In rough terms, freeze-out occurs when the rate of the density decrease becomes comparable or larger than the scattering rate [5]. The picture is the following: at the moment of scattering the particle has some finite mean-free path λ and thus would be expected to scatter after some time. However, if the density drops fast enough, after passing the length λ our particle may find itself in an environment of very low density such that no other scattering occurs.

In [6] we used the formalism of escape probabilities (conceived earlier by other authors, e.g. [7, 8]) and focused on the dependence of the escape probability on temperature and the momentum of pions. By using this formalism one *qualitatively* goes beyond probing the mean-free path and the rate of density decrease; particles now decouple from a finite four-volume. Sharp freeze-out is realised as a limiting case of this more general case, when the escape probabilities of *all* particles change from 0 to 1 in a very narrow space-time region.

The escape probability can be determined as

$$\mathcal{P}(x,\tau,p) = \exp\left(-\int_{\tau}^{\infty} d\bar{\tau}\,\mathcal{R}(x+v\bar{\tau},p)\right)\,, \tag{1}$$

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where $\mathcal{R}$ is the scattering rate. The opacity integral in this equation is evaluated along the expected trajectory of a particle with momentum p and the position characterised by x and τ if it moved straight. The integrated scattering rate gives the average number of collisions the particle would suffer on this trajectory. Assumptions about the chemical composition of the medium are included in calculation of $\mathcal{R}$, while the density decrease rate determines its time dependence and thus the value of the resulting integral.

As an example of the fireball expansion we assumed in [6] a particle with $p_t = 0$ in the centre of the fireball which does not move transversely but just waits until the surrounding matter decays. Then we only need a prescription for the time dependence of $\mathcal{R}$ for which we take an ansatz:

$$\mathcal{R}(\tau) = \mathcal{R}_0 \left(\frac{\tau_0}{\tau}\right)^{\alpha} , \tag{2}$$

where $\alpha > 1$ and $\mathcal{R}_0$ is the scattering rate at time τ_0. This corresponds to a fireball expanding longitudinally in a boost-invariant manner and transversely with the following dependence of the transverse flow rapidity on radial coordinate r:

$$\eta_t(r, \tau) = \frac{\alpha - 1}{2} \frac{r}{\tau} . \tag{3}$$

The escape probability in this toy model then reads

$$\mathcal{P}(p_t = 0, r = 0) = \exp\left(-\frac{\mathcal{R}_0 \tau_0}{\alpha - 1}\right) = \exp\left(-\frac{\mathcal{R}_0}{2\eta_t(r, \tau_0)/r}\right) . \tag{4}$$

Obviously, larger $\mathcal{R}_0$ leads to smaller escape probability, whereas stronger expansion is encoded in larger α and/or η_t and increases the value of $\mathcal{P}$.

We calculated the scattering rate of negative pions in thermal hadronic gas as

$$\mathcal{R}(p) = \sum_i \int d^3k \, \rho_i(k) \, \sigma_i'(s) \, |v_\pi - v_i|^* . \tag{5}$$

Here, $\rho_i(k)$ is the density of particles on which the pion scatters, $\sigma_i'(s)$ is the collinear cross-section, and $|v_\pi - v_i|^*$ is the relative velocity of the pion to the other scattering partner in the CMS of the pair. We integrate over the momenta of the scatterers and sum over following species as scattering partners: $i = \pi$, N, $\bar{N}$, K, ρ, Δ, $\bar{\Delta}$. For the cross-section we use a parametrisation of hadron scattering via resonances (see [6, 9]). For the temperatures we assumed values of 90, 100, and 120 MeV. We estimated the chemical potentials in such a way that we reproduced data on pion phase-space density at the SPS [10, 11] and RHIC [12] and the ratios of dN/dy of different species at midrapidity [13, 14, 15].

In Fig. 1 I compare pion scattering rates in hadronic gases corresponding to those at the SPS and at RHIC. The contribution from nucleons and antinucleons to the total scattering rate increases little when moving from SPS to the higher energy system. A higher phase-space density of pions at RHIC [12] leads to larger pion contribution, but the total scattering rate is not dominated by pions and therefore does not change much. Quantitatively, about 10% relative contribution is shifted from baryons to mesons when going from SPS to RHIC.

A summary of results is plotted in Fig. 2. The scattering rate *drops with temperature*. When we use eq. (4) and a realistic estimate of the transverse rapidity gradient [16], reasonable escape probabilities ($\sim$ 30–50%) are obtained for $T \lesssim 100\,\mathrm{MeV}$. Note that this was inferred for a particle with $p_t = 0$ in centre of the fireball.

One can see that regardless the assumed temperature and chemical potentials higher momentum particles always have smaller scattering rate and thus decouple easier. This suggests that the freeze-out can be *sequential*: high momenta first, low momenta later. A sequential freeze-out might be an additional cause of the M_t dependence of the HBT radii, if high-p_t pions decouple earlier and from a smaller fireball than the low-p_t ones. Such a scenario, however, must be studied in more detail. Nevertheless, it represents an interesting alternative to the blast-wave model, which seems to have problems in reproducing data on HBT radii [17].

Due to the momentum dependence of the scattering rate, collapsing the whole decoupling four-volume into a sharp three-dimensional freeze-out hypersurface seems an unreliable approximation.

References

[1] F. Cooper and G. Frye, Phys. Rev. D **10** (1974) 186.

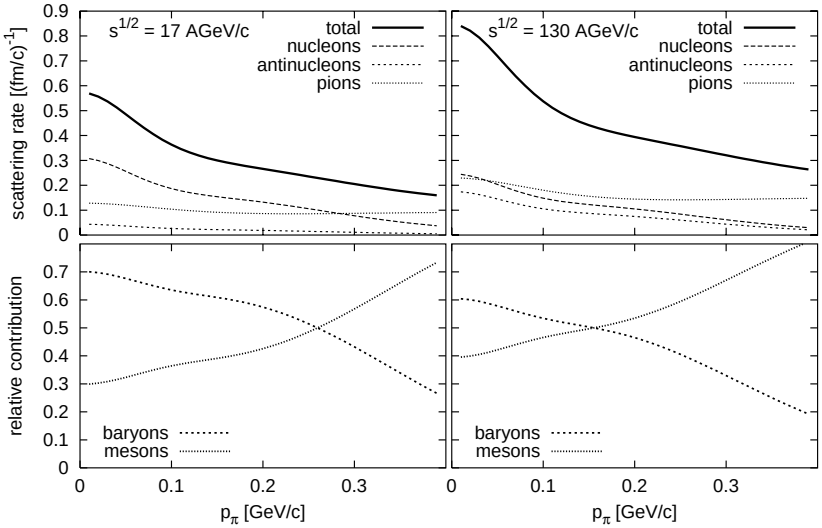

Figure 1: Scattering rate as a function of pion momentum with respect to the hadronic medium with $T = 100$ MeV. Results are obtained for SPS (left) and RHIC (right). Contributions to the scattering rate from scattering on pions, nucleons and antinucleons are indicated. The two lower panels show the baryonic and mesonic relative contributions.

[2] D. Ferenc, U.W. Heinz, B. Tomášik, U.A. Wiedemann and J.G. Cramer, Phys. Lett. B **457** (1999) 347.

[3] D. Adamová *et al.* [CERES Collaboration], Phys. Rev. Lett. **90** (2003) 022301.

[4] S. Nagamiya, Phys. Rev. Lett. **49** (1982) 1383.

[5] J.P. Bondorf, S.I.A. Garpman and J. Zimányi, Nucl. Phys. A **296** (1978) 320.

[6] B. Tomášik and U.A. Wiedemann, Phys. Rev. C **68** (2003) 034905.

[7] F. Grassi, Y. Hama and T. Kodama, Phys. Lett. B **355** (1995) 9.

[8] Y.M. Sinyukov, S.V. Akkelin and Y. Hama, Phys. Rev. Lett. **89** (2002) 052301.

[9] S.A. Bass *et al.*, Prog. Part. Nucl. Phys. **41** (1998) 225.

[10] M. Murray and B. Holzer, Phys. Rev. C **63** (2001) 054901.

[11] M.M. Aggarwal [WA98 Collaboration], Phys. Rev. C **67** (2003) 014906.

[12] R.L. Ray for the STAR Colaboration, Nucl. Phys. A **715** (2003) 45c.

[13] J. Bächler *et al.* [NA49 Collaboration], Nucl. Phys. A **661** (1999) 45.

[14] S. V. Afanasiev *et al.* [NA49 Collaboration], Phys. Rev. C **66** (2002) 054902.

[15] K. Adcox *et al.* [PHENIX Collaboration], Phys. Rev. Lett. **88** (2002) 242301.

[16] B. Tomášik, U.A. Wiedemann and U.W. Heinz, Heavy Ion Phys. **17** (2003) 105.

[17] B. Tomášik, arXiv:nucl-th/0304079.

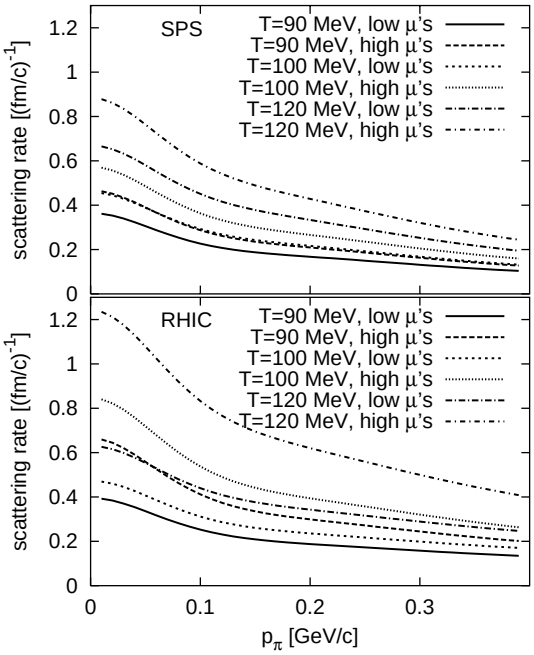

Figure 2: The pion scattering rates as functions of momentum with respect to the medium, calculated for different temperatures and sets of chemical potentials allowed by data. Details can be found in the original paper [6].