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Cosmic Ray Spectrum

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Abstract

The doubled-humped spectrum of cosmic ray particles is analyzed using a lognormal distribution derived for Fermi's mechanism of acceleration to be joined tangentially by Fermi's power law It is found that for protons and electrons measured by the AMS Collaboration, the power law is universal and the spectral index is critical, that upward protons and downward protons in the 1^{st} hump are accelerated in the same way by shock waves in the monoatomic cosmic gas with compression ratio r $\simeq$ 5/3, whereas for protons in the 2^{d} hump, r $\simeq$ 8/3 due to shock waves jump For electrons, $r \simeq 7/3$ shock wave acceleration takes place in diatomic gas

1 Introduction

The salient feature of the energy spectrum of high energy cosmic ray particles is a knee, which marks the beginning of the power law behavior $1/E^{\gamma}$ formulated by Fermi on the basis of collisions with geomagnetic fields [1] Besides this mechanism, acceleration by shock waves is also possible as pointed out by Landau [2] As energy gain expressed by energy-momentum conservation in this case is formally the same as in the case of Fermi model, this simple power law holds also for models of shock wave acceleration [3 - 7]

As is well known, the spectral index is universal $\gamma = 2\ 78 \pm 0\ 02$ for high altitude primary cosmic ray protons in energy range 0 2 to 200 GeV, according to recent experiments of the AMS Collaboration [8] Whereas the same index $2\ 80 \pm 0\ 05$ is found for protons of $E_p = 0\ 5$ to 50 TeV at 2 km sea level by the EAS-TOF Collaboration [9, 10]

We will see that this power law envelops the differential spectra at various geomagnetic latitudes measured by the AMS Collaboration [8] Therefore it represents an asymptotic distribution and that γ is a critical index for shock waves However, the power law covers only a tiny fraction of the spectrum after the knee It is a challenge to account for the remaining part of the spectrum In this regard, we note that the cosmic ray spectrum in the log-log plot as customarily presented in the literature, has a parabolic shape This implies a lognormal distribution, i e Kolgomorov's distribution for stochastic processes [11] It proves to be adequate to describe various spectra of cosmic ray particles, except the tail near the end of the spectrum, as reported before [12]

In this paper, we present a derivation of the lognormal distribution from the basic formula for energy gain of the Fermi model [1] and propose to correct the tail by the power law (Sec 2) We will use these two distributions to analyze the integral and the differential spectra of cosmic ray particles, downward protons as well as upward protons and electrons of the AMS Collaborations [8, 15]

2 Extension of the power law

Let us, following Fermi, consider energy gain of a cosmic ray particle of energy x by a collision with geomagnetic field moving with random velocity V,

$$\triangle x = xV^2 \tag{1}$$

The variate of its energy after n successive collisions is

$$X_n = X_1(1 + V^2x)^n = X_1[1 + n(V^2x) + \frac{n(n-1)}{2!}(V^2x)^2 + \quad] \tag{2}$$

If $n(V^2x)$ remains finite for $x \to \infty$, then by Stirling's formula

$$n(n-1) \quad (n-k+1) = \frac{n!}{(n-k)!} \simeq n^k$$

Therefore we may approximate the binomial series (2) by an exponential expansion and rewrite (2) as

$$X_n = X_1[1 + (nV^2x) + \frac{(nV^2x)^2}{2!} + \quad],$$

to get

$$X_n = X_1 e^{nV^2x} \tag{3}$$

Therefore

$$\text{Log X}_n = \text{Log X}_1 + n(V^2x), \tag{4}$$

and the spectrum is a lognormal distribution with variance

$$\text{L} = \frac{1}{2V^2},$$

according to Kolgomorov's theory of stochastic processes [13]

Consequently, the lognormal distribution may be used in conjunction with the power law to describe the entire spectrum As kinematic variable, instead of energy, we use rather momentum and set

$$\zeta = \text{LogP}, \tag{5}$$

which represents the phase space of the particle under consideration The parameters of these two equations are defined as follows

$$\frac{\text{dn}}{\text{dP}} = \text{N}e^{-(\zeta+\zeta^*)^2/2L}, \tag{6}$$

where ζ^* is the maximum shift, L is the width of the spectrum and N is the normalization coefficient, and

$$\frac{\text{dn}}{\text{dP}} = \text{C} \quad \text{P}^{-\gamma}, \tag{7}$$

where γ is the spectral index and C is another normalization coefficient The two distributions (6) and (7) are joined tangentially with the slope equal to γ at the point of momentum determined by

$$\text{Log P}_\times = 2\ 303 - \text{L}\gamma - \zeta^*, \tag{8}$$

so that the lognormal distribution may be continued by the power law disbtribution

Finally, it is to be noted that the fundamental equation for energy gain, Eq (1), is essentially the same as in the case of acceleration by shock waves [14] and that in this case, the spectral index depends on the adiabatic index of the medium in which propagate the shock waves we will discuss this property in Sect 5

3 The spectrum of cosmic ray protons

We now analyze cosmic ray protons, downward and upward, measured at high altitude 380 km and geomagnetic latitude between 0 and 1 rad by the AMS Collaboration [15] Let us begin with the integrated momentum distribution as shown in Fig 1, composed of the differential distributions with $\Delta\theta = 0\ 2$ rad in Fig 2 The solid curve represents the power law fit for P > 23 08 GeV/c, the parameters are (C in 10^3 protons/(m^2 sec sr MeV))

$$\gamma_{\rm p} = 2\ 781 \pm 0\ 030, \quad {\rm C_p} = 99\ 8 \pm 9\ 8$$

The spectral index γ_p here estimated agrees well with $2\ 80 \pm 0\ 02$ reported by the AMS Collaboration [8] within $\sim 1/2$ standard deviation

The dotted line represents least-squares fit with the lognormal distribution, the parameters are (N in 10^3protons /(m^2 sec sr MeV))

$$\zeta_p^* = 0\ 1001 \pm 0\ 017, \quad {\rm L}_p = 0\ 257 \pm 0\ 002, \quad {\rm N}_p = 1\ 091 \pm 0\ 004$$

A comparison with the data indicates that the fit is rather satisfactory except the tail The deviations in the low momentum region are probably due to the jacobian dE/dP = P/E, since they do not appear in the dn/dK plot of the AMS Collaboration [8]

However, the deviations near the end of the spectrum are systematic, characteristic of the lognormal distribution Now, the tail may be corrected by the power law as shown in the figure by the solid line, the two distributions are in contact tangentially at the point marked by a cross corresponding to

$$\rm P_\times = 35\ 07 \pm 1\ 88\ GeV/c$$

as determined by Eq (8) The slope of the tangent according to the lognormal fit is

$$\frac{{\rm Log\ P_\times} - \zeta^*}{2\ 303\,{\rm L}} = 2\ 781 \pm 0\ 028$$

exactly equal to γ_p of the power law fit as mentioned above

By integration, we find for the flux

$$\phi_p = (4\ 81 \pm 0\ 04)10^3, \ {\rm protons}/(m^2 sec\, sr)$$

and the average momentum

$$< {\rm P} >_p = 6\ 343 \pm 1\ 830 \ {\rm GeV/c}$$

Consider next the components of the integrated proton spectrum we have analyzed above, namely the differential spectrum as a function of the latitude presented in the log-log plot of Fig 2 As a fit to the double-humped spectrum requires a two-lognormal distribution, we therefore write

$$\frac{\mathrm{dn}}{\mathrm{dP}} = \sum_{i=1}^{2} \mathrm{N}_i \, e^{-(\zeta+\zeta_i^*)^2/2L}, \tag{9}$$

where the suffix refers to the regions of the 1^{st} and the 2^{d} maximum The parameters of least-squares fits are summarized in Table I, together with the parameters of the power law fits in the 2^{d} region Some of the fits are shown in Fig 2, others being omitted for clarity Note that at $\theta = 0\ 95$ rad, the 1^{st} maximum is so attenuated that a single lognormal distribution fits rather well the entire spectrum, as shown by the solid line in the figure

As regards the power fits, we find this remarkable property of universality that the spectral indices and the normalization coefficients are practically all the same within ≤ 2 s d , their mean values are

$$\overline{\gamma_{p\,dn\,2}} = 2\ 768 \pm 0\ 024, \quad \overline{\mathrm{C}_{p\,dn\,2}} = (16\ 11 \pm 1\ 56)10^3 \tag{10}$$

where the suffix dn stands for downward We find the same spectral index $\overline{\gamma}$ as in the case of integrated spectrum mentioned in Sect 2

Furthermore, this average power law as shown in the figure envelops all the differential spectra with $\theta = 0\ 05$ to $0\ 95$ rad Note that $C/\overline{\mathrm{C}_{p\,dn\,2}} = 6\ 18 \pm 0\ 85$ consistent with 6, namely the number of differential distributions constituting the integral distribution of Fig 1

As for the latitude dependence, we find ζ_1^* in the 1^{st} region increases with θ, whereas ζ_2^* decreases with θ, so that the two maxima move in the opposite direction On the other hand, the width parameter L depends weakly on ϑ for both lognormal distributions in the 1^{st} and 2^{d} region

The average momentum of downward protons has been computed according to the fits For $\theta = 0\ 95$ rad, we find $2\ 592 \pm 0\ 392$ GeV/c For other latitudes,we have to evaluate $< \mathrm{P} >$ separately in the 1^{st} and 2^{d} region according to $\mathrm{P} < \mathrm{P}_{min}$ or $\mathrm{P} > \mathrm{P}_{min}$ The values thus obtained are presented in Fig 3 by nablas and circles, respectively The solid lines are linear fits with

$$< \mathrm{P} >= a(\theta - b) \tag{11}$$

The parameters a (in GeV/c/radian) and b (in radian) are

$$a_{pdn1} = -0\ 62 \pm 0\ 12, \quad b_{pdn1} = -2\ 04 \pm 0\ 31 \quad \mathrm{P} < \mathrm{P}_{min},$$

$$a_{pdn2} = -12\ 51 \pm 3\ 46, \quad b_{pdn2} = -1\ 72 \pm 0\ 37 \quad \mathrm{P} > \mathrm{P}_{min}$$

As both a's are different from zero, there is a latitude effect on the acceleration of protons On the other hand, the b's are practically the same within about 1 s d , so that at a given latitude, the ratio of the average momentum of protons in the two regions is practically equal to 20, independent of the latitude

4 The upward cosmic ray protons

The AMS Collaboration has also measured the upward protons In Fig 4 are shown the differential spectra, which are analyzed by the lognormal distribution (6) The parameters are summarized in Table II, together with the estimates of $< \mathrm{P} >$, some of the least-squares fits are shown by the solid curves in the figure

These estimates as a function of the latitude θ (in radians) are shown by triangles in Fig 3 They coincide with those of downward protons (in nablas) with $\mathrm{P} < \mathrm{P}_{min}$, i e in the 1^{st} maximum region of the differential spectrum in Fig 2 The solid line represents a linear fit according to Eq (11), the parameters are

$$a_{up} = -0\ 753 \pm 0\ 077, \ \ b_{up} = -11\ 727 \pm 0\ 129$$

A comparison with the downward protons of the 1^{st} maximum region indicates

$$a_{up} \simeq a_{pdn1}; \ \ b_{up} \simeq b_{pdn1}$$

Therefore the upward protons together with the downward protons in the 1^{st} maximum region of the differential spectrum in Fig 2 belong to the same family, they are accelerated in the same way by the geomagnetic fields On the other hand, the downward protons in the 2^{d} region are accelerated to much higher energy We will discuss this point in the next section

Furthermore, the value of b_{up} is comparable to those of downward protons of both the 1^{st} and the 2^{d} maximum regions, b_{pdn1} and b_{pdn2} mentioned above (Sect 3) Therefore the threshold of acceleration is given by the mean value of these b's

$$\bar{b} = 1\ 829 \pm 0\ 205 \text{ rad} = 104\ 8 \pm 11\ 5 \text{ degrees}$$

consistent with $\pi/2$, i e the zenith direction along which has been oriented the positive z-axis of the AMS setup [8]

As the rate $a = \triangle < \mathrm{P} > /\triangle\theta \neq 0$, there is, in general, a latitude effect of slight anisotropy due to the relative motion of the system of observation with respect to the shock wave frame, where the accelerated protons are expected to be isotropic, in view of the invariance of the formula for energy gain, Eq (1)

This difference in acceleration with the downward protons in the 2^{d} maximum region of the differential spectrum implies a difference in spectral index of the power law for the integrated spectrum Indeed, we have already seen in the case of protons in the 2^{d} region of the spectrum in the last section Let us consider the integrated spectrum of upward protons as shown in Fig 5 The mixed line shows the lognormal fit with

$$\zeta^{*}_{up} = 0\ 416 \pm 0\ 018, \ \ \mathrm{L}_{up} = 0\ 090 \pm 0\ 002, \ \ \mathrm{N}_{up} = 0\ 307 \pm 0\ 017$$

Whereas the solid curve represents a power law fit with

$$\gamma_{up} = 5\ 672 \pm 0\ 104, \ \ \mathrm{C}_{up} = 2\ 520 \pm 0\ 442$$

The cross marks the tangent point, its coordinates according to Eq (8) are

$$\mathrm{P}_\times = 4\ 592\ \mathrm{GeV},\ \ \frac{\mathrm{dn}}{\mathrm{dP}} = 5\ 21710^{-4}$$

We have also shown by nablas in the same figure, the downward protons in the 1^{st} maximum region and by the dotted line, the lognormal fit with the parameters

$$\zeta^*_{pdn1} = 0\ 345 \pm 0\ 018,\ \ \mathrm{L}_{pdn1} = 0\ 084 \pm 0\ 003,\ \ \mathrm{N}_{pdn1} = 0\ 253 \pm 0\ 010$$

comparable to those of upward protons mentioned above Lack of data, it is not possible to make a reliable estimate of γ_{pdn1} However, as their flux are the same,

$$\phi_{up} = 191\ 0 \pm 10\ 6,\ \ \phi_{up} = 190\ 4 \pm 7\ 5$$

We therefore assume their spectral indices to be the same Their relationship with the index of the integrated spectrum analyzed in Sect 3 is as follows

$$\gamma_{up} = \gamma_{pdn\,1} \simeq 2\gamma_p \tag{12}$$

We will discuss this important property of cosmic ray acceleration in the next section

5 Properties of the shock wave acceleration

We have seen the universality property of the simple power law for the cosmic ray spectrum For protons in the momentum region of the 2^d maximum, the spectral index is found to be independent of the latitude, in spite of the complexity of the double-humped structure of the differential spectrum

However, for protons in the 1^{st} maximum region, the spectral index is about twice larger, the same is true for upward protons This difference is due to the difference in the acceleration attributed to the *jump* of shock waves [5] Indeed, the shock wave acceleration predicts also a power law of the form (see Ref [14])

$$\frac{\mathrm{dn}}{\mathrm{dP}} \sim \mathrm{P}^{-(r+2)/(r-1)} \tag{13}$$

where $r = v_1/v_2$ is the compression ratio of the shock, v_1 and v_2 being the upstream and the downstream velocity Therefore r is related to the spectral index γ of Fermi's power law (7) by a homographic transformation, namely

$$r = \frac{\gamma + 2}{\gamma - 1} \tag{14}$$

For the upward protons and the downward protons in the 1^{st} maximum region, $\gamma_{up} = 5\ 672 \pm 0\ 104$, therefore

$$r_{pdn1} = 1\ 642 \pm 0\ 043 \simeq \frac{5}{3},$$

i e equal to the adiabatic index of monoatomic cosmic gas It is interesting to note that the power law in this case is just like Kolpogomorov's law for the isotropic turbulence [16]

For the downward protons in the 2^d maximum region, $\gamma_{pdn\,2} = \gamma_p = 2\ 801 \pm 0\ 031$, therefore

$$r_{pdn\,2} = 2\ 660 \pm 0\ 043 = 1 + r_{pdn1}$$

implying

$$v_1 \to v_1 + v_2, \tag{15}$$

i e , the upstream velocity acquires a *jump* from the downstream velocity

Finally, for negative particles, namely electrons, the spectrum measured by the AMS Collaboration [17] is shown in Fig 6 The dotted line represents a lognormal fit with

$$\zeta_e^* = 0\ 137 \pm 0\ 004, \ \ \mathrm{L}_e = 0\ 124 \pm 0\ 001, \ \ \mathrm{N}_e = 25\ 0 \pm 1\ 1$$

and the solid line corresponds to a power law fit with

$$\gamma_e = 2\ 340 \pm 0\ 135, \ \ \mathrm{C}_e = 296 \pm 35$$

We find $\gamma_e \gg \gamma_p$ and

$$r_e = 2\ 340 \pm 0\ 135 \simeq \frac{7}{3}$$

which is the adiabatic index of diatomic gas

It is interesting to note that the width parameter is related to that of protons mentioned in Sect 2, namely

$$\mathrm{L}_e = \frac{1}{2}\ \mathrm{L}_p,$$

as in the case of protons and electrons observed in inclusive e^+e^- annihilations as reported before [12(b)] This property implies that cosmic ray electrons arise from decays of $\pi^o \to \gamma + \gamma$ then $\gamma \to e^+ + e^-$ and that equipartition of energy prevails also in the acceleration of cosmic ray particles

6 Conclusion

A lognormal distribution Eq (6) has been derived on the basis of the fundamental formula for energy gain of the Fermi mechanism for cosmic ray particle acceleration [1] This lognormal distribution intercepts tangentially Fermi's power law distribution Eq (7) Their application allows us to analyze the entire spectrum, especially the double-humped differential spectrum of cosmic ray protons and electrons measured by the AMS Collaboration [3, 8, 17]

As regards the spectral index of integrated proton spectrum, it is found $\gamma_p = 2\ 781 \pm 0\ 030$ The average momentum of proton is $< \mathrm{P} >_p = 6\ 343 \pm 1\ 830$ GeV/c and the flux $\phi_p = (4\ 81 \pm 0\ 04)10^3$ in protons/$(m^2$ sec sr)

For the downward protons in the 2^d maximum region of the differential spectrum in Fig 2, the power law is practically independent of the latitude (cf parameters in Table I) The average $\gamma_{pdn2} = 2\ 768 \pm 0\ 024$ is equal to γ_p. However, the the normalization coefficients are quite different, their average amounts to $\simeq 1/6$ of C_p of Fig 1, because the latter has been integrated over 6 components of the differential spectrum in Fig 2 Therefore, the power law represents a limiting distribution, it envelops all the differential spectral as shown in Fig 2

The average momentum of protons estimated according to the lognormal fits is shown in Fig 3, circles for protons in the 2^d maximum region and nablas for protons in the 1^{st} maximum region, together with $< P >$ of upward protons listed in Table II The solid lines represents linear fits with Eq (11) There is a latitude effect, due to the motion of the observation with respect to the geomagnetic fields It is found $< P > \to 0$ as $\theta \to \pi/2$, i e the zenith direction, as should be

The upward protons are shown in Fig 4, the parameters of lognormal fits and the average momentum are listed in Table II Their integrated spectrum is shown by triangles in Fig 3 It is the same as that in nablas for downward protons in the 1^{st} maximum region of Fig 2 Therefore these upward and downward protons are accelerated in the same way, but different from the downward protons in the 2^d maximum region of Fig 2

Therefore, the double-humped structure of the differential spectrum of protons in Fig 2 reflects the difference in their acceleration As the power law holds also for shock wave acceleration, in this case, the exponent is determined by the compression ratio of shock waves, known as the adiabatic index r, and may be expressed in terms of the spectral index γ, Eq (14) We find for upward protons and downward protons in the 1^{st}, $r_{pdn1} = 1\ 642 \pm 0\ 043 \simeq 5/3$ of monoatomic cosmic gas On the other hand, for downward protons in the $2d$ maximum region, $r_{pdn2} = 2\ 660 \pm 0\ 043 \simeq 1 + r_{pdn1}$ so that there is a *jump* of shock wave acceleration as pointed out by Bell [5]

Finally, as for negative ions, namely electrons of the AMS measurements [17], as shown in Fig 6, we find $\gamma_e = 3\ 239 \pm 0\ 053$ and $r_e = 2\ 340 \pm 0\ 135 \simeq 7/3$ equal to adiabatic index for diatomic cosmic gas The width of the electrons spectrum is much narrower, about one half of that of the proton spectrum, indicatin that they are decays of neutral pions, a property of the lognormal distribution for decay particles

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References and Fotnotes

[1] E Fermi, Phys Rev 75, 1169 (1949) and Astrophys J 119,1 (1954)

[2] L D Landau, Izv Akad Nauk SSSR,sr fiz 17, 51 (1953), footnote

[3] W I Axford, E Lear and G Skadron, Proc 15th Int Cosmic Ray Conf, Plovliv, vol 11, 132 (1977) Budapest, Center Resaerch for Physics

[4] R D Blandford and J P Ostriker, Astrophys J 211, L29 (1978)

[5] A R Bell, Mon Note R Astr Soc 182, 147 (1978)

[6] F C Michel, Astrophys J 247, 664 (1981)

[7] L O'C Drury, Rep Prog Phys vol 46, 973 (1983)

[8] AMS Collaboration J Alcraz et al , Phys Lett B 490, 27 (2000) and ibid 472, 215 (2000)

[9] EAS-TOP Collaboration M Agliweetta et al , AstroPhys J 19, 329 (2003)

[10] J R Horondel, Astrophys L 19, 193 (2003)

[11] A N Kolgomorov, C R Acak Sci (Doklady) URSS, XXXI, 99 (1941)

[12] T F Hoang, (a) Scaling particle-antiparticle in cosmic rays LBNL-49064 92002) and (b) Cosmic ray plasma of leptons, protons and ^{4}He LBNL-52116 (2002)

[13] M G Kendall and A Stwart, The Advanced Theory of Statistics, Hafner Publ Co , vol 1 (1969) p 169 et sq

[14] B T Draine and C F McKee, Ann Rev Astrophys 31, 373 (1993)

[15] AMS Collaboration, Phys Lett B, 472, 215 (2000)

[16] A N Kolgomorov, Dokl Akad Nauk SSR 30, 299 (1941)

[17] AMS Collaborationm J Alcaraz et al , Phys Lett B, 484, 10 (2000)

Table I - Parameters of the two-lognormal and the power law fits to the differential spectra of high altitude downward protons at latitudes θ (in radians)with Eqs (7) and (9), the normalization coefficients N and C are in protons (m^2 sec sr MeV)

θ i	ζ_i^*	L_i	N/10^3	C/10^3	γ
0 05 1	0 150 0 012	0 068 ± 0 002	54 4 ± 1 2		
2	*−1 211 ± 0 001*	*0 016 ± 0 006*	*6 1 ± 0 01*	*2 792 ± 0 021*	*19 05 ± 1*
0 25 1	0 232 ± 0 022	0 063 ± 0 050	30 8 ± 1 8		
2	*−1 246 ± 0 002*	*0 020 ± 0 003*	*5 2 ± 0 1*	*2 732 ± 0 022*	*16 86 ± 0*
0 45 1	0 347 ± 0 043	0 076 ± 0 007	30 5 ± 3 2		
2	*−1 072 ± 0 003*	*0 022 ± 0 001*	*12 0 ± 0 01*	*2 765 ± 0 007*	*15 67 ± 1*
0 65 1	0 345 ± 0 039	0 053 ± 0 006	41 7 ± 4 1		
2	*−0 825 ± 0 010*	*0 017 ± 0 002*	*41 7 ± 4 1*	*2 780 ± 0 024*	*15 95 ± 1*
0 85 1	0 417 ± 0 077	0 066 ± 0 025	49 3 ± 4 8		
2	*−0 814 ± 0 008*	*0 016 ± 0 002*	*54 7 ± 3 0*	*2 795 ± 0 027*	*15 95 ± 1*
0 95 1	-0 059 ± 0 005	0 101 ± 0 003	964 ± 11	2 744 ± 0 014	13 84 ± 0

Table II - Geomagnetic dependence of upward protons at altitude 380 km above earth Parameters of lognormal fits to the differential spectra measured by the AMS Collaboration [15], the normalization coefficient N in protons(m^2 sec sr MeV)

θ	ζ^*	L	N/10^3	< P > GeV/c
0 05	0 156 ± 0 011	0 069 ± 0 002	0 052 ± 0 001	1 259 ± 0 004
0 25	0 303 ± 0 036	0 082 ± 0 007	0 033 ± 0 002	1 077 ± 0 005
0 45	0 830 ± 0 033	0 076 ± 0 009	0 080 ± 0 010	1 006 ± 0 005
0 65	0 870 ± 0 198	0 125 ± 0 028	0 165 ± 0 114	0 735 ± 0 008
0 85	0 526 ± 0 087	0 053 ± 0 050	0 153 ± 0 054	0 637 ± 0 010
0 95	0 673 ± 0 165	0 074 ± 0 019	0 248 ± 0 169	0 644 ± 0 009

Figure Captions

[1] Integrated spectrum of downward protons at high altitude 380 km measured by the AMS Collaboration [8] The solid curve represents a power law fit with Eq (7), the spectral index is $\gamma = 2\ 781 \pm 0\ 030$ The dotted line represent a lognormal fit according to Eq (6), the width is $L_p = 0\ 257 \pm 0\ 001$ The two fits are in contact tangentially, marked by a cross, the slope of the tangent being equal to the spectral index γ_p

[2] Differential spectrum of downward protons at high altitude 380 km as a function of latitude θ (in radian), data of the AMS Collaboration [8] The solid curves represent least-squares fits with a two-lognormal distribution Eq (9) The parameters are summarized in Table I, together with the power law fits, which are practically the same, the spectral index being consistent with γ_p of the integrated spectrum

[3] Plots of the average momentum of downward and upward protons as a function of the latitude, AMS Collaboration [8] An latitude effect is shown by $< P >= a(\theta - b)$, due to the relative motion of the observation with respect to the geomagnetic field The parameter b is practically the same $b = 1\ 829 \pm 0\ 725$ rad $\simeq \pi/2$, i e the direction towards the zenith

[4] Differential spectrum of upward protons of the AMS Collaboration [8] The curves are fits with the lognormal distribution Eq (6) The parameters are summarized in Table II, together with the estimated average momentum The latitude dependence of $< P >$ is shown by triangles in Fig 3

[5] Integrated spectrum of upward protons (in triangles) and downward protons (in nablas)in the 1^{st} maximum region in Fig 2 The curves represent power law and lognormmal fits, Eqs (6) and (7) They are practically the same, the spectral index $\gamma_{pup} = 5\ 672 \pm 0\ 104 \simeq 2\gamma_{pdn2}$ the downward protons in the 2^d maximum region

[6] Integrated spectrum of electrons of the AMS Collaboration [17] The curves are fits with the power law and the lognormal distribution The spectral index $\gamma_e = 2\ 340 \pm 0\ 135$ The width $L_e = 0\ 124 \pm 0\ 001 \simeq \frac{1}{2} L_p$ of protons in Fig 1, indicating the electrons are from decays of π^o and that energy equipartition holds in the acceleration of cosmic ray particles, see text

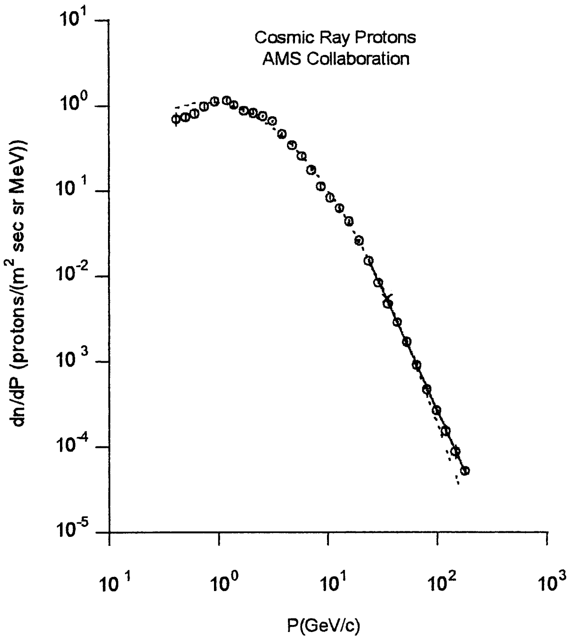

Cosmic Ray Protons
AMS Collaboration
dn/dP (protons/(m² sec sr MeV))
P(GeV/c)

Fig 1

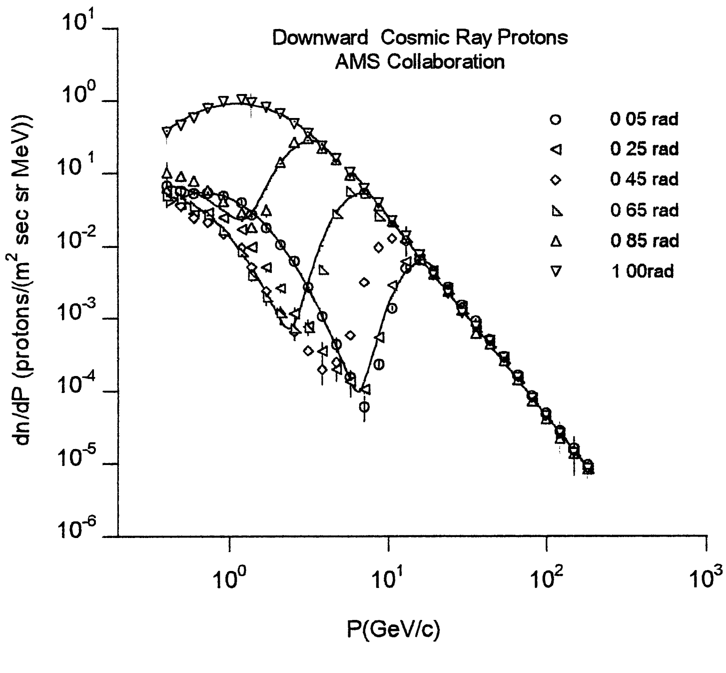
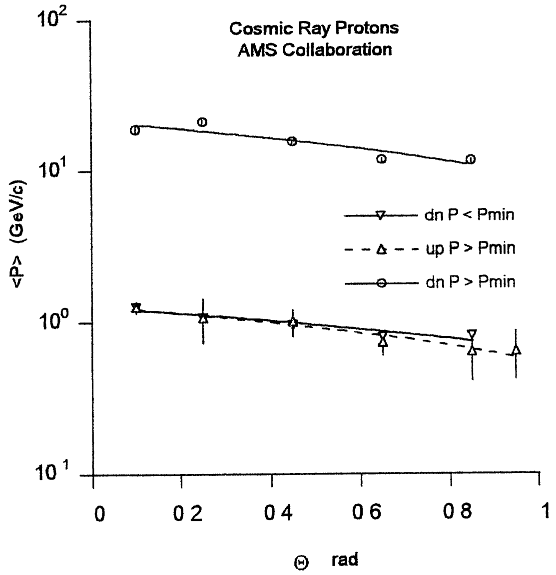
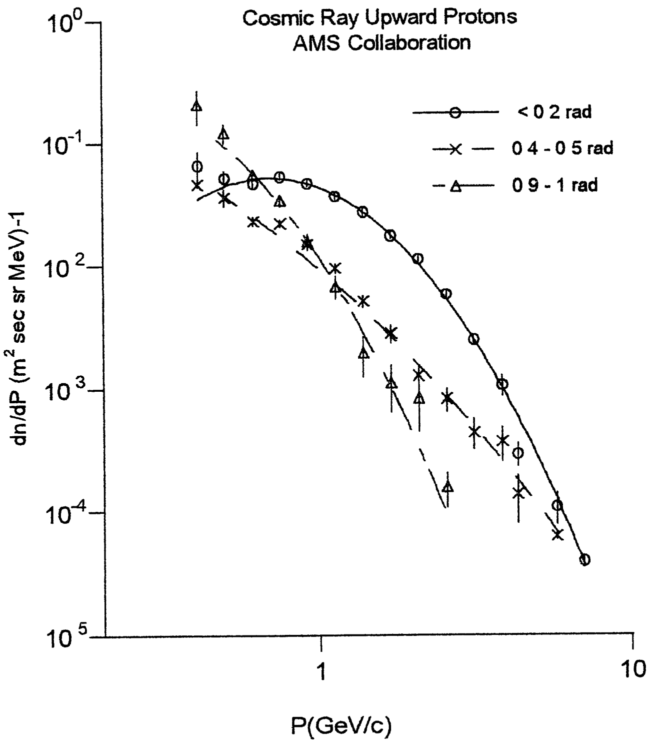

Downward Cosmic Ray Protons
AMS Collaboration
0 05 rad
0 25 rad
0 45 rad
0 65 rad
0 85 rad
1 00rad
dn/dP (protons/(m² sec sr MeV))
P(GeV/c)

Fig 2



Cosmic Ray Protons
AMS Collaboration
10^2
10^1
10^0
10 1
<P> (GeV/c)
dn P < Pmin
up P > Pmin
dn P > Pmin
0
0 2
0 4
0 6
0 8
1
Θ rad

Fig 3



Cosmic Ray Upward Protons
AMS Collaboration
< 0 2 rad
0 4 - 0 5 rad
0 9 - 1 rad
10^0
10^{-1}
10 2
10 3
10^{-4}
10 5
dn/dP (m^2 sec sr MeV)-1
1
10
P(GeV/c)

Fig 4

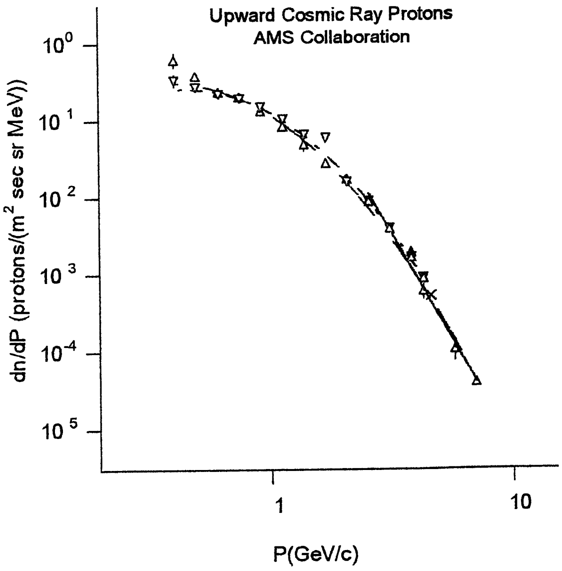

Upward Cosmic Ray Protons
AMS Collaboration
dn/dP (protons/(m^2 sec sr MeV))
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
1
10
P(GeV/c)

Fig 5

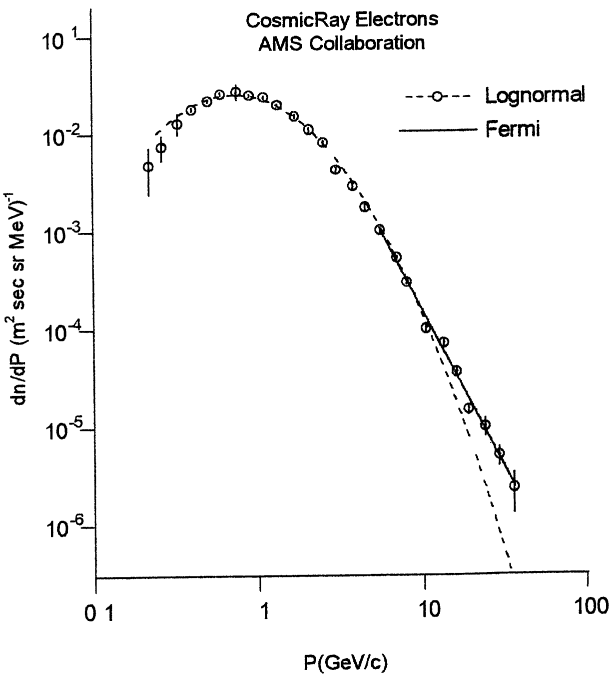

CosmicRay Electrons
AMS Collaboration
Lognormal
Fermi
dn/dP (m^2 sec sr MeV)$^{-1}$
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
0.1
1
10
100
P(GeV/c)

Fig 6