A new, exact, gauge-invariant RG-flow equation

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We define a simple scale-dependent effective action and derive the exact RG-flow equation it obeys. We then use the Vilkovisky-De Witt geometrical approach to improve the previous definition and obtain a reparametrization-invariant RG-flow equation. When extended to gauge theories, our procedure provides a manifestly gauge-invariant, exact RG-flow equation.

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The idea of renormalization, originally introduced to remove infinities from perturbative calculations, has evolved into a powerful tool that helps understanding the global behaviour of quantum and statistical systems under changes of the observation scale [1,2].

The search for new, non-perturbative methods to handle problems out of the reach of perturbation theory has prompted in recent years a renewed and growing interest [3–7] in the "old" subject [1,8] of "exact" renormalization group (RG) equations. One typically defines a scale-dependent effective action, Γ_k, which interpolates between the classical (bare) action S at $k = \Lambda$ (the UV cutoff) and the effective action Γ at $k = 0$. The free term is modified by the introduction of a suitable (but largely arbitrary) cutoff function that effectively kills the contribution to the functional integral from momenta below the running scale k. The implementation of such a procedure for gauge theories poses however a major problem: the presence of the cutoff function prevents the possibility of defining a gauge invariant Γ_k (see [9] for earlier attempts to circumvent this problem).

In this letter we present a new definition of Γ_k and derive the exact RG-flow equation it obeys by following the spirit of Ref. [1,8], i.e. with the help of direct integration over successive shells of degrees of freedom. For the pourposes of the present work, the main advantage of our procedure is that it can be extended in a very natural way through the geometrical approach pioneered by Vilkovisky and De Witt [10,11] (see also [12]) in order to define a gauge-invariant, more generally a reparametrization-invariant (see below), scale-dependent effective action.

To illustrate our procedure, and to set up the tools for our following analysis, we begin by defining Γ_k for a simple scalar field theory. If Λ is the UV cutoff, we introduce the notation ϕ_0^Λ for the field, to indicate that it contains "modes" in the range $[0, \Lambda]$, and write the classical (bare) action as $S[\phi_0^\Lambda]$. For any given scale k, we divide ϕ_0^Λ into the "low-frequency" and "high-frequency" components, ϕ_0^k and ϕ_k^Λ respectively, where ϕ_0^k contains the modes ϕ_p with $0 < p < k$, and ϕ_k^Λ those in the range $[k, \Lambda]$. Even though for the scalar theory it is always possible to define the RG flow in Fourier space, it is well known that the notion of RG flow is much more general. Neither k nor Λ must necessarily have the meaning of momenta (this observation is important for the following where we have to implement a gauge invariant flow for gauge theories).

Let us now introduce the notion of "shell", described by δk, denote the fields $\phi_0^{k-\delta k}$, $\phi_{k-\delta k}^k$ and ϕ_k^Λ by $\phi_<$, ϕ_S and $\phi_>$, respectively, and use de Witt's [11] condensed notation whereby an index such as i denotes all indices (Fourier, Lorentz, spinor, space-time coordinate x, ...). Repeated indices will denote summation over internal indices as well as integration over space-time (or momenta). The components of ϕ_S and $\phi_>$ will be indicated by ϕ^s and ϕ^a (same for $\bar\phi$), and differentiation w.r.t. any ϕ^i ($\bar\phi^i$) by a comma followed by the index i. Later on we will also use $A, B, \dots$ to denote fields with components in the slightly larger interval $[k - \delta k, \Lambda]$.

The effective action $\Gamma[\bar\phi]$, a functional of the "classical" (or "mean") fields $\bar\phi$, can be defined as the solution of the functional-integral equation:

$$e^{-\Gamma[\bar\phi]} = \int [D\phi] e^{-S[\phi] + (\phi^i - \bar\phi^i)\Gamma[\bar\phi]_{,i}} . \tag{1}$$

The scale-dependent generalization of (1) that we propose to use, and later generalize, is simply obtained from (1) after inserting under the integral a product of δ-functions, $\Pi_0^k \delta(\phi_p - \bar\phi_p)$, i.e.:

$$e^{-\Gamma_k[\bar\phi]} = \int [D\phi_>] e^{-S[\bar\phi_<, \phi_>] + (\phi^a - \bar\phi^a)\Gamma_k[\bar\phi]_{,a}} . \tag{2}$$

This very natural definition of a scale-dependent effective action clearly interpolates between the classical and the quantum action, $\Gamma_\Lambda[\bar\phi] = S[\bar\phi]$ and $\Gamma_0[\bar\phi] = \Gamma[\bar\phi]$, and can be obtained by a partial Legendre transform [13] of a functional $W_k[\bar\phi_0^k, J_k^\Lambda]$ in which the low-frequency fields $\bar\phi_0^k$ are kept as parameters, while the high frequency degrees of freedom are Legendre-transformed.

We now derive some identities that will be useful in the following. By differentiating Γ_k in Eq.(2) w.r.t. $\bar\phi^a$, we find (for a non-singular 2nd-derivative matrix of Γ_k)

$$< \phi^a >= \bar{\phi}^a \ , \tag{3}$$

where the average is computed with the weight in Eq. (2). Thus, as we expect, $\bar{\phi}^a$ is the mean value of ϕ^a. Differentiating Eq.(3) w.r.t. $\bar{\phi}^s$ we get:

$$\begin{aligned} &< S(\bar{\phi}_<, \bar{\phi}_S, \phi_>)_{,s}(\phi^a - \bar{\phi}^a) > \\ &= \Gamma_{k,\,sb} < (\phi^b - \bar{\phi}^b)(\phi^a - \bar{\phi}^a) >= \Gamma_{k,\,sb}(\Gamma_{k,ba})^{-1} \ , \end{aligned} \tag{4}$$

where $-(\Gamma_{k,ba})^{-1}$ is the propagator for modes above the shell. A second useful relation comes from differentiating Γ_k w.r.t. $\bar{\phi}^s$:

$$< S(\bar{\phi}_<, \bar{\phi}_S, \phi_>)_{,s} >= \Gamma_{k,\,s} \ . \tag{5}$$

Finally, differentiating Γ_k once more w.r.t. $\bar{\phi}^s$, and making use of Eq.(4), we obtain the relation:

$$\begin{aligned} &< S_{,ss'} > - < S_{,s} S_{,s'} > + < S_{,s} >< S_{,s'} > \\ &= \Gamma_{k,\,ss'} - \Gamma_{k,\,sa}(\Gamma_{k,ab})^{-1}\Gamma_{k,\,bs'} \ . \end{aligned} \tag{6}$$

Let us consider now the effective action Γ_k at a slightly lower scale $k - \delta k$. From Eq.(2) we have :

$$e^{-\Gamma_{k-\delta k}[\bar{\phi}_0^\Lambda]} = \int [D\phi_S] e^{(\phi^s - \bar{\phi}^s)\Gamma_{k-\delta k,\,s}} e^Y \ , \tag{7}$$

where

$$e^Y = \int [D\phi_>] e^{-S[\bar{\phi}_<, \phi_S, \phi_>] + (\phi^a - \bar{\phi}^a)\cdot\Gamma_{k-\delta k,a}} \ . \tag{8}$$

We are interested in computing the difference between Γ_k and $\Gamma_{k-\delta k}$ to $O(\delta k)$ and thus start expanding to first order $\Gamma_{k-\delta k,a}$ around $\Gamma_{k,a}$ in Eq.(7). At the same time we expand $S[\bar{\phi}_<, \phi_S, \phi_>]$ around $\phi_S = \bar{\phi}_S$. Denoting the fluctuations $(\phi^s - \bar{\phi}^s)$ and $(\phi^a - \bar{\phi}^a)$ by η^s and η^a respectively, we get :

$$e^Y = e^{-\Gamma_k} < e^{-[S_{,s}\eta^s + \frac{1}{2} S_{,ss'}\eta^s \eta^{s'} + \ldots + \delta k \frac{\partial \Gamma_{k,a}}{\partial k}\eta^a]} > \ , \tag{9}$$

where the (omitted) arguments of $S_{,s}$ and $S_{,ss'}$ are $[\bar{\phi}_<, \bar{\phi}_S, \phi_>]$.

Following the classic arguments of [8], we know that, in order to collect all terms up to $O(\delta k)$, we only need to keep terms up to $O((\eta^s)^2)$, and thus we neglect the ellipses. The r.h.s. of Eq.(9) can be now computed using the identity

$$\langle e^{-f} \rangle = e^{-<f> + \frac{1}{2}(<f^2> - <f>^2) + O(f^3)}. \tag{10}$$

Thanks to (3), the last term in (9) can only contribute $O((\delta k)^2)$, so we also neglect this term. Then, with the help of the relations (5) and (6), we immediately compute the r.h.s. of Eq.(9) and find that (7) becomes :

$$e^{-\Gamma_{k-\delta k}} = e^{-\Gamma_k} \int [D\eta_S] e^{\Delta\Gamma_{k,\,s}\eta^s - \frac{1}{2} K_{ss'}\eta^s \eta^{s'}} \ , \tag{11}$$

where $\Delta\Gamma_{k,\,s} = \Gamma_{k-\delta k,\,s} - \Gamma_{k,\,s}$ and $K_{ss'}$ is nothing but the r.h.s. of Eq.(6), i.e. :

$$K_{ss'} = \Gamma_{k,\,ss'} - \Gamma_{k,\,sa}(\Gamma_{k,ab})^{-1}\Gamma_{k,\,bs'} \ . \tag{12}$$

As $\Delta\Gamma_{k,\,s}$ is $O(\delta k)$, it would contribute an $O((\delta k)^2)$ term after performing the gaussian integral. Neglecting again this higher order term, we finally find that the difference between $\Gamma_{k-\delta k}$ and Γ_k (evaluated at the same values of their arguments) consists, to $O(\delta k)$, of just a determinant, i.e.

$$\Gamma_{k-\delta k} = \Gamma_k + \frac{1}{2}\mathrm{Tr}\ln K_{ss'}. \tag{13}$$

Using standard properties of determinants, Eq. (13) can be rewritten in a form that will be more useful for our subsequent generalizations, i.e.

$$\Gamma_{k-\delta k} - \Gamma_k = \frac{1}{2}\ln\left(\frac{\det\Gamma_{k,AB}}{\det\Gamma_{k,ab}}\right) \ , \tag{14}$$

where we recall that the indices A, B span the region $[k - \delta k, \Lambda]$ (see [14] for a different rederivation of (14), equation that appeared in a previous version of this paper).

Let us now discuss how one can extend our results to the general case, including gauge theories. It was first noted by Vilkovisky [10] that the usual definition of the effective action, Eq.(1), is in general not invariant under a reparametrization of the classical fields. Obviously this holds true also for our definition (2) of Γ_k at any scale k. He also pointed out that, in the case of gauge theories, the gauge dependence of the off-shell effective action is just a manifestation of such a reparametrization dependence.

The origin of the problem can be seen easily from the definition of the effective action (1). Let us think of (field) configuration space as a manifold $\mathcal{M}$ endowed with a metric g_{ij} and assume that Γ, like S is a scalar field on $\mathcal{M}$. While the functional integration measure can be made reparametrization invariant through the introduction of a $\sqrt{g}$, the second term in the exponential has bad transformation properties since the gradient is a covariant vector while the "coordinate difference" $(\phi - \bar{\phi})$ is a contravariant vector only for very trivial (flat) spaces. In the case of gauge theories there is an additional complication coming from the fact that the physical space is the quotient space $\mathcal{M}/\mathcal{G}$ ($\mathcal{G}$ is the gauge group) rather than $\mathcal{M}$.

Vilkovisky and De Witt have shown that a meaningful definition of the effective action can be given also in the general (curved) case in terms of geodesic normal fields, $\sigma^i[\varphi_*, \phi]$, based at a point φ_* in $\mathcal{M}$ [10,11]. The $\sigma^i[\varphi_*, \phi]$ are the components of a vector tangent at φ_* to the geodesic from φ_* to ϕ. Its length is the distance between the two points along the geodesic itself. Under coordinate transformations $\sigma^i[\varphi_*, \phi]$ transforms as a vector at φ_* and as a scalar at ϕ. A useful property of the σ fields is that any scalar function $A[\phi]$ can be expanded in a covariant Taylor series [10,11] (the semicolon denotes covariant derivatives w.r.t. ϕ) :

$$A[\phi] \equiv A[\varphi_*, \sigma] = \sum_{n=0}^{\infty} \frac{1}{n!} A_{;a_1 \cdots a_n}[\varphi_*] \sigma^{a_1} \cdots \sigma^{a_n} . \quad (15)$$

As emphasized before, the definitions of the upper space, of the shell, and of the lower space are completely general and can be obtained with the help of any mode decomposition of the fields. From now on we denote by λ these generic modes. As before we introduce the notation $\sigma^i = (\sigma_<, \sigma_S, \sigma_>)$. The submanifold spanned by $\sigma_>$ we denote by $\mathcal{M}_>$ and the one spanned by $(\sigma_S, \sigma_>)$ by $\mathcal{M}_\geq$. The metric in σ coordinates is related to the original metric by

$$\hat{g}_{lm}(\varphi_*, \sigma) = \frac{\partial \phi^i}{\partial \sigma^l} \frac{\partial \phi^j}{\partial \sigma^m} g_{ij}(\phi) . \quad (16)$$

The induced metric on $\mathcal{M}_>$ ($\mathcal{M}_\geq$) is just the restriction of $\hat{g}_{lm}$ to the appropriate set of indices, $\hat{g}_{ab}$ ($\hat{g}_{AB}$).

Given the arbitrary coordinates (fields) ϕ^i, the base point φ_*, and the gaussian normal coordinates σ^i in $\mathcal{M}$, we can now define, following [11], the scale (i.e. λ)-dependent effective action, $\hat{\Gamma}_\lambda$, as :

$$e^{-\hat{\Gamma}_\lambda[\varphi_*, \bar{\sigma}]} = \int [D\sigma_>] \sqrt{\hat{g}} \; e^{-S + (\sigma^a - \bar{\sigma}^a) \hat{\Gamma}_\lambda[\varphi_*, \bar{\sigma}]_{,a}} , \quad (17)$$

where $\hat{g} = \det \hat{g}_{ab}$. S is the classical action expanded as in (15), where, as in the analogous Eq.(2), the $\sigma_<$ are replaced by the mean values $\bar{\sigma}_<$: $S = S[\varphi_*; \bar{\sigma}_<, \sigma_S, \sigma_>]$. Since φ_* is kept fixed, the steps that lead from Eq.(2) to the RG equation (14) can be repeated (with slight modifications due to the presence of the non-trivial metric) and we get:

$$\hat{\Gamma}_{\lambda - \delta\lambda}[\varphi_*, \bar{\sigma}] = \hat{\Gamma}_\lambda[\varphi_*, \bar{\sigma}] + \frac{1}{2} \ln \left(\frac{\det \hat{\Gamma}_{\lambda, A}{}^B}{\det \hat{\Gamma}_{\lambda, a}{}^b} \right) , \quad (18)$$

where the indices are raised with the help of the corresponding induced metrics on each submanifold*.

We now wish to rewrite Eq.(18) in general coordinates. Define:

$$\Gamma_\lambda[\varphi_*, \bar{\phi}] = \hat{\Gamma}_\lambda[\varphi_*, \sigma(\varphi_*, \bar{\phi})] = \hat{\Gamma}_\lambda[\varphi_*, \bar{\sigma}] . \quad (19)$$

It is rather straightforward, though tedious, to connect the partial derivatives of $\hat{\Gamma}$ with respect to the $\bar{\sigma}$'s to the partial *covariant* derivatives of Γ with respect to the $\bar{\phi}$'s (both taken, of course, at fixed φ_*). Consider first these relations at the level of the full effective actions $\hat{\Gamma}$ and Γ.

For the first derivatives the result is simply:

$$\Gamma_{,i} = D_i^k \hat{\Gamma}_{,k} , \quad (20)$$

*To be precise in Eq. (18) the determinants of the metrics appear under an expectation value sign rather than being computed at the expectation value of the field. We expect the difference to be insignificant.

where, following [11], we have introduced:

$$D_i^k = \frac{\partial \bar{\sigma}^k}{\partial \bar{\phi}^i} . \quad (21)$$

The bi-vector D_i^k has the property that, once contracted with a covariant vector at φ_*, converts it into a covariant vector at $\bar{\phi}$, as exemplified indeed in (20).

The relation connecting second derivatives can be put in the form:

$$\hat{\Gamma}_{,kl} = (D^{-1})_k^i (D^{-1})_l^j \bar{\Gamma}_{ij} , \quad (22)$$

where

$$\bar{\Gamma}_{ij} \equiv \Gamma_{;ij} - \sigma^l_{;ij} (D^{-1})_l^k \Gamma_{,k} \quad (23)$$

is a second-rank tensor at $\bar{\phi}$. The quantity $\sigma^l_{;ij}$ has a covariant expansion [11] in the distance between φ_* and $\bar{\phi}$.

The above formulae can be easily generalized to the case in which the derivatives are restricted to lie on the $\mathcal{M}_>$ (or $\mathcal{M}_\geq$) manifold. Indeed the derivatives of $\hat{\Gamma}_\lambda$ with respect to $\bar{\sigma}^a$ will be related to the derivatives of Γ_λ with respect to generic coordinates ξ^a on $\mathcal{M}_>$ by exactly the same formulae (20), (22) where now:

$$D_b^a = \frac{\partial \bar{\sigma}^a}{\partial \xi^b} . \quad (24)$$

Using Eq. (16) we obtain our final result:

$$\Gamma_{\lambda - \delta\lambda}[\varphi_*, \bar{\phi}] = \Gamma_\lambda[\varphi_*, \bar{\phi}] + \frac{1}{2} \ln \left(\frac{\det \bar{\Gamma}_{\lambda A}{}^B}{\det \bar{\Gamma}_{\lambda a}{}^b} \right) , \quad (25)$$

where indices and covariant derivatives are all now defined in terms of the induced metrics (16) on the relevant submanifold.

Let us see now how the previous steps can be repeated in the case of a gauge theory. As it was shown by Vilkovisky and DeWitt [10,11], we first need to reduce the gauge theory to a "non-gauge" one. Let us indicate as before by $\mathcal{M}$ the field space, by ϕ^i the gauge fields, with g_{ij} the associated metric, by σ^m a complete set of gauge-invariant coordinates, and by R^i_α the generators of the gauge transformations:

$$\delta \phi^i = R^i_\alpha d\epsilon^\alpha , \quad (26)$$

where ϵ^α are coordinates on the gauge orbits. The metric decomposes into the block diagonal form [15]

$$ds^2 = h_{mn} d\sigma^m d\sigma^n + \gamma_{\alpha\beta} d\epsilon^\alpha d\epsilon^\beta , \gamma_{\alpha\beta} = R^i_\alpha g_{ij} R^j_\beta, \quad (27)$$

where

$$h_{mn} = \frac{\partial \phi^i}{\partial \sigma^m} \frac{\partial \phi^j}{\partial \sigma^n} \Pi_{ij} , \quad (28)$$

and we defined the projector on physical orbit space

$$\Pi_i^j = \delta_i^j - g_{ik} R_\alpha^k \gamma^{\alpha\beta} R_\beta^j \ . \tag{29}$$

Although the σ^m were so far arbitrary, we used an important result of [10] to take them as gaussian normal coordinates both in the induced metric h_{mn} and in the full space (provided geodesics are defined, in the latter, with respect to Vilkovisky's connection [10]).

Instead of using ϵ^α to parametrize points on orbits one can start with the "gauge fixing" coordinates χ^α and write the definition of the effective action a la Faddeev-Popov:

$$e^{-\Gamma[\varphi_*,\bar\phi]} = \int [D\phi^i]\sqrt{g}\delta(\chi^\alpha)\det\left(\frac{\partial\chi^\alpha}{\partial\epsilon^\beta}\right)\cdot e^{-S(\phi)+(\sigma^m-\bar\sigma^m)D_m^{-1\,n}\Gamma_{,n}} \ . \tag{30}$$

Changing integration variables to σ^m, ϵ^α we get

$$e^{-\hat\Gamma[\varphi_*,\bar\sigma]} = \int [D\sigma^m][D\epsilon^\alpha]\sqrt{h}\sqrt{\gamma}\delta(\chi^\alpha)\det\left(\frac{\partial\chi^\alpha}{\partial\epsilon^\beta}\right)\cdot e^{-S(\phi_*,\sigma)+(\sigma^m-\bar\sigma^m)\hat\Gamma_{,m}} = \int [D\sigma^m]\sqrt{h}e^{-\tilde S(\phi_*,\sigma)+(\sigma^m-\bar\sigma^m)\hat\Gamma_{,m}} \ , \tag{31}$$

where

$$\tilde S = S - \frac{1}{2}\ln\det(\gamma) \ . \tag{32}$$

With the gauge effective action written in this form we can directly apply the procedure followed from (17) to (18) and obtain, as before,

$$\hat\Gamma_{\lambda-\delta\lambda}[\varphi_*,\bar\sigma] = \hat\Gamma_\lambda[\varphi_*,\bar\sigma] + \frac{1}{2}\ln\left(\frac{\det\hat\Gamma_{\lambda,A}{}^B}{\det\hat\Gamma_{\lambda,a}{}^b}\right) \ . \tag{33}$$

We can now repeat the steps (19)-(25) and, following [10–12], write (33) in arbitrary coordinates $\bar\phi$ as

$$\Gamma_{\lambda-\delta\lambda}[\varphi_*,\bar\phi] = \Gamma_\lambda[\varphi_*,\bar\phi] + \frac{1}{2}\ln\left[\frac{\det(P_\geq \Pi\bar\Gamma\Pi P_\geq)}{\det(P_> \Pi\bar\Gamma\Pi P_>)}\right] \ , \tag{34}$$

where $\bar\Gamma$ is defined as in (23) but in terms of the original connection, Π stands for the projector on the physical space (29), and $P_>$ ($P_\geq$) is a projector on $\mathcal{M}_>$ ($\mathcal{M}_\geq$).

Eq.(34) is our desired gauge-invariant RG-flow equation for $\Gamma_\lambda[\varphi_*,\bar\phi]$. We believe, instead, that no such a closed RG-equation holds for the original Vilkovisky-De Witt effective action $\Gamma_{VDW}[\bar\phi] \equiv \Gamma[\bar\phi,\bar\phi]$. This is probably related to the fact that, unlike $\Gamma[\varphi_*,\bar\phi]$, $\Gamma_{VDW}[\bar\phi]$ does not generate 1PI vertex functions [11,15].

As a first application of (34) we can compute the one-loop effective action at $\varphi_* = \bar\phi$ to compare it with [12]. Within this approximation we have to set $\bar\Gamma_{ij} = S_{;ij}$ inside the brackets in (34) and then integrate the evolution from $\lambda = \Lambda$ to $\lambda = 0$. Using $\Gamma_\Lambda = \tilde S$, together with (32), we get:

$$\Gamma_0 = S + \frac{1}{2}\ln\frac{\det(\Pi_i^k S_{;k}^l \Pi_l^j)}{\det\ \gamma} \ , \tag{35}$$

in agreement with the one-loop result of [12].

Beyond one-loop, our evolution equations should be useful in a variety of problems pertaining to non-abelian gauge theories and to quantum gravity. In practice, one will necessarily have to resort to some form of truncation of Γ_k, so that our exact equations become approximate RG-flow equations for a finite set of gauge-invariant low-energy parameters. We plan to come back soon to these applications elsewhere.

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[1] K. G. Wilson, Phys. Rev. **B4** (1971) 3184; K. G. Wilson, J. Kogut, Phys. Rep. **12** (1974) 75.

[2] S. Weinberg, *Critical Phenomena for Field Theorists*, Erice Subnucl. Phys. 1 (1976).

[3] J. Polchinski, Nucl. Phys. **B231** (1984) 269.

[4] A. Hasenfratz, P. Hasenfratz, Nucl. Phys. **B270** (1986) 685.

[5] C. Wetterich, Phys. Lett. **B301** (1993) 90.

[6] M. Bonini, M. D'Attanasio, G. Marchesini, Nucl. Phys. **B409** (1993) 441.

[7] T.R. Morris, Int. J. Mod. Phys. **A9** (1994) 2411.

[8] F. Wegner, A. Houghton, Phys. Rev. **A8** (1973) 401.

[9] M. Reuter, C. Wetterich, Nucl. Phys. **B417** (1994) 181; M. Bonini, M. D'Attanasio, G. Marchesini, Nucl. Phys. **B421** (1994) 429.

[10] G. A. Vilkovisky, in *B. S. De Witt Sixtieth Aniversary Volume*, ed. S. Christensen (Adam Hilger, Bristol, 1983); Nucl. Phys. **B234** (1984) 125.

[11] B. De Witt, in *Quantum Field Theory and Quantum Statistics, Essays in Honor of the 60th Birthday of E.S. Fradkin* (I.A. Batalin, C.J. Isham and G.A. Vilkovisky editors), IOP publishing Ltd. (1987), p. 191; *The Global Approach to Quantum Field Theory*, Oxford University Press (2003).

[12] E.S. Fradkin, A.A. Tseytlin, Nucl. Phys. **B234** (1984) 509.

[13] B. Zumino, Brandeis lectures 1970 (Vol. 2, p. 441, S. Deser ed., M.I.T. Press).

[14] F.P. Correia, hep-th/0207282.

[15] G. Kunstatter, Class.Quant.Grav. **9** (1992) 157.