Implementation of the Hough Transform
for 3D Track Reconstruction in Drift Chambers

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This paper is devoted to the method developed in Ref. [1] for 3D re-construction of the straight tracks in the tracking system consisting of the drift-chamber stereo layers. The method is based on the Hough-transform approach [2] – the discrete case of more general Radon transform [3] – and takes into account both coordinates of the hit wires and drift distances not only for the measurements in one projection, but also in the rotated stereo layers. The proposed method allows one to resolve the right-left ambiguity and provides the accordance between vertical and horizontal projections of the track.

Let the straight track of charged particle be detected by the system of *drift chambers* consisting of the cylindrical tubes placed in such a way that their anode wires stretched along the tube axes are parallel to the vertical coordinate axis Y. In this case the signals from the drift chambers give the information about the track projection onto the horizontal plane XZ. The track projection is described by $x(z) = x_0 + t_x(z - z_0)$, where x_0 and $t_x = \tan\theta_x$ are the projection offset in z and slope, respectively.

The single measurement from the hit wire includes its coordinates x_i, z_i and measured distance r_i from the wire to the track. Assume that error of r_i measurement is uniformly distributed within the range $[-c, c]$, where $c \approx (2 \div 3)\sigma$ is a tuning parameter of the algorithm while σ is a space resolution of the drift chamber. At each value of t_x, *the Hough image of a single measurement* (x_i, z_i, r_i) in the space P_x of projection parameters (x_0, t_x) is given by two ranges of possible values of x_0 (see Fig. 1):

$$x_0^u \in \left[x_i - (z_i - z_0)t_x + (r_i - c)\sqrt{1 + t_x^2} \; ; \; x_i - (z_i - z_0)t_x + (r_i + c)\sqrt{1 + t_x^2} \right] \quad (1)$$

is an upper branch M^u,

$$x_0^d \in \left[x_i - (z_i - z_0)t_x - (r_i + c)\sqrt{1 + t_x^2} \; ; \; x_i - (z_i - z_0)t_x - (r_i - c)\sqrt{1 + t_x^2} \right] \quad (2)$$
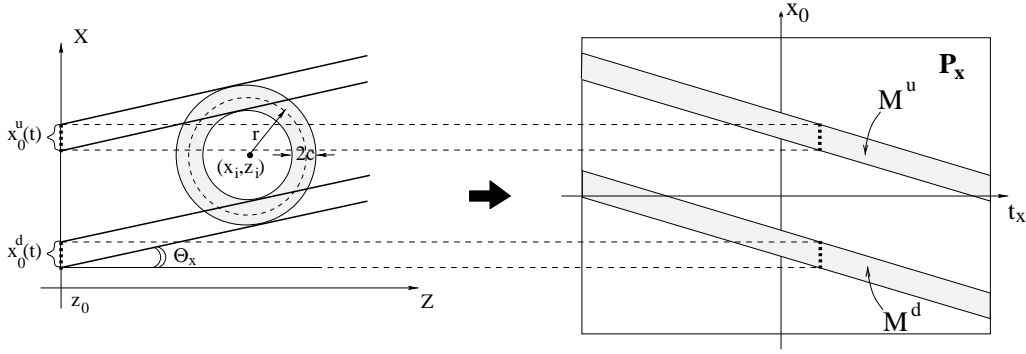
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Figure 1: The Hough image of the single measurement (x_i, z_i, r_i) in the space P_x of projection parameters (x_0, t_x)

is a lower branch M^d.

For chamber operation in the *proportional regime* (no measurement of r_i), the Hough image of a single hit (x_i, z_i) is given in the parameter space P_x by a range of possible values of x_0 at each value of t_x:

$$x_0 \in \left[x_i - (z_i - z_0)t_x - R\sqrt{1 + t_x^2} \, ; \, x_i - (z_i - z_0)t_x + R\sqrt{1 + t_x^2} \right], \quad (3)$$

where R is the tube radius.

Assume $M_1, M_2, \ldots, M_n$ – are Hough images of single measurements from n hits belonging to the same track. In this case *the Hough image of the track*, $M^{(n)}$, is defined in the parameter space P_x as an intersection of set of Hough images M_k, $k = 1, \ldots, n$: $M^{(n)} = \bigcap_{k=1}^n M_k$. To characterize the track reliability level, let us introduce the criterion $J(M)$ which can be determined as number of hits having produced the Hough image M of the track. In particular, $J(M^{(n)}) = n$. For the reconstruction we use only those M for which $J(M) \geq J_{min}$, where a threshold value of J_{min} is a tuning parameter of the algorithm. If the intersection M is found, then the track parameters x_0, t_x are estimated as coordinates of center of gravity of M in the parameter space P_x.

For 3D reconstruction, it is also necessary to determine the track projection onto the vertical plane YZ. For this aim, the set of vertical tubes (*"0" stereo-layers*) with set of tubes rotated by the angle α around the axis Z (*"α" stereo-layers*) can be used as it is shown in Fig. 2. The YZ track projection is described by $y(z) = y_0 + t_y(z - z_0)$, where y_0 and $t_y = \tan\theta_y$ are the projection offset in z and slope, respectively.

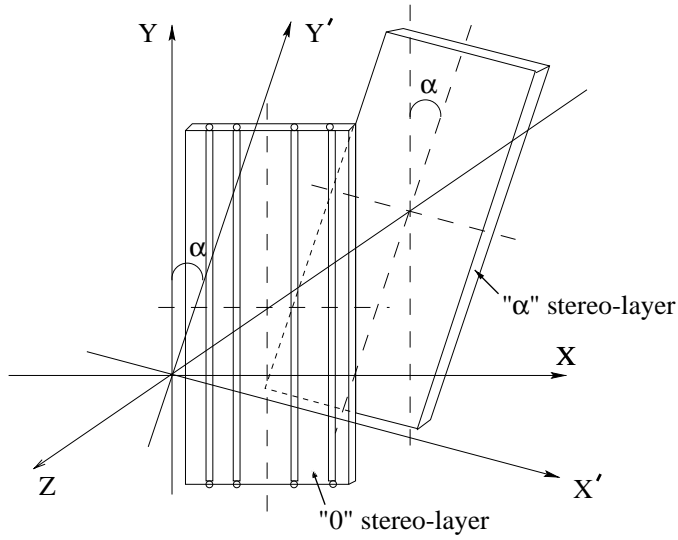Assume parameters x_0, t_x for the XZ track projection have already

Figure 2: Stereo-layers in the XYZ and rotated $X'Y'Z$ coordinate systems

been determined using measurements in the "0" stereo-layers and Hough-transform approach described above. Then, the single measurements (x'_i, z_i, r_i) in "α" stereo-layers with hit-wire coordinates determined in the rotated system $X'Y'Z$ (see Fig. 2) can be used for reconstruction of track projection onto the plane YZ. The corresponding Hough images of single measurements are defined in the space P_y of the parameters y_0, t_y by the following boundaries of possible values of y_0 at fixed t_y (see Ref. [1] for more detail):

$$
\begin{aligned}
y_0(t_y; x_0, t_x) &= \frac{x_0 + t_x(z_i - z_0)}{\tan \alpha} - \frac{x'_i}{\sin \alpha} - t_y(z_i - z_0) \\
&\pm \frac{r_i \pm c}{\sin \alpha} \sqrt{1 + (t_x \cos \alpha - t_y \sin \alpha)^2}
\end{aligned}
\tag{4}
$$

– for the drift chambers, or

$$
\begin{aligned}
y_0(t_y; x_0, t_x) &= \frac{x_0 + t_x(z_i - z_0)}{\tan \alpha} - \frac{x'_i}{\sin \alpha} - t_y(z_i - z_0) \\
&\pm \frac{R}{\sin \alpha} \sqrt{1 + (t_x \cos \alpha - t_y \sin \alpha)^2}
\end{aligned}
\tag{5}
$$

- for chamber operation in proportional regime.

For the program realization of the discussed algorithm, for example, for XZ-projection finding, the space P_x of the track parameters x_0, t_x can be treated as a *discrete two-dimensional raster* $n \times m$, which is described

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by array of its cells $\rho(l, k)$ with indices $l = 1, \ldots, n$ and $k = 1, \ldots, m$. The Hough image of each hit can be consequently constructed as a strip on the raster: the current value of the raster cell is increased by a unit if the cell is located within the limits given by Eqs. (1), (2) or (3). After completing this procedure the value of each cell of the raster becomes equal to the number of the Hough stripes having passed the cell. Each local maximum on the raster exceeding some threshold J_{min}, can be identified with the XZ projection having the corresponding values of parameters x_0, t_x.

The selection algorithm of reliable tracks includes ordering the raster cells according to the criterion $J(M)$ by using a special structure – the *array of bidirectional lists* determining the *hierarchy of the raster cells* filled in. The given array of the lists is a vector-column where each element is a pointer to the list and corresponds to a certain meaning of the criterion $J(M) > J_{min}$. After filling the raster, the hierarchy of its cells is built: if $\rho(l, k) \geq J_{min}$, then the point (l, k) is added into the list for $J = \rho(l, k)$.

At the first step of 3D track reconstruction, the global XZ raster and its hierarchy are built by using the hits only in the "0" stereo-layers. The further steps of the algorithm include:

- estimating of the initial values of x_0, t_x corresponding to the first maximum in the global XZ-raster hierarchy;

- building the local XZ raster and its hierarchy by using the hits in the "0" stereo-layers within a corridor around the track projection defined by the initial values of x_0, t_x;

- improvement of the estimates of x_0, t_x as coordinates of center of gravity of the local-raster maximum;

- building the YZ raster and its hierarchy assigned to the found XZ projection (x_0, t_x), by using hits only in the "α" stereo-layers;

- finding of YZ projection (y_0, t_y) assigned to the found XZ projection (x_0, t_x), by using the YZ-raster hierarchy;

- subtraction of the hits in "0" and "α" stereo-layers, which belong to the 3D-track $(x_0, t_x; y_0, t_y)$, and consecutive subtraction of the corresponding Hough strips from the global raster and its hierarchy.

The procedure described above should be repeated iteratively until there remain the cells of the global XZ raster exceeding some threshold J_{min}.

The examples of global and local XZ rasters for Monte-Carlo tracks are shown in Fig. 3. The small crosses on the global raster mark Monte-Carlo tracks on the (x_0, t_x)-plane of the track projection parameters while
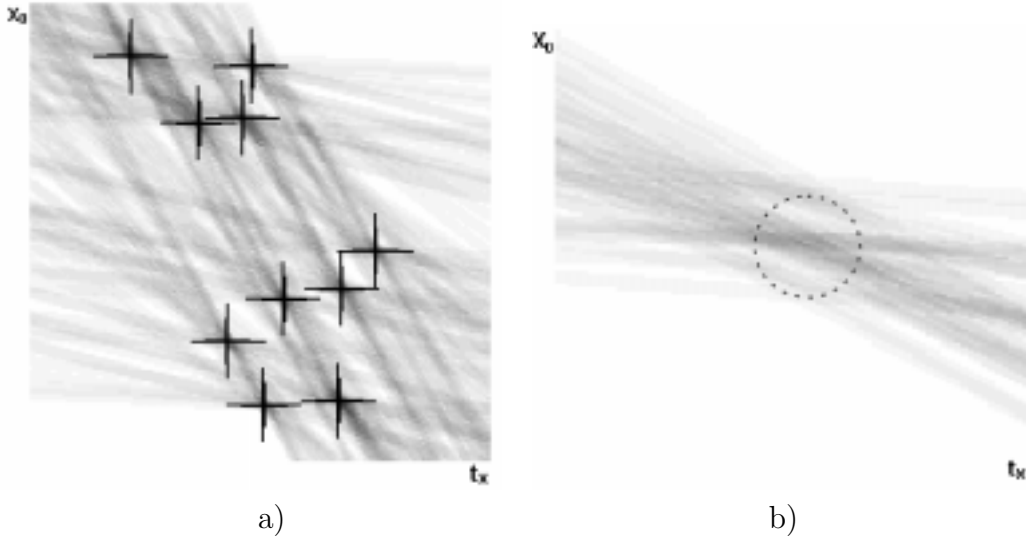
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Figure 3: Examples of global (a) and local (b) XZ rasters

the large crosses correspond to the values x_0, t_x of reconstructed projections. Fig. 3a shows a good precision of track reconstruction by the Hough-transform algorithm described above.

This algorithm has been realized in the program *Htr* developed for track finding in the PC chambers (Pattern Tracker) of the HERA-B Outer Tracker [4]. The program *Htr* was integrated into the program environment of $ARTE$ – the general software for event processing at HERA-B – and tested both with Monte-Carlo and real data. The tests showed the stable *Htr* performance with average track finding efficiency of about 90% and rate of ghosts at the level 23% under real conditions of PC-chamber operation. The proposed program realization of the described algorithm provides the time-consuming optimization of event processing and high efficiency of the track finding under large track-occupancy of the detector as well as under high level of noisy and dead channels.

This work was done in the HERA-B software group at DESY, Hamburg.

References

[1] Ar. Belkov, JINR Communication P10-2001-182, Dubna, 2001.

[2] Hough P.V.C. A Method and Means for Recognizing Complex Patterns: US Patent 3,069,654, 1962.

[3] J. Radon, Ber. Ver. Sächs. Akad. Wiss.: Leipzig, Math-Phys. Kl., Vol. 69 (1917) p. 262.

[4] E. Hartouni et al., HERA-B Design Report: DESY-PRC 95/01, 1995.