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Ground–γ band mixing and $\Delta L = 1$ staggering in heavy deformed nuclei

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The odd–even staggering effect (OES), observed in the collective γ- bands of heavy deformed nuclei, represents the relative displacement of the odd angular momentum levels of the band with respect to their neighboring levels with even angular momentum [1]. It is traditionally considered in terms of a plot of the moment-of-inertia parameter versus the angular momentum L.

In the present work we propose that the OES effect can be reasonably characterized by the quantity:

$$Stg(L) = 6\Delta E(L) - 4\Delta E(L-1) - 4\Delta E(L+1) + \Delta E(L+2) + \Delta E(L-2) \ , \qquad (1)$$

which is the discrete approximation of the fourth derivative of the function $\Delta E(L) = E(L+1) - E(L)$, i.e. the fifth derivative of the γ- band energy $E(L)$.

In the case of a rigid rotor one can easily see that $Stg(L)$ is equal to zero. Moreover the terms of the second power in $L(L+1)$ also give zero in Eq. (1). Therefore, any non-zero values of the quantity $Stg(L)$ will indicate a deviation of order higher than $(L(L+1))^2$ from the regular rotational motion of the nuclear system. Eq. (1) is introduced by analogy with the $\Delta L = 2$ staggering observed in superdeformed nuclei and diatomic molecules [2].

On the above basis it is natural to apply the quantity $Stg(L)$ to study the OES in the γ-bands of heavy deformed nuclei, i.e. to interpret this effect in the form of $\Delta L = 1$ discrete derivation. Such an approach could be very useful in providing a unified analysis of the different kinds of staggering effects as well as in comparing their physical explanations.

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Since for the γ-bands the experimental energy values are known, Eq. (1) can be written in the form:

$$\begin{aligned} \mathrm{Stg}(L) &= 10E(L+1)+5E(L-1)+E(L+3) \\ &- [10E(L)+5E(L+2)+E(L-2)] , \end{aligned} \tag{2}$$

i. e. the quantity $\mathrm{Stg}(L)$ is simply determined by six band level energies (with $L-2$, $L-1$, ..., $L+3$).

We have found that Eq. (2) provides a well developed $\Delta L = 1$ staggering pattern (zigzagging behavior of the function $Stg(L)$) in all nuclei for which the gamma band is long enough ($L \geq 10$), ^{156}Gd [3], ^{156,160}Dy [3], ^{162}Dy [4], $^{162-166}$Er [3], ^{170}Yb [5], ^{228}Th [6], ^{232}Th [4]. The experimental staggering patterns for ^{164,166}Er are illustrated in Fig. 1 (black circles).

We propose that OES can be interpreted reasonably as the result of the interaction of the γ band with the ground (g) band in the framework of a Vector Boson Model (VBM) with SU(3) dynamical symmetry. In the VBM the two bands are coupled into the same split (λ, μ) multiplet of SU(3) [7, 8]. The effective model interaction which reduces the SU(3) symmetry to the rotational group SO(3) has the form:

$$V = g_1 L^2 + g_2 L \cdot Q \cdot L + g_3 A^+ A , \tag{3}$$

where g_1, g_2 and g_3 are free model parameters; L and Q are the angular momentum and quadrupole operators respectively; and A^+A is a fourth order vector-boson operator.

For the multiplets of the type $(\lambda, 2)$ the model provides a simple expression for the γ-band energy levels:

$$\begin{aligned} E^{\gamma}(L) &= 2B + AL(L+1) + B[\sqrt{1+aL(L+1)+bL^2(L+1)^2} \\ &- CL(L+1) - 1]\left(\frac{1+(-1)^L}{2}\right) , \end{aligned} \tag{4}$$

where the quantities A, B, C , a and b are determined by the effective interaction (3) and depend on the model parameters as well as on the SU(3) quantum number λ. The last factor in Eq. (4) switches over E^{γ} between the odd and the even states and thus gives a natural possibility to reproduce the parity effects in the γ-band structure. It is important to remark that such a result is a direct consequence of the SU(3) dynamical symmetry mechanism.

We have applied the above model formalism to reproduce theoretically the $\Delta L = 1$ discrete derivatives (1) and (2). After introducing Eq. (4) into Eq. (2), we obtain the following model expression for the function $Stg(L)$:

$$\begin{aligned} Stg(L) &= \frac{B}{2}\left(10R(L+1)+5R(L-1)+R(L+3)\right)[1+(-1)^{L+1}] \\ &- \frac{B}{2}\left(10R(L)+5R(L+2)+R(L-2)\right)[1+(-1)^{L}] , \end{aligned} \tag{5}$$

where

$$R(L) = \sqrt{1+aL(L+1)+bL^2(L+1)^2} - CL(L+1) - 1 . \tag{6}$$

One can easily verify that the right-hand side of (5) has alternative signs as a function of the angular momentum values $L = 2, 3, 4, ...$, i.e. it gives a regular model staggering pattern. In addition, the amplitude of the staggering increases monotonously with L. The signs and the amplitude are determined by the terms $BR(x)$, $(x = L-2, L-1, \ldots, L+3)$ which depend on the high order effective interactions in (3).

Eq. (5) allows one to study the OES effect in terms of the VBM. Moreover the obtained result provides a reasonable theoretical tool to interpret the OES effect in the meaning of a $\Delta L = 1$ staggering effect. In addition one could estimate analytically the behavior of this effect in dependence on the nuclear collective characteristics.

So we found that Eq. (5) reproduces successfully the zigzagging of the quantity $Stg(L)$ in all considered nuclei up to $L = 12-13$. In Fig. 1 the experimental (black circles) and the theoretical (open circles) staggering patterns for ^{164}Er (Fig. 1(a)) and ^{166}Er (Fig. 1(b)) are compared.

Based on the experimental and theoretical staggering patterns in rare earth nuclei and actinides we obtained the following information about the $\Delta L = 1$ staggering effect in these regions.

For the considered nuclei the experimental $\Delta L = 1$ staggering amplitude varies in rather wide range. For example, for $L = 8$ the absolute value of the quantity $Stg(8)$ is observed in the limits $0.01-0.5$ MeV ($Stg(L) = -0.013$ MeV for ^{166}Er and $Stg(L) = 0.467$ MeV for ^{156}Gd).

The analysis of the staggering patterns in rare earth nuclei shows that the amplitude generally decreases towards the midshell nuclei. The best example is the group of the three Er isotopes, ^{162}Er (with $Stg(8) = 0.425$ MeV), ^{164}Er ($Stg(8) = 0.251$ MeV) and ^{166}Er (with $Stg(8) = -0.013$ MeV).

This observation is consistent with the general behavior of the nuclear rotational properties in the limits of the valence shells. It is well known that towards the midshell region these properties are better revealed so that any kind of deviations from the regular rotational band–structures should be smaller. In this respect the weaker $\Delta L = 1$ staggering effect observed in the midshell isotopes of rare earth nuclei is quite natural.

On the other hand, such a behavior of the staggering effect can be reasonably interpreted in terms of the ground–γ band interaction. It has been shown in the SU(3) dynamical symmetry framework that the mixing of the two bands systematically decreases towards the middle of a given rotational region [9]. This is associated with the corresponding increase in the splitting of the SU(3) multiplet i.e. in the energy separation between the bands, as well as with the increase in the SU(3) quantum number λ. The splitting is measured by the ratio [8, 9]:

$$\Delta E_L = \frac{E_L^\gamma - E_L^g}{E_2^g} , \tag{7}$$

which characterizes the magnitude of the energy differences between the even angular momentum states of the g- and the γ- band.

On the above basis it has been established that for nuclei with a weak SU(3) splitting ($\Delta E_2 = 5-12$ for rare earth nuclei and $\Delta E_2 = 13-15$ for actinides) the g and the γ bands are strongly coupled in the framework of the SU(3) dynamical symmetry, with λ obtaining favored values in the region $\lambda = 14 - 20$. The nuclei of the present study indeed belong to this kind of SU(3) dynamical symmetry nuclei. In such a way the relatively strong g–γ band interaction in these nuclei can be considered as the reason causing the observed OES ($\Delta L = 1$ staggering).

For the midshell nuclei where strong SU(3) splitting is observed ($\Delta E_2 = 12 - 20$ for rare earth nuclei, and $\Delta E_2 = 15 - 25$ for actinides) the g and the γ bands are weakly coupled in the framework of the SU(3) dynamical symmetry, with $\lambda > 60$. So, the weaker mutual perturbation of these two bands in the midshell region is consistent with the respectively good rotational behavior of the γ-band.

The above consideration is consistent with the recently suggested possibility [8, 9] for a transition from the g–γ band coupling scheme of the VBM, which is more appropriate near the ends of the rotational regions, to the Interacting Boson Model classification scheme [10] with β–γ band coupling, which is more relevant in the midshell nuclei.

The dynamical mechanism causing such a transition should be founded on the so called SU(3) contraction process, in which the algebra of SU(3) goes to the algebra of the semi-direct product $T_5 \wedge SO(3)$. It has been shown that in the SU(3) contraction limit $\lambda \to \infty$, with μ finite the g–γ band mixing gradually disappears so that the corresponding SU(3) multiplets are disintegrated into distinct noninteracting bands [8, 9]. It follows that all fine spectroscopic effects based on the band-mixing interactions, such as the OES effect, should be reduced towards this limit.

So one can verify that the staggering amplitude determined in Eqs. (2) and Eq. (4) goes to zero when λ increases to infinity. In fact, for this limit we have deduced analytically that the terms of the type $BR(L)$ which determine the quantity $Stg(L)$ in Eq. (5) go to zero as

$$\lim_{\lambda\to\infty} BR(L) = \lim_{\lambda\to\infty} \frac{3(g_2 g_3 - 3g_2^2)}{g_3 \lambda^2} L(L+1) = 0 \ . \qquad (8)$$

This is illustrated in Fig. 2, where the model staggering pattern is plotted for $\lambda = 20$, $\lambda = 40$ and $\lambda = 60$ and fixed (overall) values of the model parameters $g_2 = -0.2$ and $g_3 = -0.25$.

The above considerations explain the presence of the well developed OES patterns in nuclei with relatively weak SU(3) splitting (strong g–γ band coupling) as well as the decrease in the staggering amplitude (even the absence of the effect) for the strongly split SU(3) multiplets in midshell nuclei.

Further, it has been established that for the nuclei under study the experimental staggering amplitude generally increases with the increase of the angular momentum L up to $L = 12 - 13$. This result is well reproduced by the theoretical expression Eq. (4) which provides a monotonous increase of the function $Stg(L)$.

Interesting behavior of the staggering amplitude is observed in the nuclei ^{164}Er (See Fig. 1(a)) and ^{170}Yb. In these cases the amplitude initially increases up to $L = 8 - 10$ and then begins to decrease. Further, at $L = 14$, an irregularity in the alternative signs of the quantity $Stg(L)$ occurs. Actually, at this angular momentum the structure of the γ-band in these nuclei is changed due to a crossing with another band and a backbending effect is observed. It is remarkable that the experimentally determined quantity $Stg(L)$ gives an excellent indication for the presence of bandcrossing effects.

The results presented show that the use of the fourth derivative of the odd-even ($\Delta L = 1$) energy differences gives a rather accurate quantitative measure to estimate the magnitude of the OES effect in the γ- bands of heavy deformed nuclei. Hence one is able to assess this effect for a given angular momentum or given region of angular momenta. In such a way the role of the band-mixing interactions could be correctly taken into account. Moreover, the well determined staggering amplitudes together with the clearly established alternating signs pattern allow one to provide various quantitative analyses of the fine structure of nuclear collective bands as a whole.

The approach suggested gives a rather general prescription to study the fine effects based on various band mixing interactions in nuclei. It allows relevant tests and detailed comparison of different band coupling schemes with SU(3) dynamical symmetry.

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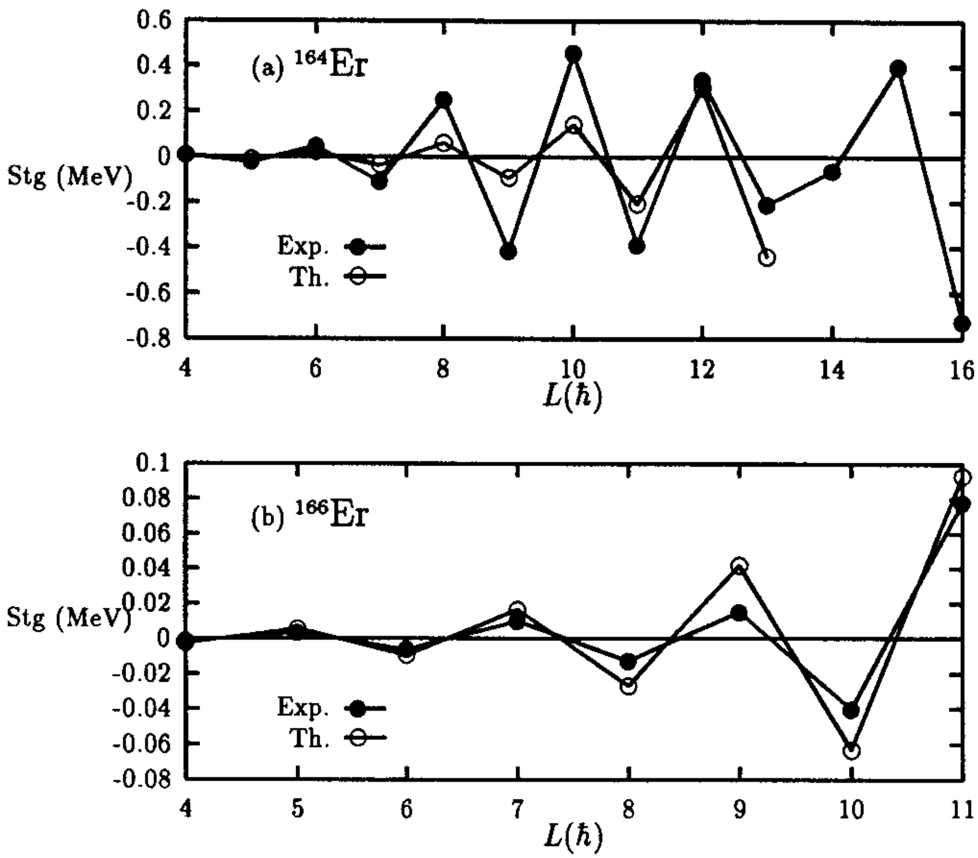

Fig. 1. Experimental and theoretical $\Delta L = 1$ staggering pattern for (a) ^{164}Er and (b) ^{166}Er (data from [3]).

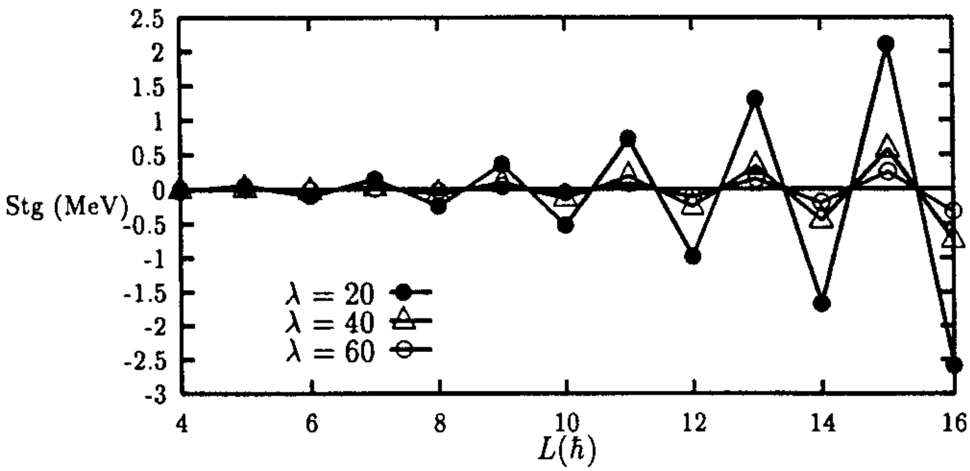

Fig. 2. The theoretical values of the quantity $Stg(L)$, Eq. (5), obtained for three different values of the SU(3) quantum number λ (λ = 20, λ = 40 and λ = 60) and fixed (overall) values of the model parameters ($g_2 = -0.2$ and $g_3 = -0.25$) are plotted as functions of the angular momentum.

References

[1] A. Bohr and B. R. Mottelson, *Nuclear Stucture* vol. II (Benjamin, New York, 1975).

[2] D. Bonatsos, C. Daskaloyannis, S. Drenska, G. Lalazissis, N. Minkov, P. Raychev and R. Roussev, Phys. Rev. A **54**, R2533 (1996).

[3] P. C. Sood, D. M. Headly and R. K. Sheline, At. Data Nucl. Data Tables **47**, 89 (1991).

[4] M. Sakai, At. Data Nucl. Data Tables **31**, 399 (1984).

[5] D. E. Archer, M. A. Riley, T. B. Brown, D. J. Hartley, J. Döring, G. D. Johnes, J. Pfohl, S. L. Tabor, J. Simpson, Y. Sun, and J. L. Egido, Phys. Rev. C 57, 2924 (1998).

[6] T. Weber, J. Gröger, C. Günther and J. deBoer, Eur. Phys. J. A **1**, 39 (1998).

[7] S. Alisauskas, P. P. Raychev and R. P. Roussev, J. Phys. G **7**, 1213 (1981); P. P. Raychev and R. P. Roussev, J. Phys. G **7**, 1227 (1981).

[8] N. Minkov, S. Drenska, P. Raychev, R. Roussev and D. Bonatsos, Phys. Rev. C **55**, 2345 (1997).

[9] N. Minkov, S. Drenska, P. Raychev, R. Roussev and D. Bonatsos, Phys. Rev. C **60**, 034305 (1999).

[10] F. Iachello and A. Arima, *The Interacting Boson Model* (Cambridge University Press, Cambridge, 1987).