BOUND STATES OF PRIMORDIAL BLACK HOLES AND DARK MATTER

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ABSTRACT

We show that stable bound states of black holes, called holeums, having asymptotic freedom, an exclusion property and a nearly parameter – free gravitational analogue of the hydrogen atom spectrum would be formed in the early universe and in accelerator experiments. They are massive components of dark matter, a new source of gravitational waves and would form impregnable haloes around galaxies.

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Recently a great deal of attention is being focused on the gravitational interaction. On the one hand experiments are being set up to detect gravitational waves and on the other theorists are being urged to present new theoretical results. The twenty first century is being dubbed the century of gravitation. In this letter we consider stable gravitational bound states of two or more primordial black holes, called holeums. They would be a component of dark matter and a source of gravity waves. In the followng, unless stated otherwise, primordial black holes will be referred to as black holes. An isolated black hole is subject to evaporation of its mass due to the vacuum fluctuations in its vicinity. However, when the temperature T of the big-bang universe is much higher than $T_b = mc^2 / k_B$ where m, c, k_B are the mass of the black hole, the speed of light in vacuum and the Boltzman constant respectively, the number density of such black holes is proportional to $(k_BT)^3$ and they are in intense fields. These conditions are conducive to the formation of their bound states, holeums. Once a part of a stable holeum a black hole is immune to decay just like a neutron in a stable nucleus. Since the holeums give off only the gravitational radiation they constitute a component of the dark matter in the universe. We work within the frame work of Newtonian gravity and Schrodinger equation. The former is a weak field limit of general relativity and possesses unsurpassed simplicity and transparency [1] and the latter gives exact solution to the problem at hand.

Consider two black holes of masses m_1 and m_2 interacting through the gravitational potential.

$$V(r) = -\hbar c \alpha_g / r \quad (1),$$

$$\alpha_g = m_1 m_2 / m_P^2 \quad (2),$$

$$m_P^2 = \hbar c / G \quad (3),$$

where m_P is the Planck mass and $\hbar$ and G are the Planck's constant h reduced by 2π and the universal constant of gravity, respectively. The Schrodinger equation is exactly solvable for this potential and the energy eigen values are given by

$$E_2 = -\mu_2 c^2 \alpha_2^2 / (2 n_2^2) \quad (4),$$

where the reduced mass μ_2 is given by

$$1 / \mu_2 = 1 / m_1 + 1 / m_2 \quad (5),$$

$$\alpha_2 = \alpha_g \quad (6),$$

Formally these are the energy eigen values of hydrogen atom : n_2 is the principal quantum number of the holeum, $n_2 = 1, 2, 3, \ldots \infty$. The size r_2 and the mass M_2 of the holeum are given by

$$r_2 = (R_1 + R_2)\, n_2 / (2\alpha_2^2) \quad (7),$$

$$M_2 = (m_1 + m_2)\,(1 - m_1 m_2 \alpha_2^2 / (2n_2^2(m_1 + m_2)^2)) \quad (8),$$

where

$$R_i = 2m_i G / c^2, \qquad i = 1,2 \quad (9),$$

are the radii of the black holes. Since the area $4\pi R^2$ of a black hole of radius R never decreases and since the black holes must not overlap, therefore we must have

$$r_2 \geq R_1 + R_2 \quad (10),$$

for all n_2 and at all times. From Eqs. (7) and (10) we can prove the following theorem.

Theorem : A holeum will have a stable ground state, $n_2 = 1$, only if

$$\alpha_2^2 < ½ = \alpha_C^2 \quad (11),$$

otherwise it can exist only in excited states.

$$n_2 > 2\alpha_2^2 > 1 \quad (12).$$

The latter case will eventually result in coalescene of the black holes and destruction of the holeum[2]. Using Eqs. (2) and (6) we can rewrite Eq.(11) as

$$(m_1.m_2)^{½} < m_C = 2^{-¼}\, m_P \quad (13),$$

α_C and m_C are called the cosmic limits for the formation of stable holeums. In reference[2] we considered unstable holeums satisfying Eq.(12). In this paper we consider only the stable holeums satisfying Eqs. (11) and (13). Now a holeum of mass M_2, Eq.(8), picks up a black hole of mass m_3 to form a holeum of mass M_3 and so on. After a holeum has gobbled up k black holes the following situation obtains :

$$E_k = -\mu_k c^2 \alpha_k^2 / (2\, n_k^2) \quad (14),$$

where the reduced mass μ_k is given by

$$1 / \mu_k = 1 / m_k + 1 / M_{k-1} \quad (15),$$

$$\alpha_k = m_k M_{k-1} / m_P^2 \quad (16).$$

Here a bound state of k -1 black holes of mass M_{k-1} has picked up a black hole of mass m_k to form a bound state of mass M_k. Eq.(14) gives a "global view" of a k–holeum. It shows what is formally a hydrogen atom spectrum except that the fine

structure constant $\alpha = e^2 / \hbar c$ is replaced by the gravitational coupling constant α_k. In a "local view", the following equations reveal all the k black holes it has gobbled up :

$$E_k = -(mc^2 \alpha_g^2 / 2\, n_k^2)\, ((k-1)^3 / k)\, g_{k-1}^3 / f_{k-1} \quad (17),$$

$$g_{k-1} = 1 - \alpha_g^2 Z_{k-1} / 2(k-1) \quad (18),$$

$$f_{k-1} = 1 - \alpha_g^2 Z_{k-1} / 2k \quad (19),$$

$$Z_{k-1} = \sum_{j=2}^{k-1} ((j-1)^3 / j n_j^2)\, g_{j-1}^3 / f_{j-1} \quad (20),$$

$$r_k = (kRn_k / 2\alpha_g^2 (k-1)^2)\, (f_{k-1} / g_{k-1}^2) \quad (21),$$

$$M_k = km g_k \quad (22),$$

$$\alpha_k = (k-1)\, \alpha_g\, g_{k-1} \quad (23),$$

where we have used the recursion relations, Eqs. (15) and (16) to arrive at Eq.(17) from Eq.(14). For simplicity here we have specialized to the case of k identical black holes each of mass m and radius R. To produce the most compact nonoverlapping bound state, in Eq.(21) we must have

$$2(k-1)2\, \alpha_g^2 = 1 \quad (24),$$

Then it can be shown that

$$f_{k-1} \geq g_{k-1}^2 \quad (25),$$

So, that from Eqs. (21) and (25), we have

$$r_k > kRn_k \quad (26),$$

for all n_k and k. Eq. (26) is a condition for the stability of a k-Holeum. It is an exclusion condition similar to the Pauli exclusion principle. No two black holes can overlap each other at any time. It is amusing to note that Eq.(24) may be rewritten as

$$(m_P / m)^4 = 2\,(k-1)^2 = 2, 8, 28, 32, \ldots \quad (27),$$

Eqs. (24) and (27) signify the quantization of the coupling constant and the mass of the black holes respectively and at the same time Eq.(27) gives the magic number sequence of atomic physics. The latter arises from the spin ½ of the electrons and gives the total number of electrons in a closed shell. Our black holes have no spin.

Their overlap and the maximum compactness of their bound states give Eqs. (24) and (26). Now if the size of a compact bound state is at all times greater than or equal to kRn_k for arbitrary n_k then it can only be a straight linear chain or a straight string. In an endless, ring-like, structure all the black holes, in an identical quantum state, are equivalent. By reductio ad absurdum it can be proved that a holeum is not an endless, ring-like structure. Now we make Taylor series expansions of f_{k-1} and g_{k-1} treating α_g^2 as a small parameter. Keeping only the first two terms we get,

$$E_{nn'} = -(R_g / n^2)(1 - 1/6n'^2 + o(k^{-1})) \quad (28),$$

$$M_{nn'} = M_g (1 - 1/12n'^2 - 1/(4k^{1/2} . n^2) + o(k^{-1})) \quad (29),$$

$$\alpha_{nn'} = \alpha_C (1 - 1/12n'^2 + o(k^{-1})) \quad (30),$$

$$r_{nn'} = nr_g (1 + 1/12n'^2 + o(k^{-1})) \quad (31),$$

$$R_g = m_c c^2 (k-1)^{1/2} / 4k \quad (32),$$

$$M_g = m_c k^{1/2} \quad (33),$$

$$r_g = R_c k^{1/2} \quad (34),$$

$$R_C = 2m_c G / c^2 \quad (35),$$

$$\alpha_{jn'} = \alpha_C (k-1)^{-1} (j-1)(1 - 1/12n'^2 + o(k^{-1})) \quad (36),$$

Here $n_2 = n_3 = ... = n_{k-1} = n'$ and $n_k = n$. R_g, Eq. (32), is the gravitational Rydberg constant. Here we have assumed $k >> 2$ but exact equations can be written down for $k = 2,3,4$, after which they become cumbersome. Every hydrogen atom-like energy level characterized by $n_k = n$ is spread out into a band whose lower and upper bounds correspond to $n' = \infty$ and $n' = 1$, respectively. Here $n' = 1$ corresponds to all the inner subholeums being in their ground states and $n' = \infty$ corresponds to their complete dissociation leaving only the two outer most black holes to define the holeum. For k=2 there is no spreading of energy levels into bands and the spectrum is formally identical with the line spectrum of the hydrogen atom. Spreading begins with k=3 where it is about 3.5 % and reaches an upper limit of 16.7 % for k>>2. Normally all the inner subholeums will be in their ground states, n'=1, and the system will be described by only one quantum number n. Greater excitation may lead to the raising of n_{k-1} to values greater than unity. And the system will be described by n_k and n_{k-1} and all the other n_i's set equal to unity and so on. From Eq.(36) it is clear that the gravitational coupling $\alpha_{jn'}$ of the jth subholeum from the centre goes like k^{-1} for small values of $j << k$ and that it tends to the cosmic limit

α_c as $j \rightarrow k$ for $k>>2$. This means that the black holes near the centre of the holeum display asymptotic freedom while those near the edge, $j \rightarrow k$, show a saturation of their coupling strength to the upper bound α_c. In this theory there are no free parameteres except k which is related to an appropriate temperature of the big-bang universe. The temperature at which the rate of reaction of holeum with black holes falls below the rate of expansion of the big-bang universe the holeum stops picking up more black holes and its k value freezes. In a treatment similar to that of nucleosynthesis we assume that the k – holeums are in thermal equilibrium at a temperature $T << M_k c^2 / k_B$ and their number density N_k is given by

$$N_k = g_k (M_k k_B T / (2\pi\hbar^2))^{3/2} \exp((\mu_k - M_k c^2) / k_B T) \quad (37),$$

where g_k is the degeneracy factor and $\mu_k = k\mu$ where μ is the chemical potential of a black hole interacting with another one. Eliminaing μ between Eq.(37) and a similar one for N, the number density of black holes of mass m, we get

$$N_k = (N/4)\, g_k\, k^{3/2}\, \xi^{k-1} \exp(B_k / k_B T) \quad (38),$$

where,

$$\xi = (N/4)(2\pi\hbar^2 / (mk_B T)^{3/2} \quad (39),$$

$$B_k = kmc^2 - M_k c^2 \cong m_c c^2 k^{1/2} / 12 n'^2 \quad (40).$$

The smallness of ξ^{k-1} is compensated for by the largeness of $\exp(B_k / k_B T)$ at a temperature T_k given by

$$T_k = B_k / (k_B (k-1) \mid \ln \xi \mid) \quad (41).$$

Since no empirical data exists on the reaction rates or the number densities of holeums and black holes we consider two widely different values : $N = 10^{-10}$ per cm^3 and $N = 10^{10}$ per cm^3 at 3° k. This is to be compared with 10^{-6} per. cm^3 which is the number density of photons at 3° k. For $n' = n = 1$ and $k_B T \doteq 10$ eV, say we calculate from Eq.(41) k to be 5.4449×10^{47} and 1.5178×10^{48} for the two values of N referred to above, respectively. This is like a galaxy of black holes in a holeum. The former value of k corresponds to M_k=1.2391 x 10^{16} kg, r_k=0.22 A and m=14.00 eV/c^2. Thus, we have an atom-sized holeum having a mass that is a hundred millonth that of the earth and containing a galaxy of black holes of mass 14 keV / c^2 each. These holeums have only the gravitational interactions and will, therefore, constitute an important compenent of dark matter in the unniverse. As opposed to this the particles of ordinary matter have all the four fundamental interactions. Hence, clouds of holeums will seperate from those of the ordinary matter and will form invisible haloes around the galaxies. The enormous mass of holeums and their submicroscopic size will present formidable obstacles to intergalactic travel as they would form impregnable walls around the galaxies. The lower and the upper bounds for the frequencies of gravitational radiation emitted by a holeum in the Lyman series $n \to 1$ are $7\nu_o / 12$ and ν_o, respectively, where

$$\nu_o = R_g / h = 6.22 \times 10^{40} (k-1)^{1/2} k^{-1} \qquad (42).$$

For the atom-sized holeum mentioned above, this is in the range 10^{15} Hz. It can be shown that the maximum population density of holeums with $k >> 2$ and $k_B T = 10$ eV, say, occurs at $n=5$ for $n' = \infty$ and at $n' = 4.56 \approx 5$ for $n' = 1$. Thus, the quantum numbers of subholeums make little difference here. For small holeums having $k = 2,3,4$ etc. the maximum population density occurs at about $n = 10^{10}$ and the values of n' make little difference. Thus, the smaller holeums are in much greater states of excitation and are more prone to radiate than the larger ones. In the transitions of the type $n + 1 \to n$ with $n >> 1$ the maximum frequency emitted is given by

$$\nu_{max.} \approx 2 R_g / n^3 = 1.244 \times 10^{41} (k-1)^{1/2} k^{-1} n^{-3} \qquad (43),$$

and this could well be in the low frequency domain ν = 1500 to 1600 Hz where detectors are being built[3]. Thus, the holeums would be an important source of gravitational radiation. k can be determined, for example, by experimentally finding the lower or the upper limits of the Lyman series. This, in turn, completely determines all the attributes of the holeums emitting this radiation.

In the high energy accelerator experiments involving the collisions of particles and antiparticles highly excited states of vacuum, mini bigbangs, are produced. This will lead to the production of micro black holes and their stable holeums which will be manifested in the form of missing mass, momentum etc.

Giving spin, charge etc. to the black holes may lead to stable holeums having properties of quarks, leptons etc. This is receiving attention and will be reported elsewhere.

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