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ENHANCEMENT OF NEUTRINO CONVERSION IN MEDIUM AND THE EARTH EFFECT

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Abstract

We analyze the mechanisms (MSW and parametric resonance etc.) of neutrino resonance when neutrinos propagate in matter. All enhancements are classified in only two types. One is induced by the mixing angle via factor $\sin^2 2\theta_m$ in the probability, which is well known as the MSW effect. The other is caused by the oscillating phase ϕ which appears in the factor $(1 - \cos\phi(t))$. When the solar and atmospheric neutrinos pass through the earth, this second type enhancement plays an important role in distinguishing different oscillation channels.

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I. INTRODUCTION

Matter can play an important role when the 'Natural' neutrinos pass through the earth. It was found that one of the enhancements is not due to MSW resonance but because the oscillating phases inside the earth satisfy a certain relationship [1,2]. When the high energy (e.g., $E_\nu = 25$ GeV) atmospheric neutrinos cross both the core and the mantle of the earth, an obvious dip appears in the $\nu_\mu \rightarrow \nu_s$ survival probability with respect to the zenith angle. It corresponds to the position where three phases in three layers are approximately π. This can be explained as an "extended parametric resonance". This effect also results in the deep night solar neutrino events when they pass through the core of the earth, which was explained with another name NOLR resonance [3]. In this paper we will try to resolve these different explanations.

For two neutrino species, a neutrino state vector v_i can be described by an angle α_i:

$$v_i = \nu_a \cos\alpha_i \ + \ \nu_b \sin\alpha_i, \tag{1}$$

where ν_a and ν_b are a pair of orthogonal states, e.g., flavor eigenstates. They are admixtures of energy eigenstates in vacuum(matter) with a parameter: the mixing angle $\theta_{(m)}$,

$$\begin{pmatrix} \nu_a \\ \nu_b \end{pmatrix} = \begin{pmatrix} \cos\theta_{(m)} & \sin\theta_{(m)} \\ -\sin\theta_{(m)} & \cos\theta_{(m)} \end{pmatrix} \times \begin{pmatrix} \nu_{1(m)} \\ \nu_{2(m)} \end{pmatrix} \tag{2}$$

II. PHASE ENHANCEMENT VERSUS MIXING ANGLE (MSW) ENHANCEMENT

In a constant density medium, after the neutrinos pass a distance $x \equiv t$ [†], a phase appears in eq. (2)

$$\begin{pmatrix} \nu_a(t) \\ \nu_b(t) \end{pmatrix} = \begin{pmatrix} \cos\theta_{(m)} & \sin\theta_{(m)} \\ -\sin\theta_{(m)} & \cos\theta_{(m)} \end{pmatrix} \times \begin{pmatrix} \nu_{1(m)} \\ \nu_{2(m)} \cdot e^{i\phi(t)} \end{pmatrix}. \tag{3}$$

This phase is caused by either the energy or the momentum difference between $\nu_{1(m)}$ and $\nu_{2(m)}$. The survival amplitude is then

$$A \ = \ 1 + \sin^2\theta_{(m)}(e^{i\phi(t)} - 1) \tag{4}$$

[†]The neutrino is extremely relativistic.

and the survival probability reads

$$P \;=\; 1 - \frac{1}{2}\sin^2 2\theta_{(m)}(1 - \cos\phi(t)). \tag{5}$$

These two equations have two degrees of freedom, the mixing angle $\theta_{(m)}$ and the phase $\phi(t)$. Thus there are two ways of enhancing the oscillation: via mixing angle (MSW) and/or via phase enhancements. In fact, eq. (5) can be written as

$$P \;=\; 1 - \sin^2 2\theta_{(m)} \sin^2 \frac{\phi}{2}. \tag{6}$$

Here we see that $2\theta_{(m)}$ and $\phi/2$ look symmetric to each other. $\sin^2 2\theta_{(m)}$ takes its maximal value 1 when the MSW resonance happens. On the other hand, neutrino transition can be big via certain combinations of phases in different density medium layers. Especially when the mixing angles are small, a resonance via the phase, which is called parametric resonance, can lead to a large transition which is of the order of 1. However, eqs. (5) (6) are valid only in constant medium. Thus constraints on $2\theta_m$ and $\phi/2$ are different.

III. NEUTRINOS IN A MULTI-LAYER MEDIUM

Consider a neutrino state v traveling in a fluctuating medium which contains n layers, where it is a constant density structure in each layer (Fig. 1). One can get a coupling angle between a reference vector ν_b and the final state vector at resonance,

$$\alpha_n = (-1)^n \left[\sum_{k=1}^{n} (-1)^k \cdot 2\theta_m^k + \alpha_0 \right]. \tag{7}$$

Here θ_m^k is the matter mixing angle in the number k-th layer, while α_0 is an initial coupling angle. At this resonance, all phases take values of an odd multiple of 2π, $\phi_i = (2k+1)\pi$. The transition probability is then

$$P(v \to \nu_b; t) = \sin^2 \alpha_n. \tag{8}$$

The fluctuating-like density profile is needed to enhance angle α_n, which is amplified by keeping all $(2\theta_m^{2l} - 2\theta_m^{2l-1})$ in same sign for all pairs of steps.

To study resonance/enhancement, the conversion between orthogonal states is essential. The reason is, when α_0 is close to $\pi/2$, one can get a large value of P in eq. (8) even if there isn't any oscillation at all. Thus, adding up an initial angle α_0 is just making the problem more complicated. Here we will choose $\alpha_0 = 0$.

• The series of angle α_i are the angles for the i-th layer in eq. (7). If all α_i are less than $\pi/2$, from eq. (8) an enhancement of angle α_n is equivalent to an enhancement of the oscillation. Otherwise, the enhancement is overleapt due to the *SINE* function.

• If $(2\theta_m^{2l} - 2\theta_m^{2l-1})$ is at the order of $\pi/2$, or if some $2\theta_m^i > \pi/2$, after a few steps P would lose the order of enhancement and become chaotic. One needs to vary the phase $\phi^i(R_i, L_m^i)$ from $(2k+1)\pi$ in order to get a maximal P. For a 2-3 layer medium profile, the condition of $P = 1$ in this chaotic region has been studied in [4]. In a graphic representation of neutrino oscillation, the more general condition can easily be obtained . An earlier paper [5] had also studied that an jump in a density profile can result in an interference peak in P, which can be of the order of 1.

IV. PARAMETRIC RESONANCE AND NEUTRINOS IN EARTH

In a classical equation of oscillating motion of the form

$$\ddot{x} + \omega^2(t)x = 0, \tag{9}$$

if the $\omega(t)$ is periodic such that $\omega(t+T) = \omega(t)$, then eq. (9) is invariant under the transformation $t \to t+T$. One can get a solution to (9)

$$x = \mu^{t/T}\Pi(t), \ |\mu| > 1, \tag{10}$$

which increases exponentially with time. "This is called *parametric resonance*" [6]. A simple example is that of a pendulum whose point of support executes a given periodic motion in a vertical direction. The increasing function doesn't need to be always an exponential function. But the amplitude goes to infinity when $t \to \infty$.

Parametric resonance of neutrino oscillation was studied in [7–9]. In [8] a "castle wall" style density profile was considered, where the density fluctuation is a small perturbation in order to satisfy the standard parametric resonance condition

$$(2\theta_m^{2l} - 2\theta_m^{2l-1}) \ll \pi/2. \tag{11}$$

However, papers [1,2] first studied a phase enhancement when the neutrinos propagate through the earth within high energy atmospheric neutrino observations. The earth density distribution is almost spherically symmetric with two major density structures, the mantle ($\rho \approx 4.5$ g/cm^2; depth $\approx$ 2885 km) and the core ($\rho \approx 11.5$ g/cm^2; radius $\approx$ 3486 km). This earth model corresponds that neutrinos cross through three layers with the first and the third have the same structures. When the phases satisfy

$$\phi_1 \approx \phi_2 \approx \phi_3 \approx (2k+1)\pi, \tag{12}$$

One can get a parametric resonance-like transition probability for ν_μ to ν_s (sterile neutrino)

$$P(\nu_\mu \to \nu_s) \;=\; \sin^2(4\theta_m^{man} - 2\theta_m^{core}) \qquad (13)$$

which is a special case of eqs. (7) (8). For solar neutrinos, SuperKamiokande has observations on both energy and zenith angle spectra. The former is still in progress [10]. The latter is the day-night effect [11]. When the small mixing angle MSW solar neutrinos pass through the core of the earth, a phase type enhancement appears in the 5th bin of the night observation of the SuperKamiokande [12,3,13] ‡.

For these realistic 'natural' neutrinos in the earth, condition (11) is not valid. It is close to the limit of the parametric resonance. Since there were some arguments on this resonance in the literature, we give here our comments:

1) When angle $(2\theta_m^{2l} - 2\theta_m^{2l-1})$ is not small, the resonance is something different from classical parametric resonance (where the amplitude goes to infinity when $t \to \infty$). Instead it is a perfect parametric resonance of another quantity - the angle α_n in eq. (7). But since it had first been studied in correct pictures of physics [9,1,2], we follow those denominations.

2) This resonance (parametric/NOLR) needs more periods along the neutrino path. Its resonance condition inside a period is, in fact, a previous preparation for further periods. That is, it doesn't give a strongest transition mode in a single period, but a resonance for the global transition of more periods. For solar and atmospheric neutrinos passing through the earth, it is at the limit of this resonance.

3) Along the periods/layers of the neutrino trajectory, there is no need to require the same period and density difference, i.e.,

$$T^i \neq T^j \qquad \text{and} \qquad \triangle\rho^{\rm i} \neq \triangle\rho^{\rm j} \qquad (14)$$

are allowed, because parametric resonance can have different modes along the path with a same resonant effect.

In conclusion, we argue that the two parameters in two flavor neutrino oscillations: the mixing angle $\theta_{(m)}$ and the phase ϕ, cause all neutrino enhancements. There are MSW (mixing angle) enhancement/resonance and phase enhancement, of which parametric resonance is a special resonant case. These two, together with their admixture, play all amplifying oscillation effect in medium.

‡In [3] a different name for parametric resonance was suggested - neutrino oscillation length resonance, whose resonance condition satisfies general neutrino parametric resonance conditions.

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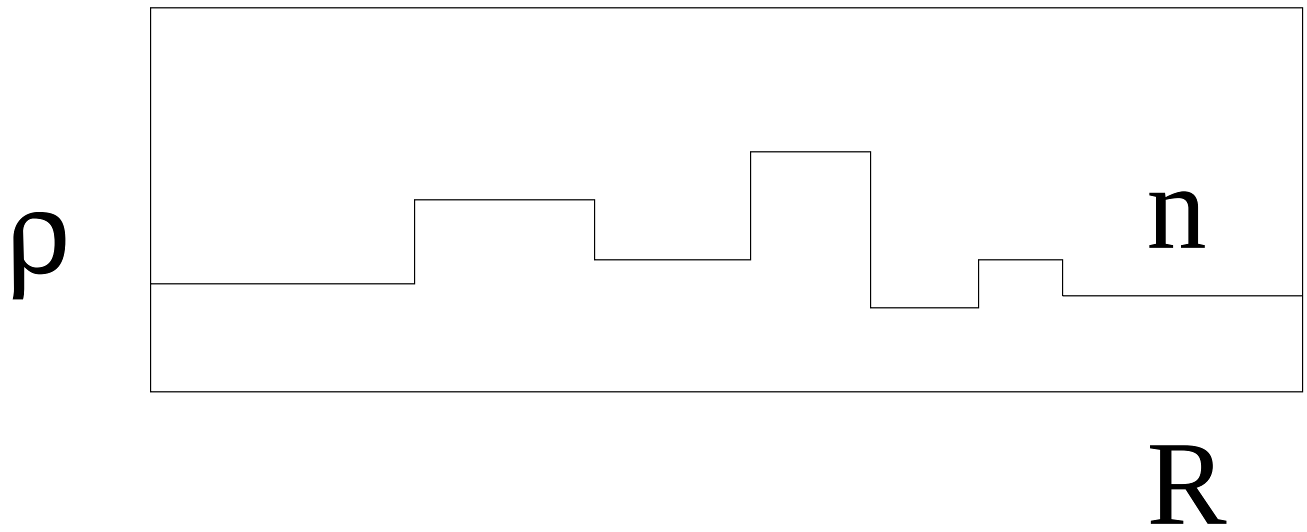

FIG. 1. A multi-layer medium with fluctuating profile.