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BUDKER INSTITUTE OF NUCLEAR PHYSICS

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$a_1(1260)\pi$ DOMINANCE IN THE PROCESS $e^+e^- \to 4\pi$ AT ENERGIES $1.05 - 1.38$ GeV

Budker INP 98-83

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Novosibirsk
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Budker Institute of Nuclear Physics, Novosibirsk, 630090, Russia

J.A.Thompson
University of Pittsburgh, Pittsburgh, PA 15260, USA

V.W.Hughes
Yale University, New Haven, CT 06511, USA

Abstract

First results of the study of the process $e^+e^- \to 4\pi$ by the CMD-2 collaboration at VEPP-2M are presented for the energy range 1.05–1.38 GeV. Using an integrated luminosity of 5.8 pb^{-1}, energy dependence of the processes $e^+e^- \to \pi^+\pi^-2\pi^0$ and $e^+e^- \to 2\pi^+2\pi^-$ has been measured. Analysis of the differential distributions demonstrates the dominance of the $a_1\pi$ and $\omega\pi$ intermediate states. Upper limits for the contributions of other alternative mechanisms are also placed.

1 Introduction.

Investigation of e^+e^- annihilation into hadrons at low energies provides unique information about interaction of light quarks and spectroscopy of their bound states. At present the energy range below J/ψ can not be satisfactorily described by QCD. Future progress in our understanding of the phenomena in this energy range is impossible without accumulation of experimental data vitally important to check the predictions of existing theoretical models. In addition, the total cross section of e^+e^- annihilation into hadrons at low energies as well as the cross sections of exclusive channels are necessary for precise calculations of various effects. These include strong interaction contributions to vacuum polarization for $(g\text{-}2)_\mu$ and $\alpha(M_Z^2)$ [1], tests of standard model by the hypothesis of conserved vector current (CVC) relating $e^+e^- \to$ hadrons to hadronic τ-lepton decays [2, 3], determination of the QCD parameters based on QCD sum rules [4] etc.

The energy behavior of the total cross section as well as that of the cross sections for exclusive channels is complicated and characteristic of various broad overlapping resonances (e.g. ρ, ω, ϕ recurrencies) with numerous common decay channels having energy thresholds just in this energy range. Presence of the broad resonances in the intermediate state necessitates consideration of the quasistationary states and makes effects of their interference important.

Until recently the investigation of e^+e^- annihilation into hadrons was restricted by measurements of the cross sections only. Appearance of the new detectors with a large solid angle operating at high luminosity colliders and providing very large data samples opens qualitatively new possibilities for the investigation of the multihadronic production in e^+e^- annihilation.

Production of four pions is one of the dominant processes of e^+e^- annihilation into hadrons in the energy range from 1.05 to 2.5 GeV. For the first time it was observed in Frascati [5] and Novosibirsk [6]. First experiments with limited data samples allowed one to qualitatively study the new phenomenon of multiple production of hadrons and estimate the magnitude of the corresponding cross sections. Subsequent measurements by different groups in Frascati, Orsay and Novosibirsk (see the references in [1]) provided more detailed information on the energy dependence of the cross sections of the processes $e^+e^- \to 2\pi^+2\pi^-$ and $e^+e^- \to \pi^+\pi^-2\pi^0$ in comparison with the previous measurements.

One of the main difficulties in the experimental studies of four pion production was caused by the existence of different intermediate states via which the final state could be produced, such as

$$e^+e^- \rightarrow \omega\pi \quad (1)$$
$$e^+e^- \rightarrow \rho\sigma \quad (2)$$
$$e^+e^- \rightarrow a_1(1260)\pi \quad (3)$$
$$e^+e^- \rightarrow h_1(1170)\pi \quad (4)$$
$$e^+e^- \rightarrow \rho^+\rho^- \quad (5)$$
$$e^+e^- \rightarrow a_2(1320)\pi \quad (6)$$
$$e^+e^- \rightarrow \pi(1300)\pi \quad (7)$$

The relative contributions of the above mentioned processes can't be obtained without the detailed analysis of the process dynamics. First attempts of this type were performed by MEA [7] and DM1 [8] in the energy range above 1.4 GeV and OLYA [9] and CMD [10] below 1.4 GeV and it was shown that the $2\pi^+2\pi^-$ final state is dominated by the $\rho^0\pi^+\pi^-$ mechanism. ND [11] measured the cross section of $\omega\pi$ production from 1.0 to 1.4 GeV with a magnitude which was confirmed by the subsequent τ decay studies at ARGUS [12] as well as by more recent results from CLEO [13] and ALEPH [14].

Later the DM2 group tried to perform partial wave analysis (PWA) of the mode with four charged pions [15] in the energy range 1.35 to 2.40 GeV. Their analysis was based on the momentum distributions only while the angular dependence as well as interference between different waves were not taken into account. Although they obtained some evidence for the presence of $a_1(1260)\pi$ and $\rho\sigma$ states, a mechanism for a substantial part of the cross section was not determined. PWA for the mode $\pi^+\pi^-2\pi^0$ was not performed because of the insufficient number of completely reconstructed events.

The abundance of various possible mechanisms and their complicated interference results in the necessity of simultaneous analysis of two possible final states ($2\pi^+2\pi^-$ and $\pi^+\pi^-2\pi^0$) which requires a general purpose detector capable of measuring energies and angles of both charged and neutral particles. The first detector of this type operating in the energy range below 1.4 GeV is the CMD-2 detector at VEPP-2M collider in Novosibirsk [16]. In this work, we present results from a model-dependent analysis of both possible channels in e^+e^- annihilation into four pions based on data collected with the CMD-2 detector. To describe four pion production we used a simple model assuming quasitwoparticle intermediate states and taking into account the important effects of the identity of the final pions as well as the interference of all possible amplitudes.

2 Data sample and event selection

The analysis described here is based on 5.8 pb^{-1} of e^+e^- data collected at center-of-mass energies $2E_{beam}$ from 1.05 up to 1.38 GeV. The data were recorded at the VEPP-2M e^+e^- collider of the Budker Institute of Nuclear Physics in Novosibirsk, Russia with the CMD-2 detector in 1997. The energy range mentioned above was scanned twice with a step of 20 MeV: first by increasing energy from 1.05 to 1.37 GeV and then by decreasing energy from 1.38 to 1.06 GeV.

The CMD-2 is a general purpose detector consisting of a drift chamber (DC) with about 250 μ resolution transverse to the beam and proportional Z-chamber used for trigger, both inside a thin (0.4 X_0) superconducting solenoid with a field of 1 T. Photons are detected in the barrel CsI calorimeter with 8-10 % energy resolution and the endcap BGO calorimeter with 6 % energy resolution. More details on the detector can be found elsewhere [16].

2.1 Event selection

The present analysis is based on completely reconstructed $\pi^+\pi^-2\pi^0$ events. At the initial stage, events with one primary vertex with two opposite sign tracks and four or more reconstructed photons were selected. Both tracks should come from the interaction region: a distance from the track trajectories to the beam axis should be less than 0.3 cm and the vertex position along the beam axis should be inside $\pm$ 10 cm. To reject background from collinear events, the acollinearity angle between tracks in the $R-\phi$ plane should be greater than 0.1 radians. Both tracks are required in the DC fiducial volume: a θ angle should be inside $0.54 \div \pi - 0.54$ radians. Clusters of energy deposition that are not matched with charged track projection are paired to form π^0 candidates. These showers must have energies greater than 20 MeV, and invariant mass of the photon pair must lie within 3σ of the π^0 mass where σ varies between $(5 \div 10)$ MeV. After that a kinematic fit was performed assuming the $\pi^+\pi^-2\pi^0$ hypothesis for all possible π^0 pairs. For further analysis the combination with $min(\chi^2)$ was selected under the condition $\chi^2/ndf < 2.5$. After such selection 22128 events remained in the energy range under study.

2.2 Final event sample

To understand the dynamics of the process we studied the distribution over the recoil mass for one of the π^0's. Figure 1 shows this distribution at the beam energy of 690 MeV. Each event gives two entries to the histogram corresponding to two π^0. A signal from the $\omega\pi^0$ final state is clearly seen. Points with errors are the data while the smooth line is our fit corresponding to the sum of a Breit-Wigner ω signal convoluted with detector resolution and a smooth combinatorial background. The detector contribution to the signal width is about 10 MeV. The number of events under the ω peak accounts for only $\sim$ 60% of the observed events that indicates at the existence of additional intermediate states.

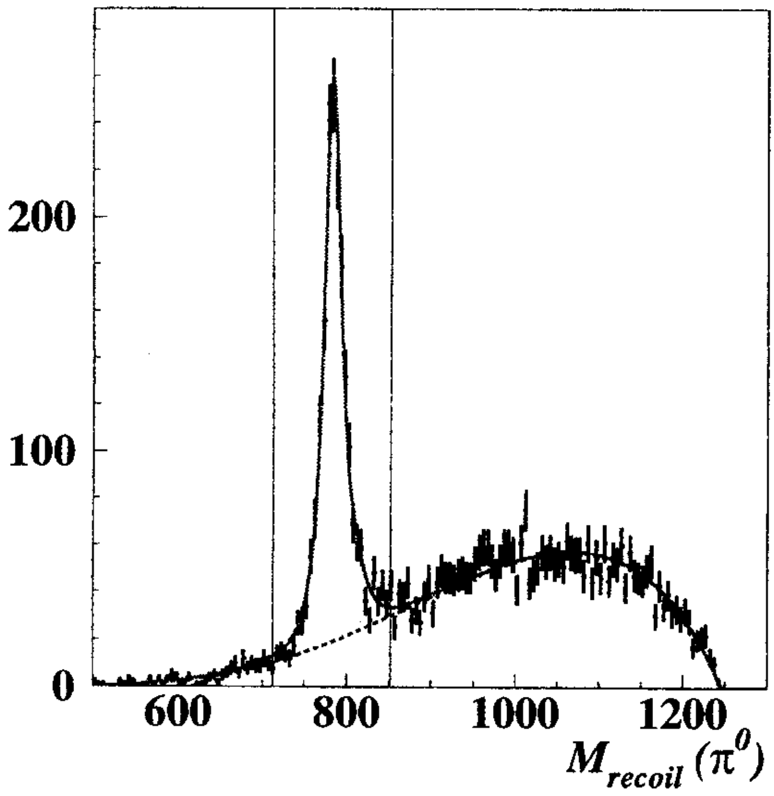

Figure 1: Distribution over the π^0 recoil mass for $\pi^+\pi^-2\pi^0$

For further analysis the data sample was subdivided into two classes:

I. $min(|M_{recoil}(\pi^0) - M_\omega|) \ < \ 70 \ MeV$

II. $min(|M_{recoil}(\pi^0) - M_\omega|) \ > \ 70 \ MeV$

where M_ω is the ω mass. The first class contains mostly $\omega\pi^0$ events while their admixture in the second class is relatively small, about $(1 \div 5)\%$ depending on the beam energy.

Figs. 2 and 3 show distributions over $M_{inv}(\pi^{\pm}\pi^0)$ and $M_{inv}(\pi^+\pi^-)$ for the events in the second class. In the spectrum of $M_{inv}(\pi^{\pm}\pi^0)$ one can see a clear signal of $\rho^{\pm}$ while that for $M_{inv}(\pi^+\pi^-)$ is relatively smooth and no signal from the ρ^0 is observed. The solid lines show our fit including smooth combinatorial background and gaussian ρ signals. Presence of the $\rho^{\pm}$ signal and absence of the ρ^0 signal lead us to the natural assumption that ρ mesons originate from an isospin 1 resonance. For the case of the resonance with $I = 0$ one expects production of both neutral and charged ρ's.

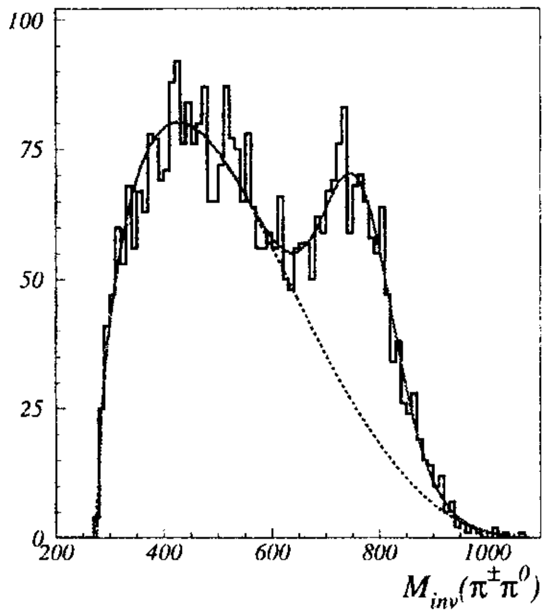

Figure 2: Distribution over $M_{inv}(\pi^{\pm}\pi^0)$ for $\pi^+\pi^-2\pi^0$ events from the class (II)

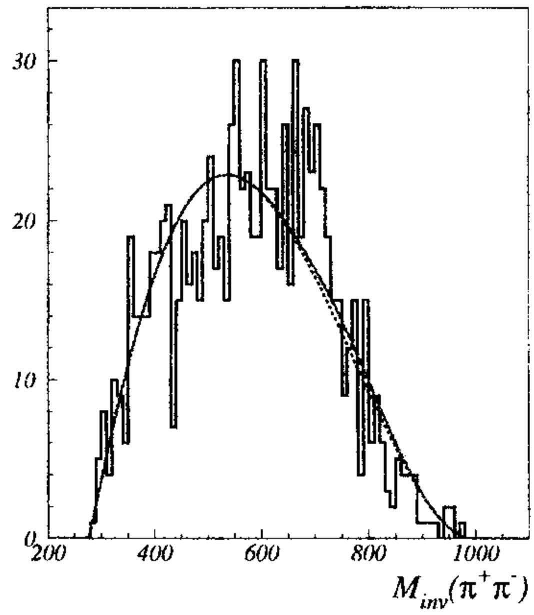

Figure 3: Distribution over $M_{inv}(\pi^+\pi^-)$ for $\pi^+\pi^-2\pi^0$ events from the class (II)

Possible candidates from the list of the processes (1)-(7) are $a_1(1260)$, $a_2(1320)$ and $\pi(1300)$. These resonances have different spin and parity resulting in different angular distributions for a recoil pion (Table 1).

Table 1: List of $I = 1$ resonances with expected angular distributions

		J^{PC}	$d\sigma/d\cos(\theta)$	χ^2/ndf
1	$a_1(1260)$	1^{++}	$const$	7.1/7
2	$\pi(1300)$	0^{-+}	$\sin^2(\theta)$	47.3/7
3	$a_2(1320)$	2^{++}	$1+\cos^2(\theta)$	37.0/7

To measure the angular distribution of the $\rho^{\pm}\pi^0$ system (or recoiled $\pi^{\mp}$) we fit the $\rho^{\pm}$ signal in eight ranges of the recoil $\pi^{\mp}$ angle with respect to the beam axis. The measured angular distribution is shown in Fig. 4. Three smooth curves show the fits corresponding to mentioned above hypotheses. The fit goodness is presented in the last column of Table 1. One can see good agreement with the hypothesis of the $a_1(1260)\pi$ intermediate state.

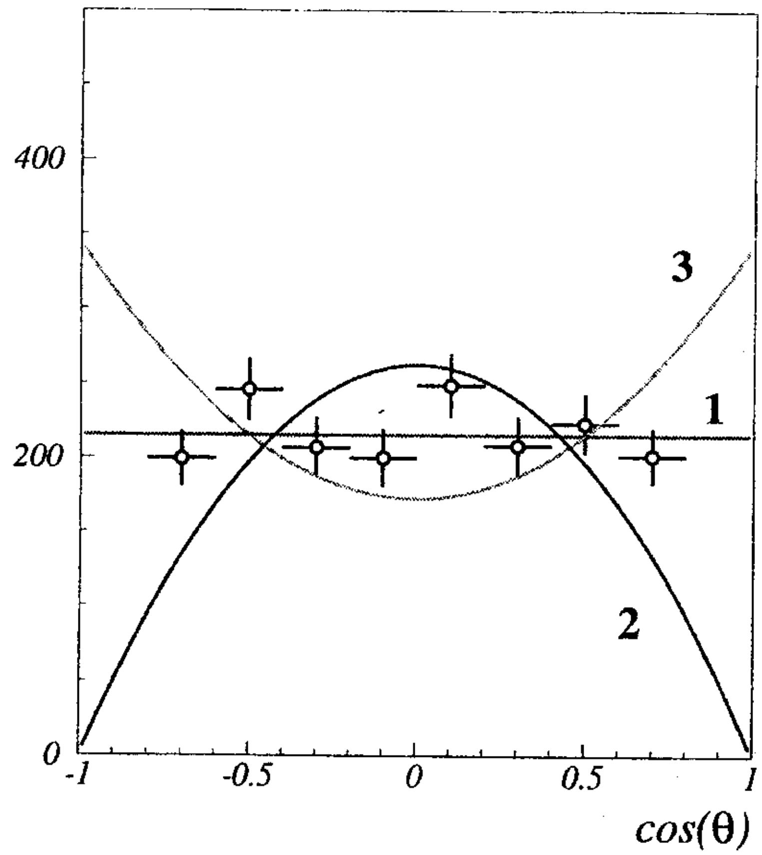

Figure 4: Angular distribution of the recoil $\pi^{\pm}$. Solid curves correspond to the following hypotheses: **1** – $a_1(1260)\pi$; **2** – $\pi(1300)\pi$; **3** – $a_2(1320)\pi$

Thus one can assume that our data can be explained by $\omega\pi^0$ and $a_1(1260)\pi$ intermediate states although small admixture of other states is not excluded.

2.3 Fitting method

For detailed comparison we perform the simulation of the processes (1)-(7) taking into account their interference as well as the interference of the diagrams differing by permutations of identical final pions (see Appendix A).

In order to extract a relative fraction of $a_1(1260)\pi$ and any other intermediate states, the unbinned maximum likelihood fit has been used [17].

Kinematics of each event is completely described by a set of measured four-momenta $p_\pi \equiv (\varepsilon, \vec{\mathbf{p}})$ of the final pions with the exception of smearing due to detector resolution and radiative effects. This set will be referred to as a vector of state $\vec{P_i} \equiv (p_{\pi^+}, p_{\pi^-}, p_{\pi^0}, p_{\pi^0})_i$ where a subscript i stands for the event number.

The theoretical probability density function for $\vec{P_i}$ depends on process dynamics and can be expressed via the matrix element $|\mathfrak{M}(\vec{P_i}, \vec{A})|^2$. Here $\vec{A}$ stands for a set of unknown model parameters like relative strength and phase of interference between the intermediate states. To obtain the optimal values of $\vec{A}$ we minimize [18] the logarithmic likelihood function

$$L(\vec{A}) = -\sum_{i=1}^{N_2} \ln w(\vec{P_i}, \vec{A}), \tag{8}$$

where $w(\vec{P_i}, \vec{A})$ is the probability to observe an event in state $\vec{P_i}$, and N_2 is the number of events in the second class. Since the detector resolution for the invariant mass of three pions is about 10 MeV, i.e. comparable to the ω width, smearing effects are significant for the events of the first class. Therefore we use the events of the second class only. To fix the fraction of events in the first class we add the following term to the function (8)

$$\frac{(r(\vec{A}) - r_1)^2}{2\sigma_{r_1}^2},$$

where $r_1 = \frac{N_1}{N_1+N_2}$ is a measured fraction, N_1 is the number of events in the first class, and $r(\vec{A})$ is the expectation of r_1.

The normalized probability density function is expressed via the matrix element

$$w(\vec{P_i}, \vec{A}) = \frac{|\mathfrak{M}(\vec{P_i}, \vec{A})|^2}{\sigma_{vis}(\vec{A})} \cdot \frac{1}{\prod_j 2\varepsilon_j},$$

where j is the particle number, and $\sigma_{vis}(\vec{A})$ is a model dependent visible cross section:

$$\sigma_{vis}(\vec{A}) = \int |\mathfrak{M}(\vec{P_i}, \vec{A})|^2 \cdot \prod_{j=1}^{4} \frac{d^3\vec{\mathbf{p}}_j}{2\varepsilon_j}.$$

The function $\sigma_{vis}(\vec{A})$ is calculated using Monte Carlo technique. For this purpose we use a set of events sampled according to the relativistic phase space distribution. In this case, $\sigma_{vis}(\vec{A})$ is calculated as

$$\sigma_{vis}(\vec{A}) = \frac{1}{N_{MC}} \sum_{i=1}^{N_{MC}} |\mathfrak{M}(\vec{P}_i, \vec{A})|^2 ,$$

where N_{MC} is the number of second class events in a generated set.

The main goal of our fits was to find the minimal model compatible with our data. To this end, we write the matrix element as follows:

$$|\mathfrak{M}(\vec{P}_i, \vec{A})|^2 = |\mathbf{J}^{+-00}_{\omega\pi^0} + Z_{a_1} \cdot \mathbf{J}^{+-00}_{a_1\pi} + Z_X \cdot \mathbf{J}^{+-00}_X|^2 ,$$

where X stands for an admixture under study — $\rho\sigma$, $h_1(1170)\pi$, $\rho^+\rho^-$, $a_2(1320)\pi$ or $\pi(1300)\pi$. In the above equation, expressions for $\mathbf{J}^{+-00}_X$ are taken from Appendix A while complex factors Z are components of the vector $\vec{A}$.

2.4 Comparison of $\pi^+\pi^-2\pi^0$ data with simulation

Since the matrix element for the channel $\omega\pi^0$ has a well-known structure, one can use the events from the first class to test the adequacy of the total MC simulation [19]. Figs. 5,6 show the distributions over $M_{inv}(\pi^\pm\pi^0)$, $M_{recoil}(\pi^\pm)$, $M_{inv}(\pi^+\pi^-)$, $M_{inv}(\pi^0\pi^0)$, $cos(\psi_{\pi^\pm\pi^0})$, $cos(\psi_{\pi^+\pi^-})$, $cos(\psi_{\pi^0\pi^0})$ and $M_{recoil}(\pi^0)$ for the events of the first class at the beam energy of 690 MeV. Points with errors are the data, while the histograms correspond to the simulation of the processes $\omega\pi^0$ and $a_1(1260)\pi$. Good consistence of the data and MC makes us confident that MC simulation adequately reproduces both the kinematics of produced particles and the detector response to them.

Similar distributions for the events of the second class are shown in Figs. 7,8. One can see that the process $\pi^+\pi^-2\pi^0$ is satisfactorily described in the minimal model in which there are two intermediate states $\omega\pi^0$ and $a_1(1260)\pi$ only. Similar consistence is observed at other energies: we have also examined the energy points $2E_{beam} = (1.28 \pm 0.01)$ and (1.18 ± 0.03) GeV.

To determine the admixture of other possible mechanisms we extend the minimal model above by adding each of the other states one by one and performing the fit. The results of these fits are shown in Table 2. From its third column one can see that the relative fractions r_X of the additional intermediate state X with respect to the $a_1(1260)\pi$ are small. This confirms our assumption of the $a_1(1260)\pi$ dominance.

Because of the theoretical uncertainty of the $a_1(1260)\pi$ matrix element, we do not consider the nonnegligible magnitude of the $\rho^+\rho^-$ and $\pi(1300)\pi$

Table 2: The results of fit to different models with upper limits

Model	L_{min}/N_{ev}	r_X, %	Upper limit, %
$\omega\pi^0 + a_1\pi$	1264/891	—	—
$\omega\pi^0 + a_1\pi + \rho\sigma$	1256/891	$2.1^{+1.2}_{-0.9}$	4.3
$\omega\pi^0 + a_1\pi + h_1\pi$	1263/891	$0.1^{+0.2}_{-0.1}$	0.4
$\omega\pi^0 + a_1\pi + a_2\pi$	1263/891	$0.2^{+0.4}_{-0.2}$	0.8
$\omega\pi^0 + a_1\pi + \pi'\pi$	1250/891	$9.5^{+3.2}_{-2.8}$	15.
$\omega\pi^0 + a_1\pi + \rho^+\rho^-$	1246/891	$4.7^{+2.0}_{-1.6}$	7.7

contributions as significant. Instead, we prefer to set upper limits for the fraction of these intermediate states. The values of the likelihood function (8) for optimal parameters (L_{min}) do not contradict to our expectation based on the simulation of $\omega\pi^0$ and $a_1(1260)\pi$.

2.5 Comparison of $2\pi^+2\pi^-$ data with simulation

Consider now the $2\pi^+2\pi^-$ channel. In this case we don't have $\omega\pi^0$ in the intermediate state and therefore no additional complicacy to check the assumption of the $a_1(1260)\pi$ dominance arises. To this end four-track events were selected. All tracks should come from the interaction region: a distance from track trajectories to the beam axis should be less than 1 cm and the vertex position along the beam axis should be inside ± 15 cm. After that a kinematic fit was performed assuming the $2\pi^+2\pi^-$ hypothesis and events with $\chi^2/ndf < 2.5$ were selected. Under these conditions 28552 events remain in the energy range under study.

Figure 9 shows distributions over $M_{inv}(\pi^+\pi^-)$, $M_{inv}(\pi^\pm\pi^\pm)$, $M_{recoil}(\pi^\pm)$ and $cos(\psi_{\pi^+\pi^-})$ for $2\pi^+2\pi^-$case. One can see that the hypothesis of the $a_1(1260)\pi$ dominance does not contradict to the data although one should note a slight deviation in the spectrum of the invariant masses of the likesign pions. This deviation can be possibly explained by the contribution of the $D-$*wave* or some final state interaction. It can't be explained by the admixture of other possible processes because their fractions compared to $a_1(1260)\pi$ are small (see Table 2). The study of more complicated cases taking into account simultaneously a few intermediate states in addition to $\omega\pi^0$ and $a_1(1260)\pi$ is now in progress.

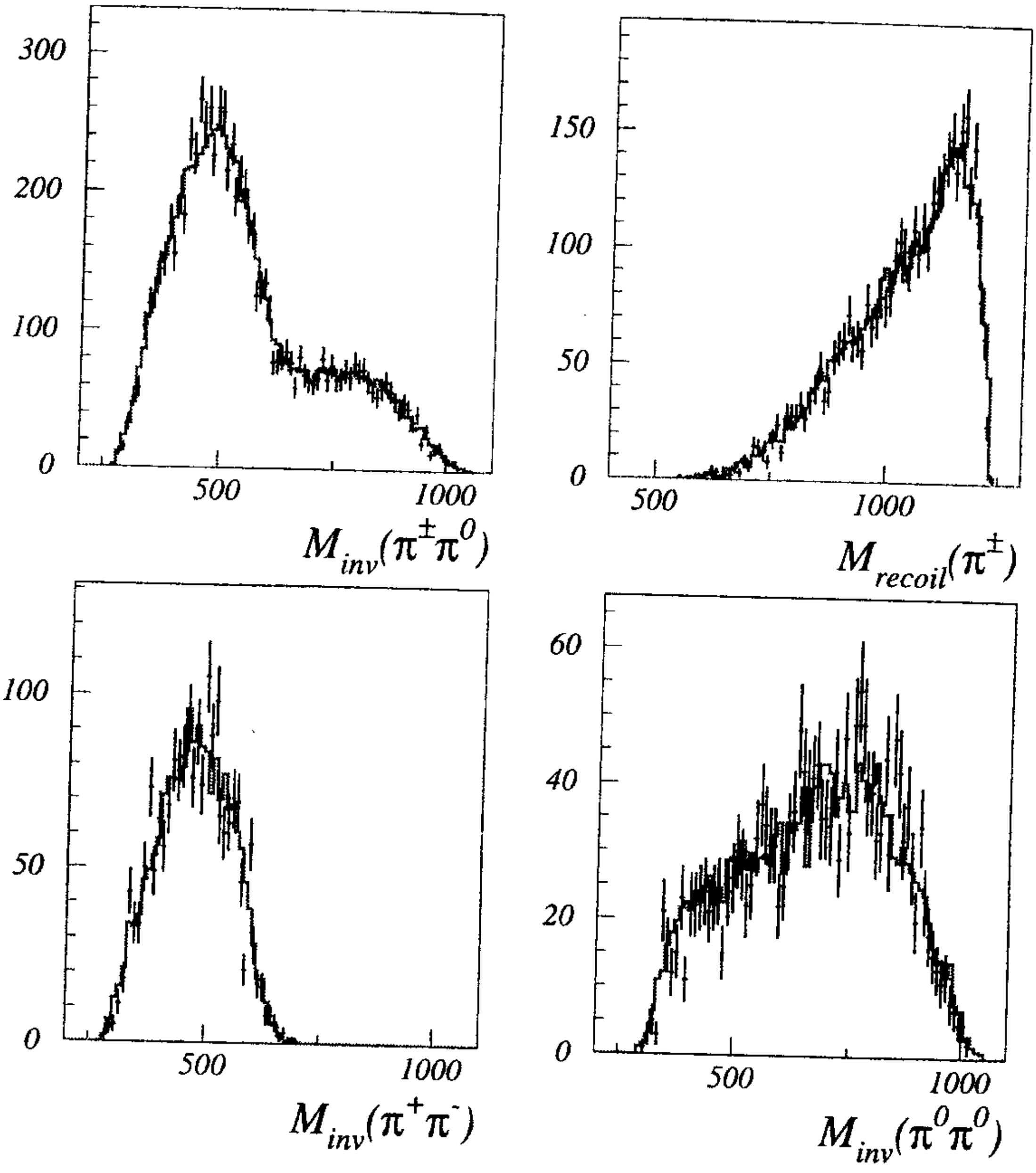

Figure 5: Distributions over $M_{inv}(\pi^{\pm}\pi^{0})$, $M_{recoil}(\pi^{\pm})$, $M_{inv}(\pi^{+}\pi^{-})$, $M_{inv}(\pi^{0}\pi^{0})$ for $\pi^{+}\pi^{-}2\pi^{0}$ events from the class (I)

where H_μ is the polarization vector of h_1. Using (21) and (12) we obtain the following representation for the h_1-meson contribution to the current $\mathbf{J}$ in the decay $\tilde{\rho} \to \pi^+(p_1)\pi^-(p_4)\pi^0(p_2)\pi^0(p_3)$:

$$\begin{aligned} \mathbf{J}_{h_1}^{+-00} &= G_{h_1}\left[\mathbf{t}_{h_1}(p_1,p_4,p_3,p_2) - \mathbf{t}_{h_1}(p_4,p_1,p_3,p_2)\right. \\ &\quad \left. - \mathbf{t}_{h_1}(p_1,p_3,p_4,p_2)\right] + (p_2 \leftrightarrow p_3), \end{aligned} \tag{22}$$

the current $\mathbf{t}_{h_1}$ is given by (15) with the change of indices $a_1 \to h_1$, and the function $g_{h_1}(s)$ in the propagator of h_1 is:

$$\begin{aligned} g_{h_1}(s) &= F_{h_1}^2(Q) \int \left| \frac{\varepsilon_2\mathbf{p}_1 - \varepsilon_1\mathbf{p}_2}{D_\rho(p_1+p_2)} - \frac{\varepsilon_2\mathbf{p}_3 - \varepsilon_3\mathbf{p}_2}{D_\rho(p_2+p_3)} - \frac{\varepsilon_3\mathbf{p}_1 - \varepsilon_1\mathbf{p}_3}{D_\rho(p_1+p_3)} \right|^2 \\ &\quad \times \frac{d\mathbf{p}_1\, d\mathbf{p}_2\, d\mathbf{p}_3\, \delta^{(4)}(p_1+p_2+p_3-Q)}{2\varepsilon_1 2\varepsilon_2 2\varepsilon_3 (2\pi)^5}. \end{aligned} \tag{23}$$

A.4 The contribution of $\rho^+\rho^-$

One more contribution to the mixed channel amplitude comes from the process $\tilde{\rho} \to \rho^+\rho^- \to 4\pi$. The matrix element corresponding to the transition $\tilde{\rho}(P) \to \rho^+(p)\rho^-(q)$ reads

$$\begin{aligned} T(\tilde{\rho} \to \rho^+\rho^-) &= F_{\tilde{\rho}\rho^+\rho^-}(P_\mu \tilde{e}_\nu - P_\nu \tilde{e}_\mu) \\ &\quad \times[(p_\mu e_\alpha^{+*} - p_\alpha e_\mu^{+*})(q_\nu e_\alpha^{-*} - q_\alpha e_\nu)^{-*}) - (\mu \leftrightarrow \nu)] \end{aligned} \tag{24}$$

where e_μ^+ and e_μ^- are the polarization vectors of ρ^+ and ρ^- respectively. Using (24), we obtain the contribution of $\rho^+\rho^-$ to the current $\mathbf{J}$ in the decay $\tilde{\rho} \to \pi^+(p_1)\pi^-(p_4)\pi^0(p_2)\pi^0(p_3)$:

$$\begin{aligned} \mathbf{J}_{\rho\rho}^{+-00} &= \frac{G_{\rho\rho}F_\rho^2(p_1+p_2)F_\rho^2(p_3+p_4)}{D_\rho(p_1+p_2)D_\rho(p_3+p_4)} \\ &\quad \times[\mathbf{p}_1(\varepsilon_3\mathbf{p}_2\mathbf{p}_4 - \varepsilon_4\mathbf{p}_2\mathbf{p}_3) - \mathbf{p}_2(\varepsilon_3\mathbf{p}_1\mathbf{p}_4 - \varepsilon_4\mathbf{p}_1\mathbf{p}_3) \\ &\quad -\mathbf{p}_3(\varepsilon_1\mathbf{p}_2\mathbf{p}_4 - \varepsilon_2\mathbf{p}_1\mathbf{p}_4) + \mathbf{p}_4(\varepsilon_1\mathbf{p}_2\mathbf{p}_3 - \varepsilon_2\mathbf{p}_1\mathbf{p}_3)] + (2 \leftrightarrow 3). \end{aligned} \tag{25}$$

A.5 The contribution of $\pi(1300)\pi$

The matrix elements for the transitions $\tilde{\rho}(P) \to \pi'(q)\pi(p)$ and $\pi'(q) \to \rho(P)\pi(p)$ are of the form (for brevity $\pi' \equiv \pi(1300)$):

$$\begin{aligned} T(\tilde{\rho} \to \pi'\pi) &= F_{\tilde{\rho}\pi'\pi}\varepsilon^{3ab}(P_\mu\tilde{e}_\nu - P_\nu\tilde{e}_\mu)q_\mu p_\nu \phi'^{a*}\phi^{b*}, \\ T(\pi' \to \rho\pi) &= F_{\pi'\rho\pi}\varepsilon^{abc}q_\mu p_\nu(P_\mu e_\nu^{b*} - P_\nu e_\mu^{b*})\phi'^{a}\phi^{c*}. \end{aligned} \tag{26}$$

Then we get for the contribution of $\pi(1300)$ to the current $\mathbf{J}$ in the decay $\tilde{\rho} \to \pi^+(p_1)\pi^+(p_2)\pi^-(p_3)\pi^-(p_4)$:

$$\begin{aligned}\mathbf{J}_{\pi'}^{++--} &= G_{\pi'}\,[\mathbf{t}_{\pi'}(p_1,p_2,p_3,p_4) + \mathbf{t}_{\pi'}(p_2,p_1,p_3,p_4) + \mathbf{t}_{\pi'}(p_1,p_2,p_4,p_3) \\ &+\mathbf{t}_{\pi'}(p_2,p_1,p_4,p_3) + \mathbf{t}_{\pi'}(p_3,p_2,p_1,p_4) + \mathbf{t}_{\pi'}(p_4,p_2,p_1,p_3) \\ &+ \mathbf{t}_{\pi'}(p_3,p_1,p_2,p_4) + \mathbf{t}_{\pi'}(p_4,p_1,p_2,p_3)]\,. \end{aligned} \tag{27}$$

In the mixed channel $\tilde{\rho} \to \pi^+(p_1)\pi^-(p_4)\pi^0(p_2)\pi^0(p_3)$ the corresponding current reads:

$$\begin{aligned}\mathbf{J}_{\pi'}^{+-00} &= G_{\pi'}\,[\mathbf{t}_{\pi'}(p_1,p_2,p_3,p_4) + \mathbf{t}_{\pi'}(p_1,p_3,p_2,p_4) \\ &+ \mathbf{t}_{\pi'}(p_4,p_1,p_3,p_2) + \mathbf{t}_{\pi'}(p_4,p_1,p_2,p_3)]\,. \end{aligned} \tag{28}$$

It (27) and (28) we use the notation

$$\mathbf{t}_{\pi'}(p_1,p_2,p_3,p_4) = \frac{F^2_{\pi'}(P-p_1)}{D_{\pi'}(P-p_1)D_\rho(p_2+p_4)}(p_2p_3-p_4p_3)(p_2+p_4)^2\,\mathbf{p}_1\,. \tag{29}$$

For the function $g_{\pi'}(s)$ in the π' propagator, one has:

$$\begin{aligned}g_{\pi'}(s) &= F^2_{\pi'}(Q)\int\left|\frac{\varepsilon_1\mathbf{p}_2\mathbf{p}_3-\varepsilon_2\mathbf{p}_1\mathbf{p}_3}{D_\rho(p_1+p_2)}+\frac{\varepsilon_3\mathbf{p}_1\mathbf{p}_2-\varepsilon_2\mathbf{p}_1\mathbf{p}_3}{D_\rho(p_2+p_3)}\right|^2 \\ &\times\frac{d\mathbf{p}_1\,d\mathbf{p}_2\,d\mathbf{p}_3\,\delta^{(4)}(p_1+p_2+p_3-Q)}{2\varepsilon_1 2\varepsilon_2 2\varepsilon_3(2\pi)^5}\,. \end{aligned} \tag{30}$$

A.6 The contribution of $\sigma\rho$

The quantum numbers of the σ resonance are $I^GJ^{PC} = 0^+0^{++}$. The matrix element of the transition $\tilde{\rho}(P) \to \sigma(q)\rho^0(p)$ is of the form:

$$T(\tilde{\rho}\to\sigma\rho^0) = F_{\tilde{\rho}\sigma\rho}(P_\mu\tilde{e}_\nu - P_\nu\tilde{e}_\mu)q_\mu e^*_\nu\phi^*_\sigma. \tag{31}$$

The corresponding contribution to the current $\mathbf{J}$ in the decay $\tilde{\rho} \to \pi^+(p_1)\pi^+(p_2)\pi^-(p_3)\pi^-(p_4)$ reads:

$$\begin{aligned}\mathbf{J}_{\sigma}^{++--} &= G_\sigma\,[\mathbf{t}_\sigma(p_1,p_2,p_3,p_4) + \mathbf{t}_\sigma(p_2,p_1,p_3,p_4) \\ &+ \mathbf{t}_\sigma(p_1,p_2,p_4,p_3) + \mathbf{t}_\sigma(p_2,p_1,p_4,p_3)]\,. \end{aligned} \tag{32}$$

In the mixed channel $\tilde{\rho} \to \pi^+(p_1)\pi^-(p_4)\pi^0(p_2)\pi^0(p_3)$ the current has the form

$$\mathbf{J}_\sigma^{+-00} = G_\sigma\mathbf{t}_\sigma(p_1,p_2,p_3,p_4)\,, \tag{33}$$

where

$$\mathbf{t}_\sigma(p_1,p_2,p_3,p_4)=\frac{F_\sigma^2(p_2+p_3)}{D_\sigma(p_2+p_3)D_\rho(p_1+p_4)}(\varepsilon_4\mathbf{p}_1-\varepsilon_1\mathbf{p}_4)\,. \qquad (34)$$

The function $g_\sigma(s)$ in the propagator of σ is equal to

$$g_\sigma(s)=(1-4m^2/s)^{1/2}\,. \qquad (35)$$

A.7 The contribution of $a_2(1320)\pi$

The quantum numbers of the $a_2(1320)$ resonance are $I^G J^{PC}=1^-2^{++}$. The matrix elements for the transitions $\tilde\rho(P)\to a_2(q)\pi(p)$ and $a_2(q)\to\rho(P')\pi(p)$ can be written in the form

$$\begin{aligned} T(\tilde\rho\to a_2\pi) &= F_{\tilde\rho a_2\pi}\varepsilon^{3ab}\varepsilon_{\mu,\nu\rho\lambda}P_\mu\tilde e_\nu p_\rho A^{a*}_{\lambda\gamma}p_\gamma q_\mu\phi^{b*}\,,\\ T(a_2\to\rho\pi) &= F_{a_2\rho\pi}\varepsilon^{abc}\varepsilon_{\mu,\nu\rho\lambda}P'_\mu e^{a*}_\nu p_\rho A^b_{\lambda\gamma}p_\gamma\phi^{c*} \end{aligned} \qquad (36)$$

where $A^a_{\mu\nu}$ is the polarization tensor of a_2 . The contributions of the a_2-meson to the current $\mathbf{J}$ in the processes $\tilde\rho\to\pi^+(p_1)\pi^+(p_2)\pi^-(p_3)\pi^-(p_4)$ and $\tilde\rho\to\pi^+(p_1)\pi^-(p_4)\pi^0(p_2)\pi^0(p_3)$ are given by the formulae (14) and (16) with the substitution $\mathbf{t}_{a_1}\to\mathbf{t}_{a2}$, where

$$\begin{aligned} \mathbf{t}_{a2}(p_1,p_2,p_3,p_4) &= \frac{F_{a2}^2(P-p_4)}{D_A(P-p_4)D_\rho(p_1+p_3)}\\ &\times\{E(\mathbf{p}_4\times\mathbf{p}_2)[\mathbf{p}_1\times\mathbf{p}_3,\mathbf{p}_2]+[p_2p_4-(p_2q)(p_4q)/m_{a2}^2]\\ &\times[\mathbf{p}_1(\varepsilon_3\mathbf{p}_2\mathbf{p}_4-\varepsilon_2\mathbf{p}_4\mathbf{p}_3)-\mathbf{p}_3(\varepsilon_1\mathbf{p}_4\mathbf{p}_2)-\varepsilon_2\mathbf{p}_4\mathbf{p}_1)\\ &+\mathbf{p}_2(\varepsilon_1\mathbf{p}_3\mathbf{p}_4-\varepsilon_3\mathbf{p}_1\mathbf{p}_4)]\}\,. \end{aligned} \qquad (37)$$

Here $q=P-p_4$, and P is the initial 4-momentum.

For the function $g_{a2}(s)$ in the propagator of a_2, we obtain:

$$g_{a2}(s)=F_{a2}^2(Q)\int\mathcal{F}_{ij}\mathcal{F}^*_{ij}\frac{d\mathbf{p}_1\,d\mathbf{p}_2\,d\mathbf{p}_3\,\delta^{(4)}(p_1+p_2+p_3-Q)}{2\varepsilon_1 2\varepsilon_2 2\varepsilon_3(2\pi)^5}\,, \qquad (38)$$

where

$$\mathcal{F}_{ij}(s)=\frac{(\mathbf{p}_2\times\mathbf{p}_3)^i\mathbf{p}_1^j}{D_\rho(p_2+p_3)}+\frac{(\mathbf{p}_1\times\mathbf{p}_3)^i\mathbf{p}_2^j}{D_\rho(p_1+p_3)}+(i\leftrightarrow j) \qquad (39)$$

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(всего 60 авторов)

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Budker INP 98-83

Ответственный за выпуск А.М. Кудрявцев
Работа поступила 13.11. 1998 г.

Сдано в набор 28.12.1998 г.
Подписано в печать 28.12.1998 г.
Формат бумаги 60×90 1/16 Объем 2.1 печ.л., 1.7 уч.-изд.л.
Тираж 150 экз. Бесплатно. Заказ N° 83

Обработано на IBM PC и отпечатано на
ротапринте ИЯФ им. Г.И. Будкера СО РАН
Новосибирск, 630090, пр. академика Лаврентьева, 11.