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Substructure recover by 3D Discrete Wavelet Transforms

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ABSTRACT

We present and discuss a method to identify substructures in combined angular-redshift samples of galaxies within Clusters. The method relies on the use of Discrete Wavelet Transform (hereafter DWT) and has already been applied to the analysis of the Coma cluster (Gambera et al. 1997). The main new ingredient of our method with respect to previous studies is the use of a 3D method instead than a 2D, i.e. we try to make use of redshift information. It is well known that redshift distortions destroy the correspondance between redshift and distance within clusters, so the physical nature of the substructures found by our method is *a priori* not obvious. By performing a series of tests on mock cluster catalogs with spatially localized substructures are that, if the subclumps are detached in phase space, our method is able to identify the existing substrtures provided that: a) one has a statistically significant number of redshifts, increasing as the distance decreases due to redshift effects; b) one knows *a priori* the scale on which substructures are to be expected. We have found that to allow an accurate recover we must have both a number of galaxies of the cluster in turn larger of about 200 for a distant cluster ($zgeq0.4$ or about 800 for a very close one, and a magnitude of completeness of our catalogues greater of $m_B = 16$.

Finally, we can conclude that the only true limitation to WTM is the necessity of knowing *a priori* the scale on which the substructure is to be found. This is an intrinsic drawback of the method and no improvement in numerical codes based on this technique could make up for it.

Key words: Galaxies: clusters of - Methods: data analysis, numerical, statistical

1 INTRODUCTION

A large fraction of Clusters of galaxies shows a very complex structure, either in the spatial and velocity distribution of the galaxies themselves (e.g. West and Bothun (1990)), or in the X-ray maps (e.g. Sarazin (1988)), or in both. Although the substructures are often clearly visible in the optical (e.g. Mellier et al. (1988) for the Coma cluster) and on the X-ray maps (Grebenev et al. (1990)) it is not always easy to make a quantitative analysis of the amount of substructure present. During the past years, many methods borrowed from statistical theory have been applied to attempt determining the number and properties of the subgroups. Most of these methods have been applied to 2D angular position galaxy maps or 1D line-of-sight velocity distributions. In a recent paper (Gambera et al. (1997), hereafter GPAB) we have tried to make use of the full 3D information available from a complete sample of galaxy projected positions and line-of-sight velocities for Abell 1656 (the Coma cluster) in order to recover and determine some morphological properties of the substructure. In this paper we will test this method on simulated cluster catalogs and we will determine the minimal requirements which have to be satisfied in order to apply it with confidence.

Our method is based on the use of the Discrete Wavelet Transform (hereafter DWT), a technique introduced during the past decade originally in turbulence studies (see e.g. Farge (1992)). Sometimes the Continuos Wavelet Transform (hereafter CWT) has been applied to characterize substructure in galaxy clusters (e.g. Escalera & Mazure (1992)), but it has been convincingly shown that the two methods are unrelated to each other (CWT is not the continuous limit of DWT), and that CWT are unsuitable to characterize the structure of clusters (Fang & Pando1992), (Pando & Fang1992). DWT admit complete, compact supported orthogonal bases (citeda, and for this reason they are suitable to the analysis of *finite* samples, as all galaxy samples are.

One point of our 3D DWT method deserves some clarification, i.e. the fact that we are using heterogeneous data, namely the projected position on the sky and the line-of-sight velocity. Although many authors have used the distribution function of the projected velocity in their statistical analysis of substructure, one can argue that in the highly nonlinear environment of a cluster both redshift distortions and relaxation effects destroy the relationship between line-of-sight

(l.o.s.) velocity and distance, thus making apparently meaningless the use of the l.o.s. velocity in a context where one tries to isolate substructures in real space. Suppose however that the substructure we ae trying to identify is made of spatially separated clumps whose inner velocity dispersion is smaller than the average l.o.s. distance among the clumps themselves. The velocity of a galaxy can be written as: $v_g = Hr + v_{pec}$, where H is the Hubble constant and v_{pec} measures the deviations induced by random velocities within the cluster due to relaxation effects and systematic infall motions. The quantity v_{pec} is a stochastic variable, described by some probability distribution, but we know from observations (e.g. the ENACS survey, Mazure et al. (1996)) and from simulations (Natarajan, Hjorth and van Kampen 1997) that its moments filtered on cluster scales are finite. So if: Hr >> v_{pec} the Hubble term dominates the l.o.s. velocity, and this quantity can then be used to characterize substructures whose relative distances within the cluster are larger than v_{pec}/H.

In order to test our 3D DWT method and to determine reliable confidence limits, we have simulated a total number of 20 clusters changing the following parameters: 1. the distance of the cluster from the observer; 2. the number of clumps making up the cluster; 3. the number of galaxies inside the cluster and the individual clumps; 4. the mean distance separating different clumps in the same cluster; 5. the completeness of our catalogues. The analysis has been performed using a *PVM* code that allows a rapid structure detection and morphological analysis in a 3-D set of data points (Pagliaro and Becciani 1998).

The plan of the paper is as follows: in §2 we describe the method of analysis, in §3 we simulate a set of clusters, in §4 we shortly come back on the usage of heterogeneous coordinates, in §5 we perform the analysis, in §6 we discuss the statistical robustness of our results and finally, in §7 we report our conclusions.

2 THE METHOD OF ANALYSIS

2.1 Method overview

The 3D DWT method can be divided in three steps. The first one is the computation of the wavelet matrices on all the scales investigated. These are computed by means of the "á trous" algorithm, as described by (Lega 1994) (hereafter L94). The same matrices are computed for the data to be analyzed and on a random distribution in the same region of space and on the same grid as the real data. On these latter matrices we calculate the threshold corresponding to a certain confidence level in the structure detection.

The second step is the segmentation analysis. The aim of this analysis is to have

all the connected pixels with a wavelet coefficients greater than the threshold labeled with an integer number different for every single structure. This allows a rapid identification of connected regions which are needed as input for the third step.

The last step of the method is the computation of a morphological parameter for every structure singled out and of a mean morphological parameter for each scale.

In the following of this section we describe the method neglecting the strictly computational features of the parallel code that we have implemented to apply it. The latter are described in details in a more technical paper (Pagliaro and Becciani 1998) and in (Pagliaro 1998).

2.2 The wavelet transform

Generally speaking, a wavelet transforms is the decomposition of a function on a basis obtained by translation and dilation of a particular function localized in both physical and frequency space. A characteristic features of this kind of analysis is that it allows a simultaneous study of both positional and scaling properties.

The method we have implemented is three-dimensional. However, for the sake of simplicity, here we describe the one dimensional reduction. The generalization to the 3-D case is straightforward.

For a one-dimensional function $f(x)$ the wavelet transform is a linear operator that can be written as:

$$\begin{aligned} w(s,t) &= \langle f|\psi\rangle \\ &= s^{-1/2}\int_{-\infty}^{+\infty} f(x)\psi^*\left(\frac{x-t}{s}\right)dx \end{aligned} \quad (1)$$

where $s(>0)$ is the scale on which the analysis is performed, $t \in \Re$ is the spatial translation parameter and ψ is the the Grossmann-Morlet ((1984),(1987)) analyzing wavelet function

$$\psi_{(s,t)}(x) = s^{-1/2}\psi\left(\frac{x-t}{s}\right) \quad (2)$$

that is spatially centered around t and has scale s. The wavelet function $\psi_{(1,0)}(x)$ is called mother wavelet. It generates the other wavelet function $\psi_{(s,t)}(x), s > 1$. We follow L94 in the choice of the mother wavelet in order to use the *à trous* algorithm in the following.

$$\psi(x) = \phi(x) - \frac{1}{2}\phi(\frac{x}{2}) \quad (3)$$

where ϕ is the cubic centred B-spline function defined by:

$$\phi(x) = \frac{|x-2|^3 - 4|x-1|^3 + 6|x|^3 - 4|x+1|^3 + |x+2|^3}{12} \quad (4)$$

The set of scales are powers of two: $s = 2^r$ and the first scale always corresponds to the size of 1 pixel.

The scale s in this kind of analysis may be considered as the resolution. In other words, if we perform a calculation on a scale s_0, we expect the wavelet transform to be sensitive to structures with typical size of about s_0 and to find out those structures.

First step of the wavelet matrices computation is the evaluation of the coefficients $c(0)$. These are defined as:

$$c(0,t) = \langle f(x)|\phi(x-t)\rangle \quad (5)$$

On the other scales the coefficients c becomes:

$$c(s,t) = \frac{1}{2^r}\langle f(x)|\phi(\frac{x-t}{2^r})\rangle \quad (6)$$

Using some properties of the B-spline we can to write:

$$c(s,t) = \sum_{n=-2}^{2} h(n)c(s-1, t+n2^{r-1}) \quad (7)$$

where $h(n) = \frac{1}{16}C_{2-n}^4$, C_n^m being the binomial coefficients. According to eqs.1 and 3 we can write the following expression for the wavelet coefficients on the various scales:

$$w(s,t) = c(s,t) - c(s-1,t) \quad (8)$$

The wavelet analysis associates to each pixel a real number, which represents the smoothed local density contrast at a given scale. At the end of this part our result is a set of matrices of wavelet coefficients; one matrix for each scale investigated.

Even if the histogram of the wavelet coefficients may suggest the presence of substructure, revealed by a queue towards the positive values, this kind of information is only visual and not easily quantifiable and spatially localizable.

2.3 The thresholding

The thresholding is made on the wavelet coefficient histogram. For a flat background, the wavelet transform yields coefficients equal to zero. The existence of structures at a given scale gives wavelet coefficient with large positive values. It is however quite obvious that this is strictly true only in an ideal case: a random distribution may have non zero coefficients even if there is no structure, due to statistical fluctuations. Moreover, the statistical behaviour of the wavelet coefficient is complex due to the correlation among nearby pixels. We

choose the threshold through a classical decision rule. We calculate the wavelet coefficients $w_{ran}(s)$ for each scale of our analysis, for a random distribution in the same region of space of our data and on the same grid. Then we calculate the probability $P[w(s) \leq w_{ran}(s)]$ and choose the value $w_{thres}(s)$ so that:

$$P[w_{thres}(s) \leq w_{ran}(s)] \leq \epsilon \quad (9)$$

Our threshold on the scale s is the value $\nu_{thres} = w_{thres}(s)$. For example, a choice for the value of ϵ of:

$$\epsilon = 0.001 \quad (10)$$

ensures a 99.9% confidence level in the structure detection. A simple flag in the input file allows an alternative choice: calculate the threshold in terms of the standard deviation of the wavelet coefficients distribution.

2.4 Structure numbering by means of segmentation

The second step of our analysis is the determination of connected pixels over a fixed threshold (*segmentation*, Rosenfeeld (1969)), the numbering of the selected structures and their morphological analysis.

The segmentation and numbering consists in the exam of the wavelet coefficients matrix; all the pixels associated with a wavelet coefficient greater than the selected threshold are labeled with an integer number. All other pixel labels are set equal to zero. Then, the same label is associated with all the pixels connected in a single structure, in a sequential way. So, the first structure individuated has the label '1' and so on.

Volume and surface of each structure singled out are calculated.

2.5 The morphological parameter

In order to perform a morphological analysis we have to introduce a morphological parameter that quantifies the sphericity of the structures. We choose the parameter:

$$L(s) = K(s)\frac{V^2}{S^3} \quad (11)$$

where V is the volume and S is the surface, as in L94, and $K(s)$ is a parameter that depends on the scale of the analysis. We want $L(s)$ to have the following behaviour: zero for very filamentary structures and 1 for spherical ones. This may be achieved putting $K = 36\pi$, but only for those scales not affected by the granular nature of the analysis. We choose the value 36π only for the scales $s = 2^r$ pixels with $r \geq 2$. For the smallest scales the constant 36π is not

adequate, since we are close to the grid resolution and the geometry of the substructures cannot be spherical. Since we want to consider as spherical a one-pixel structure, we adopt the values:

$$K(2^r) = \begin{cases} 216 & \text{if } r = 0, 1 \\ 36\pi & \text{otherwise} \end{cases} \tag{12}$$

Then, for every detection threshold we calculate the values:

$$\langle L(s) \rangle = \sum_{i=1}^{N_{obj}} \frac{L(s)}{N_{obj}} \tag{13}$$

where N_{obj} is the number of objects detected at scale s.

3 SIMULATING CLUSTERS OF GALAXIES

Our model clusters are engendered by randomly assembling a number of clumps, each characterized by the position of their centers. As coordinates we choose two angular coordinates and the recession velocity: (θ, ϕ, cz). Given the position of the center of each clump $(\theta_{cl}, \phi_{cl}, cz_{cl})$, galaxies are distributed about the center according to a gaussian distribution in real space of width σ_r. Velocities are then specified by adding to the Hubble flow a random component drawn from a maxwellian distribution with a given dispersion σ_{cl}. Each simulated cluster is then made of a collection of these clumps. This method allows a partial superposition of the clumps.

To each galaxy in the cluster we also attribute an absolute magnitude drawn randomly from a Schechter luminosity function. The values for m_* are appropriate to the central region of the Coma cluster (Biviano et al. 1995).

4 TREATING INHOMOGENEOUS COORDINATES

For analysing the simulated data we have modeled a major deficiency: observations are made in the redshift space, which causes the well known effect of the "Fingers-of-God". This effect is translated in a wavelet analysis in treating with inhomogeneous coordinates, being velocities not at all equivalent to distances. As we already discussed in the Inroduction, the smaller the velocity dispersion in groups of galaxies w.r.t. the Hubble velocity, the more reliable is the computation of distances from redshifts. Obviously, to ensure that the scale s of recover in equ. 1 has the same physical meaning on the three axis, a scale transformation is needed along the redshift axis (z).

We have transformed the angular distances from the photometric center to linear one assuming a distance from the center of the cluster equal to the mean

value of the redshift divided by H_0. Finally, to make our linear coordinates independent from the value of H_0, we renormalize them dividing by σ_{cz}.

Similar steps could invalidate all the analysis performed on such transformed data, because of "unphysicalness" of the redshift coordinate.

As a matter of fact, we have taken into account in our analysis this undesidered effect and evaluated its importance on the capability of the wavelet method to detect substructures.

Finally, the results of our method must be validated by an *a posteriori* check. As we said in the Introduction, we are making the implicit hypothesis that the clumps making up the substrctures are made up of galaxies whose velocity dispersion is much smaller than the relative distance among the clumps themselves. After having identified the clumps with the 3D DWT method it is important to verify that the galaxies in the original catalog belonging to each clump do indeed verify this constraint. In fact, in (Gambera et al. 1997) we discarded some of the substrctures found by the 3D DWT method because they did not contain enough galaxies to make a reliable statistics.

5 TESTING SUBSTRUCTURE RECOVER

5.1 Is the DWT capable to recover substructure?

In order to assess the capability of our code to recover substructures we simulate five clusters of galaxies at a fixed distance made of an increasing number of well separated clumps: from only 1 clump to 15 ones. Clusters are identified by the names C13 (containing 1 clump, 528 galaxies), C14 (3 clumps, 1224 galaxies), C1 (5 clumps, 1512 galaxies), C15 (8 clumps, 3456 galaxies) and C16 (15 clumps, 8750 galaxies). The mean separation between two clumps inside a cluster is set equal to $2.7\,h^{-1}$ Mpc. As one can see from table 1 substructures are well detected. Unfortunately, an intrinsic drawback of the DWT is the uncapability to distinguish the typical scale of substructures (Bijaoui et al.1996). Substructures "created" by our simulations are on a typical scale of about $2\,h^{-1}$ Mpc, so we can detect them looking at the right scale.

5.2 Dependence on the distance from the observer

We simulate four different clusters of galaxies with distances ranging from 8 to $68\,h^{-1}$ Mpc with step $20\,h^{-1}$ Mpc. Clusters are identified by the names C1 (set at a distance $68\,h^{-1}$ Mpc from the observer), C2 ($48\,h^{-1}$ Mpc), C3 ($28\,h^{-1}$ Mpc) and C4 ($8\,h^{-1}$ Mpc). All clusters are made of 1512 galaxies divided in

5 clumps with mean separation $2.7\,h^{-1}$ Mpc. As one can see from table 2, the recover does not depend on the distance from the observer. However at small distances the Hubble flux and the peculiar velocities of the galaxies becomes comparable, and we must take into account this effect. We have repeated the analysis on each of 20 realizations of the four clusters obtained by randomly "reshuffling" the original simulated catalogues with two different method: 1. the first 10 reshufflings were made by redistributing randomly the redshifts among the galaxies while keeping the angular coordinates fixed (see table 7); 2. the other 10 reshufflings were made varying the redshifts (namely the value of cz) of a typical value of the galaxies peculiar velocities in a random direction, while keeping the angular coordinates fixed (see table 8). In the first case, the average values of the number of structures found is always smaller than the one found in the original simulated clusters. This test, already performed in GPAB on a catalogue of clusters from the Coma cluster with the same result, strengthens our confidence on the physical significance of the structures detected in Coma. In the second case the number of structures is nearly the same as in the original catalogues. We "followed" some galaxies labeling them in the reshufflings and we have seen that the galaxies labeled are always inside the same substructure for the clusters C1, C2, C3 and reshuffled samples, but may change substructure for the case of the cluster C4. This happens because in this last case the peculiar velocities and the Hubble flux becomes comparable. However it is worth noticing that, although the galaxies may "leap" from one substructure to another, the number of these latter is always nearly the same.

5.3 Dependence on the number of galaxies

We simulate six clusters of galaxies made of an increasing number of data points (galaxies) ranging from a minimum of 118 to a maximum of 14712. Clusters are identified by the names C5 (made of 118 galaxies), C6 (330 galaxies), C7 (528 galaxies), C1 (1512 galaxies), C8 (7032 galaxies) and C9 (14712 galaxies). All these clusters are composed of 5 clumps with mean separation $2..7\,h^{-1}$ Mpc. It is clear from our results that a minimum number of at least 200 data points is required to ensure a correct substructure recover and that an increasing number of points improves the confidence of the analysis.

5.4 Dependence on the interclumps separation

Clusters are identified by the names C10 (mean separation $1.5\,h^{-1}$ Mpc), C1 ($2.7\,h^{-1}$ Mpc), C11 ($3.7\,h^{-1}$ Mpc) and C12 ($4.7\,h^{-1}$ Mpc). We note no significant variations in the number of substructures detected as the mean interclumps separation vary, as one can see from table 4, if the mean separation is greater than $1.5\,h^{-1}$ Mpc.

5.5 Dependence on completeness

Clusters are identified by the names C17 (100 % completeness at $m_B = 22$), C18 ($m_B = 20$), C19 ($m_B = 18$) and C20 ($m_B = 16$). We note a progressive decrease in the number of structures found, as the magnitude of completeness decrease. However, it is worth noticing that, till the value $m_B = 16$ included, this decrease is not dramatic and the number of structures detected is very close to the effective number of structures composing the clusters. We can consider this result as a warning to keep in mind while examining catalogues with low completeness. In this case the number of substructures detected could be less than the effective number of substructures.

6 STATISTICAL ROBUSTNESS

Drawing from a wavelet analysis of a catalogue of a combined angular-redshift distribution any conclusion about the real phase- and configuration-space structure requires that one verifies first that the catalogue does not suffer from any systematic selection biases or from other types of systematic effects like those induced by redshift distortions, as described by (Regös & Geller 1989) and (Praton & Schneider 1989). About these latter we notice that they have little significance for a distant cluster like Coma (GPAB), in which the Hubble flow term is dominant over the peculiar velocity *within* the substructures. These effects may on the other hand affect the recover in closer clusters, but our analysis shows that in the presence of a significant statistics (that we quantify with about 800 galaxies members) these effects can be neglected too.

One can reasonably argue that because the structures we find in simulated clusters of galaxies are generally well within the nonlinear virialized region on these scales we are probing a region of the phase space detached from the Hubble flow, where the linearity between redshift and distance is completely lost. On the other hand one also expects that the phase-space distribution within the nonlinear region should be enough well-mixed within each clump (if there are any) that the substructures detected correspond to substructures in velocity space.

In order to check this latter hypothesis we have repeated the wavelet analysis in

the distance dependence study (the distance is the parameter mostly affected from redshift effects) on each of 10 realizations obtained by randomly "reshuffling" the original catalogue, i.e. redistributing randomly the redshifts among the galaxies while keeping the angular coordinates fixed.

The results are consistent with those found by EM who performed a similar analysis for 2-D catalogues. The average values of the number of structures is always smaller than the one found in the original catalogue, showing that the catalogue itself is probably contaminated by some uncertainty, probably connected to the arbitrariness in the choice of the redshift limits, by some background contaminants, etc.

Another kind od "reshuffling" has been performed on the same data varying the value of the redshift by a typical peculiar velocity in a random direction and following some target galaxies. These may leap from one substructure to another, but as the number of galaxies is high the morphology of substructure is not modified.

These tests strengthen our confidence on the physical significance of most of the substructures that can be detected by means of this method, particularly when dealing with a great number of galaxies members or with fewer galaxies, but in a more distant cluster.

7 CONCLUSIONS

In the last years contradictory conclusions on various methods to detect significant substructures in clusters of galaxies have been reported by several authors (Fitchett & Webster (1997), West et al. (1988), Dressler & Shectman (1988), Mellier et al. (1988), Slezak et al. (1990), EM, Escalera et al. (1992), L94, LSABB, Lega et al. (1994), GPAB). Among the methods that have been tested and used for these purpose, we believe that the most powerful is the one based on wavelet transforms and in this paper we have investigated its dependence on various parameters that characterize a cluster of galaxies, like its distance from the observer or the number of clumps and/or of galaxies that makes it and on some selection effects. According to our analysis the wavelet transforms method is a very powerful method to recover substructure inside clusters of galaxies, rather independently from the many features that may vary in a cluster. The only serious limitation is due to the necessity to know *a priori* the scale on which the substructure is to be found. However, this is an intrinsic drawback of the method and no improvement in numerical codes based on this technique could make up for it. An interscale connectivity graph can be helpful to discrimate among the scales at which a physical object show features (Bijaoui et al. (1995), (1996)) but this technique is presently beyond the purpose of our analysis. Besides, a significant number of data points is required to perform an accurate analysis. We estimate about 200 galaxies to be a good minimum number to allow a rather accurate recover in a distant cluster. As the distance decreases, we need a larger number of galaxy members so that a statistical redistribution can compensate for redshift distortion. For the closest clusters we have taken into account ($8h^{-1}$ Mpc) 800 members should be enough. Obviously, the larger the number of data points, the more accurate the analysis. Finally, it is clear that this method can not give any kind of dynamical information on the clusters investigated and that a companion method, like high resolution N-body simulations (e.g. Becciani et al. (1996), (1997)), is required for a more complete and detailed study about evolutionary states.

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SUBSTRUCTURE RECOVERY CAPABILITY

N^a	m_B^b	N_c^c	d_o^d	N_g^e	d_i^f	0.5^g	1	2	4
$C1^*$	∞	5	68	1512	2.7	12	6	5	2
$C13$	∞	1	68	528	-	2	2	2	1
$C14$	∞	3	68	1224	2.7	5	3	3	2
$C15$	∞	8	68	3456	2.7	11	7	7	5
$C16$	∞	15	68	8750	2.7	18	18	14	9

Table 1. Substructure recover in simulated clusters. All lengths are expressed in h^{-1} Mpc;

[a] Name of the simulated cluster. The clusters marked by an asterisk have been reshuffled $10 \div 10$ times; [b] Magnitude for 100 % completeness; [c] Number of clumps making up the cluster; [d] Distance to the observer; [e] Number of galaxies making up the cluster; [f] Mean separation among clumps; [g] Number of substructures detected at scales $0.5 \div 4h^{-1}$ Mpc

RECOVERY AS DISTANCE VARIES

N^a	m_B^b	N_c^c	d_o^d	N_g^e	d_i^f	0.5^g	1	2	4
$C1^*$	∞	5	68	1512	2.7	12	6	5	2
$C2^*$	∞	5	48	1512	2.7	13	7	5	2
$C3^*$	∞	5	28	1512	2.7	13	7	4	2
$C4^*$	∞	5	8	1512	2.7	13	6	5	2

Table 2. see caption of Table 1

RECOVERY AS MEMBERS VARIES

N^a	m_B^b	N_c^c	d_o^d	N_g^e	d_i^f	0.5^g	1	2	4
$C1^*$	∞	5	68	1512	2.7	12	6	5	2
$C5$	∞	5	68	118	2.7	21	2	2	1
$C6$	∞	5	68	330	2.7	18	7	5	2
$C7$	∞	5	68	552	2.7	18	6	6	3
$C8$	∞	5	68	7032	2.7	11	5	5	2
$C9$	∞	5	68	14712	2.7	12	5	5	2

Table 3. see caption of Table 1

RECOVERY AS INTERCLUMPS VARIES

N^a	m_B^b	N_c^c	d_o^d	N_g^e	d_i^f	0.5^g	1	2	4
$C1^*$	∞	5	68	1512	2.7	12	6	5	2
$C10$	∞	5	68	1416	1.5	8	4	3	3
$C11$	∞	5	68	1512	3.7	13	10	7	5
$C12$	∞	5	68	1512	4.7	13	9	5	4

Table 4. see caption of Table 1

RECOVERY AS COMPLETENESS VARIES

N^a	m_B^b	N_c^c	d_o^d	N_g^e	d_i^f	0.5^g	1	2	4
$C17$	22	5	68	2399	2.7	8	7	5	2
$C18$	20	5	68	2399	2.7	8	7	5	2
$C19$	18	5	68	2340	2.7	7	6	5	2
$C20$	16	5	68	2280	2.7	5	5	4	1

Table 5. see caption of Table 1

SUBSTRUCTURE RECOVERY

N^a	m_B^b	N_c^c	d_o^d	N_g^e	d_i^f	0.5^g	1	2	4
$C1^*$	∞	5	68	1512	2.7	12	6	5	2
$C2^*$	∞	5	48	1512	2.7	13	7	5	2
$C3^*$	∞	5	28	1512	2.7	13	7	4	2
$C4^*$	∞	5	8	1512	2.7	13	6	5	2
$C5$	∞	5	68	118	2.7	21	2	2	1
$C6$	∞	5	68	330	2.7	18	7	5	2
$C7$	∞	5	68	552	2.7	18	6	6	3
$C8$	∞	5	68	7032	2.7	11	5	5	2
$C9$	∞	5	68	14712	2.7	12	5	5	2
$C10$	∞	5	68	1416	1.5	8	4	3	3
$C11$	∞	5	68	1512	3.7	13	10	7	5
$C12$	∞	5	68	1512	4.7	13	9	5	4
$C13$	∞	1	68	528	-	2	2	2	1
$C14$	∞	3	68	1224	2.7	5	3	3	2
$C15$	∞	8	68	3456	2.7	11	7	7	5
$C16$	∞	15	68	8750	2.7	18	18	14	9
$C17$	22	5	68	2399	2.7	8	7	5	2
$C18$	20	5	68	2399	2.7	8	7	5	2
$C19$	18	5	68	2340	2.7	7	6	5	2
$C20$	16	5	68	2280	2.7	5	5	4	1

Table 6. Substructure recover in simulated clusters. All lengths are expressed in h^{-1} Mpc;

[a] Name of the simulated cluster. The clusters marked by an asterisk have been reshuffled $10 + 10$ times; [b] Magnitude for 100 % completeness; [c] Number of clumps making up the cluster; [d] Distance to the observer; [e] Number of galaxies making up the cluster; [f] Mean separation among clumps; [g] Number of substructures detected at scales $0.5 \div 4h^{-1}$ Mpc

RESHUFFLED CLUSTERS I

N^a	d_o^b	0.5^c	1	2	4
$C1$	68	8.1 ± 1.1	4.3 ± 0.7	4.6 ± 0.8	2.0 ± 0.0
$C2$	48	9.0 ± 1.2	5.3 ± 0.8	3.9 ± 0.7	2.0 ± 0.0
$C3$	28	9.1 ± 1.3	2.9 ± 0.8	3.9 ± 0.9	2.0 ± 0.1
$C4$	8	9.4 ± 1.1	3.2 ± 0.7	3.8 ± 0.9	2.1 ± 0.1

Table 7. Results of substructure recover in totally reshuffled clusters. All lengths are expressed in h^{-1} Mpc;

[a] Name of the simulated cluster;
[b] Distance to the observer;
[c] Number of substructures detected at scales $0.5 \div 4h^{-1}$ Mpc

RESHUFFLED CLUSTERS II

N^a	d_o^b	0.5^c	1	2	4
$C1$	68	11.2 ± 1.8	5.6 ± 1.2	5.2 ± 1.2	2.0 ± 0.1
$C2$	48	13.0 ± 1.6	6.8 ± 1.0	4.6 ± 0.8	2.1 ± 0.1
$C3$	28	12.8 ± 1.8	5.4 ± 1.2	5.2 ± 1.0	2.0 ± 0.1
$C4$	8	12.4 ± 1.2	6.0 ± 1.2	4.2 ± 0.6	2.0 ± 0.0

Table 8. Results of substructure recover in reshuffled clusters with limited velocity change. All lengths are expressed in h^{-1} Mpc;

[a] Name of the simulated cluster;
[b] Distance to the observer;
[c] Number of substructures detected at scales $0.5 \div 4h^{-1}$ Mpc