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Two-point Correlators of Composite Higgs fields and Chiral Symmetry Restoration[*,†]

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Abstract

We consider the effective quasilocal quark model with two composite Higgs doublets in strong coupling (polycritical) regime below the chiral symmetry breaking energy scale Λ_{CSB}. The two-point correlators of scalar and pseudoscalar Higgs fields are calculated. The adjustment of their asymptotics at high energies allows to implement the chiral symmetry restoration in correspondence to the operator product expansion in QCD-like (technicolor, topcolor) models. The requirement of chiral symmetry restoration (CSR) at high energies above the Λ_{CSB} yields some bounds on parameters of (composite) Higgs particles and underlying effective quasilocal quark models.

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1 Introduction

If Higgs bosons are composite[1] and their masses are created by a mechanism of spontaneous chiral symmetry breaking[2, 3] one can require the chiral symmetry restoration (CSR) at high energies[4] which leads to the CSR constraints on phenomenological parameters of the Higgs model. On this way the minimal extension of the Standard Model [5] may include two (or more) doublets of Higgs particles [6, 7, 8, 9]. Recently we have developed the effective quark models including higher-dimensional vertices of fermion fields with derivatives which can serve[9] for the parameterization of unknown heavy particle dynamics beyond the Standard Model in the spirit of Wilsonian effective action approach [10]. In this paper we continue the exploration of particular Effective Quasilocal Quark Models (EQQM) which inherit main properties of a underlying vector gauge theory of QCD type (such as technicolor [11] or topcolor [12] models). The most important property turns out to be the Chiral Symmetry Breaking (CSB) at low energies and, on the other hand, the Chiral Symmetry Restoration (CSR) at high energies. The latter one is controlled by the Operator Product Expansion of quark current correlators which include a different number of parity-odd and parity-even currents. More specifically, here we deal with the two-point correlators of scalar and pseudoscalar quark densities [4, 13] which are saturated at low energies by Higgs-particle resonances of a definite parity. The difference between these correlators is decreasing rapidly in accordance to OPE of a vector-like gauge theory [14]. In the framework of either EQQM or a low-energy Higgs-field model it leads to the CSR constraints on some parameters of composite Higgs particles.

Let us remind the effective quark lagrangian of a EQQM which incorporates all higher-dimensional vertices necessary for the description of Two-Higgs Doublet Standard Model in the low-energy limit. The two-flavor, 3d generation quark models with quasilocal interaction are considered in which the t- and b-quarks are involved in the DCSB.

We restrict ourselves by examination of the Models of type I [15] with the following lagrangean[9]:

$$\begin{aligned} \mathcal{L}_I &= \bar{q}_L \not{D} q_L + \bar{t}_R \not{D} t_R + \bar{b}_R \not{D} b_R \\ &+ \frac{1}{N_c \Lambda^2} \sum_{k,l=1}^{2} a_{kl} \left(g_{t,k} J^T_{t,k} + g_{b,k} \widetilde{J}^T_{b,k} \right) i\tau_2 \left(g_{b,l} J_{b,l} - g_{t,l} \widetilde{J}_{t,l} \right). \end{aligned} \quad (1)$$

Here we have introduced the denotations for doublets of fermion currents:

$$J_{t,k} \equiv \bar{t}_R f_{t,k} \left(-\frac{\partial^2}{\Lambda^2} \right) q_L, \qquad J_{b,k} \equiv \bar{b}_R f_{b,k} \left(-\frac{\partial^2}{\Lambda^2} \right) q_L, , \qquad q_L = \begin{pmatrix} t_L \\ b_L \end{pmatrix}, \quad (2)$$

and the tilde in $\widetilde{J}_{t,k}$ and $\widetilde{J}_{b,k}$ marks charge conjugated quark currents, rotated with

τ_2 Pauli matrix

$$\tilde{J}_{t,k} = i\tau_2 J^{\star}_{t,k}, \qquad \tilde{J}_{b,k} = i\tau_2 J^{\star}_{b,k} \tag{3}$$

The subscripts t, b indicate right components of t and b quarks in the currents, the index k enumerates the formfactors:

$$f_{t,1} = 2 - 3\tau; \quad f_{t,2} = -\sqrt{3}\tau; \qquad \tau \equiv -\frac{\partial^2}{\Lambda^2};$$
$$f_{b,1} = 2 - 3\tau, \quad f_{b,2} = -\sqrt{3}\tau; \tag{4}$$

which are orthonormal on the interval $0 \leq \tau \leq 1$. In these notations coupling constants of the four-fermion interaction are represented by 2×2 matrix a_{kl} and contributed also from the Yukawa constants $g_{k,l}, g_{b,k}$.

2 Effective potential and mass spectrum of composite fields

In order to describe the dynamics of composite Higgs bosons the lagrangean density (1) of the Model I must be rearranged by means of introduction of auxiliary bosonic variables and by integrating out fermionic degrees of freedom [16]. Namely, we define two scalar $SU(2)_L$-isodoublets:

$$\Phi_1 = \begin{pmatrix} \phi_{11} \\ \phi_{12} \end{pmatrix}, \qquad \Phi_2 = \begin{pmatrix} \phi_{21} \\ \phi_{22} \end{pmatrix} \tag{5}$$

and their charge conjugates:

$$\tilde{\Phi}_1 = \begin{pmatrix} \phi^{\star}_{12} \\ -\phi^{\star}_{11} \end{pmatrix}, \qquad \tilde{\Phi}_2 = \begin{pmatrix} \phi^{\star}_{22} \\ -\phi^{\star}_{21} \end{pmatrix}. \tag{6}$$

Then the lagrangean (1) can be rewritten in the following way:

$$\mathcal{L}_I = L_{kin} + N_c\Lambda^2 \sum_{k,l=1}^{2} \Phi^{\dagger}_k (a^{-1})_{kl} \Phi_l + i \sum_{k=1}^{2} \left[g_{t,k} \tilde{\Phi}^{\dagger}_k J_{t,k} + g_{b,k} \Phi^{\dagger}_k J_{b,k} \right] + h.c. \tag{7}$$

The integrating out of fermionic degrees of freedom will produce the effective action for Higgs bosons of which we shall keep only the kinetic term and the effective potential consisting of two- and four-particles vertices. The omitted terms are supposedly small, being proportional to inverse powers of a large scale factor Λ. The Yukawa constants are chosen of the form

$$g_{t,k} = 1; \qquad g_{b,k} = g \tag{8}$$

for $k = 1, 2$. The first choice can be done because the fields Φ_1 and Φ_2 can always be rescaled by an arbitrary factor which is absorbed by redefinition of polycritical coupling constants a_{kl}. Other constants $g_{b,k}$ are taken equal for the simplicity. Their value g induces the quark mass ratio m_b/m_t.

The consistent approximation can be developed in the vicinity of (poly)critical point,

$$8\pi^2 a_{mn}^{-1} \sim \delta_{mn} + \frac{\Delta_{mn}}{\Lambda^2}, \qquad |\Delta_{ij}| \ll \Lambda^2, \tag{9}$$

which signifies the cancellation of quadratic divergences [18].

Then the effective potential of Higgs fields in this model reads:

$$\begin{aligned} V_{eff} &= \frac{N_c}{8\pi^2}\Big(- \sum_{k,l=1}^{2} (\Phi_k^\dagger \Phi_l)\Delta_{kl} - 8g^4(\Phi_1^\dagger\Phi_1)^2 \ln g^2 \\ &+(1+g^4)\left[8(\Phi_1^\dagger\Phi_1)^2 \left(\ln\frac{\Lambda^2}{4(\Phi_1^\dagger\Phi_1)} + \frac{1}{2}\right) + \right. \\ &-\frac{159}{8}(\Phi_1^\dagger\Phi_1)^2 + \frac{9}{8}(\Phi_2^\dagger\Phi_2)^2 + \frac{3}{4}(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \\ &+\frac{3}{4}(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \frac{3}{8}(\Phi_1^\dagger\Phi_2)^2 + \frac{3}{8}(\Phi_2^\dagger\Phi_1)^2 - \\ &-\frac{5\sqrt{3}}{4}(\Phi_1^\dagger\Phi_1)\Big((\Phi_1^\dagger\Phi_2) + (\Phi_2^\dagger\Phi_1)\Big) + \\ &\left. +\frac{\sqrt{3}}{4}(\Phi_2^\dagger\Phi_2)\Big((\Phi_1^\dagger\Phi_2) + (\Phi_2^\dagger\Phi_1)\Big)\right]\Big) + O\left(\frac{\ln\Lambda}{\Lambda^2}\right), \end{aligned} \tag{10}$$

where the bilinear "mass" term is in general non-diagonal and represented by the real, symmetric 2×2 matrix Δ_{kl}. This is the two-Higgs potential in the large N_c approach and a more realistic potential should involve the true Renormalization Group flow of Two-Higgs Doublet SM [1] with initial conditions at high energies taken from (10).

We assume the electric charge stability of the vacuum, *i.e.* that only neutral components of both Higgs doublets may have nonzero v.e.v.[1, 7, 17]. Hence, one can deal with only neutral components of the Higgs doublets in the effective action for studying DCSB.

In general there exists a special regime where the ratio of v.e.v. of neutral Higgs fields is complex [18]. Here let us consider the phase where CP-parity is not broken spontaneously. Then one can relate real fields ϕ_1, ϕ_2 to the neutral components of Higgs doublets:

$$\phi_{12} = \phi_1; \qquad \phi_{22} = \phi_2. \tag{11}$$

The condition of minimum of the potential (10) with the charged components of Higgs doublets put to zero values: $\phi_{11} = \phi_{21} = 0$, brings the mass-gap equations for them:

$$\frac{\delta V_{eff}(\phi,\phi^*)}{\delta\phi_m^*} = 0 = \frac{\delta V_{eff}(\phi,\phi^*)}{\delta\phi_m}, \tag{12}$$

[1] see the updated one-loop RG equations in [19] and references therein.

which solution may cause the DCSB if it is an absolute minimum. In the explicit form they are:

$$\begin{aligned}
\Delta_{11}\phi_1 + \Delta_{12}\phi_2 &= 16\phi_1^3 \ln\frac{\Lambda^2}{4\phi_1^2} + 16\phi_1^3 g^4 \ln\frac{\Lambda^2}{4g^2\phi_1^2} \\
&+ (1+g^4)\Big[-\frac{159}{4}\phi_1^3 - \frac{15\sqrt{3}}{4}\phi_1^2\phi_2 + \frac{9}{4}\phi_1\phi_2^2 + \frac{\sqrt{3}}{4}\phi_2^3\Big], \\
\Delta_{22}\phi_2 + \Delta_{12}\phi_1 &= (1+g^4)\Big[\frac{9}{4}\phi_1^2\phi_2 + \frac{3\sqrt{3}}{4}\phi_1\phi_2^2 + \frac{9}{4}\phi_2^3 - \frac{5\sqrt{3}}{4}\phi_1^3\Big]
\end{aligned} \tag{13}$$

The solution of the mass-gap equation of Gross-Neveu-type is:

$$\phi_1^2 \approx \frac{\det\Delta}{16(1+g^4)\Delta_{22}\ln\left(\frac{\Lambda^2}{m_t^2}\right)}, \quad \phi_2 \approx -\frac{\Delta_{12}}{\Delta_{22}}\phi_1. \tag{14}$$

There are also abnormal and special solutions discussed in [16, 18] in details.

The true minimum is derived from the positivity of the second variation of the effective action around a solution of the mass-gap equation,

$$\phi_k = <\phi_k> + \sigma_k(x) + i\pi_k(x). \tag{15}$$

This variation reads:

$$\begin{aligned}
\frac{16\pi^2}{N_c}\delta^2 S_{eff} &\equiv \Big(\sigma, (A^{\sigma\sigma}p^2 + B^{\sigma\sigma})\sigma\Big) \\
&+ 2\Big(\pi, (A^{\pi\sigma}p^2 + B^{\pi\sigma})\sigma\Big) + \Big(\pi, (A^{\pi\pi}p^2 + B^{\pi\pi})\pi\Big),
\end{aligned} \tag{16}$$

where two symmetric matrices - for the kinetic term $\hat{A} = \left(A^{ij}_{lm}\right), \quad i,j = (\sigma,\pi)$ and for the constant, momentum independent part, $\hat{B} = \left(B^{ij}_{lm}\right)$ - have been introduced. In the CP-conserving phase: $\hat{A}^{\sigma\pi} = \hat{B}^{\sigma\pi} = 0$.

The mass spectrum of related bosonic states is determined by the solutions of the secular equation :

$$\det(\hat{A}p^2 + \hat{B}) = 0, \tag{17}$$

at $-m^2 = p^2 < 0$ in both scalar and pseudoscalar channels.

The "kinetic" matrix $\hat{A}$ as being multiplied by p^2 is derived in the soft-momentum expansion in powers of p^2:

$$\mathcal{L}_{\rm kin} = \frac{N_c}{16\pi^2}\sum_{l,m=1}^{2}\left(I^{(1)}_{l,m}(\partial_\mu\phi^\star_{l1})(\partial_\mu\phi_{m1}) + I^{(2)}_{l,m}(\partial_\mu\phi^\star_{l2})(\partial_\mu\phi_{m2})\right) \tag{18}$$

and $I^{(1)}_{mn}$ contributes to the kinetic term for charged components of Higgs doublets and $I^{(2)}_{mn}$ defines the latter one for the neutral components:

$$\begin{aligned} I^{(1)}_{lm} &\simeq I^{(2)}_{l,m} \simeq A^{\sigma\sigma}_{l,m} \simeq A^{\pi\pi}_{l,m} \simeq \left(f_{t,l}(0)f_{t,m}(0) + g^2 f_{b,l}(0)f_{b,m}(0)\right)\ln\frac{\Lambda^2}{m_t^2} \\ &+ \int\limits_0^1 (f_{t,l}(\tau)f_{t,m}(\tau) + g^2 f_{b,l}(\tau)f_{b,m}(\tau) - \\ &- f_{t,l}(0)f_{t,m}(0) - g^2 f_{b,l}(0)f_{b,m}(0))\frac{d\tau}{\tau} + O\left(\frac{\ln\frac{\Lambda^2}{m_t^2}}{\Lambda^2}\right). \end{aligned} \tag{19}$$

The related integrals for $I^{(1),(2)}_{lm}$ have been calculated at large values of Λ, in the large-log limit $\ln(\Lambda^2/m_t^2) \simeq \ln(\Lambda^2/m_b^2) \gg 1$ and for the CP conserving phase. After substitution of (4) kinetic matrices $\hat{A}$ take the form:

$$A^{\sigma\sigma}_{l,m} \simeq A^{\pi\pi}_{l,m} \simeq (1+g^2)\begin{pmatrix} 4\ln\frac{\Lambda^2}{m_t^2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{3}{2} \end{pmatrix} \tag{20}$$

Let us now obtain the matrix of second variations of the effective potential for the Model I :

$$B^{\pi\sigma}_{kl} = \frac{8\pi^2}{N_c}\frac{\partial^2}{\partial\phi_l\partial\phi_m}V_{eff}, \qquad (l,m = 1,2). \tag{21}$$

As we consider the CP-conserving case we can split this matrix in the $B^{\sigma\sigma}_{kl}$ matrix for scalars and $B^{\pi\pi}_{kl}$ matrix for pseudoscalars:

$$\begin{aligned} B^{\sigma\sigma}_{11} &= (1+g^4)\left[64\phi_1^2\ln\left(\frac{\Lambda^2}{m_t^2}\right) - 223\phi_1^2 - \frac{15\sqrt{3}}{2}\phi_1\phi_2 - \frac{\sqrt{3}}{2}\frac{\phi_2^3}{\phi_1}\right] + \\ &+ 2\Delta_{12}\frac{\phi_2}{\phi_1} - 64\phi_1^2 g^4 \ln g^2, \end{aligned} \tag{22}$$

$$B^{\sigma\sigma}_{12} = -2\Delta_{12} + (1+g^4)\left[9\phi_1\phi_2 - \frac{15\sqrt{3}}{2}\phi_1^2 + \frac{3\sqrt{3}}{2}\phi_2^2\right], \tag{23}$$

$$B^{\sigma\sigma}_{22} = (1+g^4)\left[9\phi_2^2 + \frac{3\sqrt{3}}{2}\phi_1\phi_2 + \frac{5\sqrt{3}}{2}\frac{\phi_1^3}{\phi_2}\right] + 2\Delta_{12}\frac{\phi_1}{\phi_2}, \tag{24}$$

$$B^{\pi\pi}_{11} = (1+g^4)\left[\frac{5\sqrt{3}}{2}\phi_1\phi_2 - \frac{\sqrt{3}}{2}\frac{\phi_2^3}{\phi_1} - 3\phi_2^2\right] + 2\Delta_{12}\frac{\phi_2}{\phi_1}, \tag{25}$$

$$B^{\pi\pi}_{12} = -2\Delta_{12} + (1+g^4)\left[-\frac{5\sqrt{3}}{2}\phi_1^2 + \frac{\sqrt{3}}{2}\phi_2^2 + 3\phi_1\phi_2\right], \tag{26}$$

$$B^{\pi\pi}_{22} = (1+g^4)\left[-3\phi_1^2 - \frac{\sqrt{3}}{2}\phi_1\phi_2 + \frac{5\sqrt{3}}{2}\frac{\phi_1^3}{\phi_2}\right] + 2\Delta_{12}\frac{\phi_1}{\phi_2}, \tag{27}$$

$$B^{\sigma\pi}_{kl} = 0 \qquad k,l = (1,2). \tag{28}$$

where the mass-gap equation (13) has been employed.

On solutions of the Gross-Neveu type (14) one finds:

$$\begin{aligned} B^{\sigma\sigma}_{11} &\simeq \frac{4\det\Delta - 2\Delta^2_{12}}{\Delta_{22}}; \quad B^{\sigma\sigma}_{12} \simeq -2\Delta_{12}; \quad B^{\sigma\sigma}_{22} \simeq 2\Delta_{22}; \\ B^{\pi\pi}_{11} &\simeq -\frac{2\Delta^2_{12}}{\Delta_{22}}; \quad B^{\pi\pi}_{12} \simeq -2\Delta_{12}; \quad B^{\pi\pi}_{22} \simeq -2\Delta_{22}. \end{aligned} \tag{29}$$

After substituting the matrices $\hat{A}, \hat{B}$ into (17) one can get the mass spectrum for the Higgs bosons in Model I. In particular, the Gross-Neveu-type solution brings the spectrum for scalars:

$$m^2_\sigma \approx -\frac{\det\Delta}{(1+g^2)\Delta_{22}\ln\left(\frac{\Lambda^2}{\mu^2}\right)} = 4m^2_d; \qquad m^2_{\sigma'} \approx -\frac{4\Delta_{22}}{3(1+g^2)}, \tag{30}$$

and for pseudoscalars:

$$m^2_\pi = 0; \qquad m^2_{\pi'} \approx -\frac{4\Delta_{22}}{3(1+g^2)}. \tag{31}$$

3 Chiral symmetry restoration in QCD-like models

In large N_c QCD [20] the leading contributions into two-point correlators of scalar and pseudoscalar quark densities are given by sums over an (infinite) number of meson poles

$$\begin{aligned} \Pi_\sigma(p^2) &= -\int d^4x \, \exp(ipx) \, \langle T\,(\bar{q}q(x)\;\bar{q}q(0))\rangle \\ &= \sum_n \frac{Z^\sigma_n}{p^2 + m^2_{\sigma,n}} + C^\sigma_0 + C^\sigma_1 p^2, \\ \Pi^{ab}_\pi(p^2) &= \int d^4x \, \exp(ipx) \, \langle T\left(\bar{q}\gamma_5\tau^a q(x)\;\bar{q}\gamma_5\tau^b q(0)\right)\rangle \\ &= \delta^{ab}\left(\sum_n \frac{Z^\pi_n}{p^2 + m^2_{\pi,n}} + C^\pi_0 + C^\pi_1 p^2\right). \end{aligned} \tag{32}$$

C_0 and C_1 are contact terms required for the regularisation of infinite sums.

The high-energy asymptotics are given by perturbation theory and operator product expansion taking into account the asymptotic freedom of QCD. As well the nonperturbative generation of gluon and quark condensates [14] is assumed to determine subleading power-like corrections to perturbative asymptotics . The long derivative in QCD, $i\partial_\mu \to iD_\mu = i\partial_\mu + G_\mu$, contains gluon fields $G_\mu \equiv g_s\lambda^a G^a_\mu$, where

$tr(\lambda^a\lambda^b) = 2\delta^{ab}$ and g_s is the QCD coupling constant. The related gluon field strength is defined as $G_{\mu\nu} \equiv -i[D_\mu, D_\nu]$. Respectively, in the chiral limit ($m_q = 0$) the scalar and pseudoscalar correlators have the following OPE motivated behavior [14] at large p^2:

$$\begin{aligned}
\Pi_{\sigma,\pi}(p^2)|_{p^2\to\infty} &\simeq \frac{N_c}{4\pi^2}\left(1+\frac{11N_c\alpha_s}{8\pi}\right)p^2 \ln\frac{p^2}{\mu^2} \\
&+\frac{\alpha_s}{4\pi p^2}\langle(G^a_{\mu\nu})^2\rangle + \frac{2\pi\alpha_s}{3p^4}\langle\bar{q}\gamma_\mu\lambda^k q\bar{q}\gamma_\mu\lambda^k q\rangle \\
&\mp\frac{\pi\alpha_s}{p^4}\langle\bar{q}\sigma_{\mu\nu}\lambda^k q\bar{q}\sigma_{\mu\nu}\lambda^k q\rangle + \mathcal{O}(\frac{1}{p^6}),
\end{aligned} \tag{33}$$

for $N_f = 2$, in Euclidean notations. Herein $\alpha_s \equiv g_s/4\pi = \alpha_s(\mu) \ll 1; \quad \mu > \Lambda_{CSB}$. In the large-$N_c$ limit

$$\left(\Pi_P(p^2) - \Pi_S(p^2)\right)_{p^2\to\infty} \equiv \frac{\Delta_{SP}}{p^4} + \mathcal{O}\left(\frac{1}{p^6}\right); \quad \Delta_{SP} \simeq \frac{24(N_c^2-1)}{N_c^2}\pi\alpha_s(<\bar{q}q>)^2, \tag{34}$$

where the vacuum dominance hypothesis[14] has been exploited. This rapidly decreasing asymptotics is a consequence of the chiral invariance of the lagrangean of massless QCD.

When comparing (32) and the first term of (33) one can see that in order to reproduce the perturbative asymptotics the infinite number of resonances with the same quantum numbers should exist in each channel. On the other hand, in the difference of scalar and pseudoscalar correlators the saturation may be successfully delivered by a finite number of low-lying resonances due to CSR.

Thus CSR at high energies may be thought of as a possible constraint on the EQQM to be a manifestation of compositeness in the Higgs model [18, 21]. We shall demand that, at the compositeness scale Λ, the relation (34) is approximately fulfilled.

Following the planar limit of the QCD, eq.(32) one can make the two-resonance ansatz for scalar and pseudoscalar correlators provided by Two-channel EQQM model,

$$\begin{aligned}
\Pi_\sigma(p) &= \frac{Z^\sigma}{p^2+m_\sigma^2} + \frac{Z^{\sigma'}}{p^2+m_{\sigma'}^2} + C_0^\sigma; \\
\Pi_\pi(p) &= \frac{Z^\pi}{p^2} + \frac{Z^{\pi'}}{p^2+m_{\pi'}^2} + C_0^\pi.
\end{aligned} \tag{35}$$

We remark that for this type of models the constants can be taken $C_1^\sigma = C_1^\pi = 0$. From the requirement of asymptotic CSR (34) it follows that,

$$C_0^\sigma = C_0^\pi \equiv C\left(= \frac{2<\bar{q}q>}{M_0} < 0\right); \tag{36}$$

$$Z^\sigma + Z^{\sigma'} = Z^\pi + Z^{\pi'}; \qquad Z^\sigma m_\sigma^2 + Z^{\sigma'} m_{\sigma'}^2 \simeq Z^\pi m_{\pi'}^2, \tag{37}$$

if one neglects with a supposedly small Δ_{SP} in (34) (see [13]). The first two relations can be fulfilled in the conventional NJL model which corresponds to the one-resonance ansatz, $Z^{\sigma',\pi'} = 0$, whereas the last one can be provided only in a two-resonance model.

The soft-momentum limit of the correlators is connected to the structural constants of the effective chiral lagrangian [22] (in our case of the EW lagrangian in the large Higgs mass limit [24]),

$$\begin{aligned} \frac{Z^\sigma}{m_\sigma^2} + \frac{Z^{\sigma'}}{m_{\sigma'}^2} + C &= 16B_0^2(2L_8 + H_2); \\ \frac{Z^{\pi'}}{m_{\pi'}^2} + C &= 16B_0^2(-2L_8 + H_2). \end{aligned} \tag{38}$$

These relations together with the empirical estimations on the corresponding vertex may serve for the determination of the size of the (techni-,top-)quark condensate $C_q = -B_0 F_0^2$. When eliminating the unobservable constant H_2 one comes to the large-N_c sum rule [4, 13] for meson parameters based on the phenomenological constant L_8 [22, 23],

$$\frac{Z^\sigma}{m_\sigma^2} + \frac{Z^{\sigma'}}{m_{\sigma'}^2} - \frac{Z^{\pi'}}{m_{\pi'}^2} = 64B_0^2 L_8. \tag{39}$$

4 CSR sum rules in Quasilocal Quark Model

In order to build systematically quark correlators we introduce conventionally external sources which couple to the scalar and pseudoscalar quark densities. As these densities in our model are doublets, eq.(2), the relevant sources $X_1 = (\chi_{1j}), X_2 = (\chi_{2j})$ are taken as doublets as well. The sources are complex: $\chi_{kj} = s_{kj} + ip_{kj}$, generating both scalar and pseudoscalar densities. The structure of the corresponding vertices in the quark lagrangian is similar to the Yukawa vertices in (7). Let us simplify our analysis and set $g = 0$ as $m_t \gg m_b$. Then the introduction of external sources can be performed by shifting the Higgs fields in the quark part of the lagrangian (7):

$$\phi_{kj} \longrightarrow \bar{\phi}_{kj} + \chi_{kj}. \tag{40}$$

The dynamical boson fields arise as fluctuations around the solutions of the mass-gap equation (13):

$$\bar{\phi}_{k1} = \phi_{k1} - \chi_{k1}; \quad \bar{\phi}_{k2} = \phi_{k2} - \chi_{k2} + < \bar{\phi}_{k2} > . \tag{41}$$

In terms of doublets X_k, Φ_k the supplementary lagrangian takes the form:

$$\begin{aligned} \Delta\mathcal{L}_I &= N_c\Lambda^2 \sum_{k,l=1}^{2} \Big[X_k^\dagger (a^{-1})_{kl} X_l - X_k^\dagger (a^{-1})_{kl}\Phi_l \\ &\quad -\Phi_k^\dagger (a^{-1})_{kl} X_l - X_k^\dagger (a^{-1})_{kl}\langle\Phi_l\rangle - \langle\Phi_k^\dagger\rangle (a^{-1})_{kl} X_l\Big]. \end{aligned} \tag{42}$$

Let us restrict ourselves with neutral components only and neglect terms linear in external sources irrelevant for correlators. Then with notations:

$$\phi_{k2} = \sigma_k + i\pi_k; \qquad \chi_{k2} = s_k + i\, p_k, \tag{43}$$

one obtains the quadratic part of the lagrangian $\mathcal{L}_I$ consisted of (16) and (42). As it is gaussian one can easily unravel the dependence on external fields with the help of classical Eqs. of motion:

$$\begin{aligned}
\sigma_k^{\rm cl} &= 16\pi^2\Lambda^2 \sum_{l,m=1}^{2} \left(A^{\sigma\sigma}p^2 + B^{\sigma\sigma}\right)^{-1}_{kl} a^{-1}_{lm} S_m \simeq 2\Lambda^2 \sum_{l=1}^{2} \left(A^{\sigma\sigma}p^2 + B^{\sigma\sigma}\right)^{-1}_{kl} S_l \\
\pi_k^{\rm cl} &= 16\pi^2\Lambda^2 \sum_{l,m=1}^{2} \left(A^{\pi\pi}p^2 + B^{\pi\pi}\right)^{-1}_{kl} a^{-1}_{lm} P_m \simeq 2\Lambda^2 \sum_{l=1}^{2} \left(A^{\pi\pi}p^2 + B^{\pi\pi}\right)^{-1}_{kl} P_l, \qquad (44)
\end{aligned}$$

which is simplified in the vicinity of polycritical point, $8\pi^2 a^{-1}_{kl} \simeq \delta_{kl}$.

The resulting effective action for generating of two-point correlators is given by:

$$\begin{aligned}
S^{(2)} &\simeq \frac{N_c\Lambda^2}{8\pi^2} \sum_{k,l=1}^{2} \left(S_k \Gamma^{(\sigma)}_{kl} S_l + P_k \Gamma^{(\pi)}_{kl} P_l\right) \\
\Gamma^{(\sigma)}_{kl} &= \delta_{kl} - 2\Lambda^2 \left(A^{\sigma\sigma}p^2 + B^{\sigma\sigma}\right)^{-1}_{kl}; \quad \Gamma^{(\pi)}_{kl} = \delta_{kl} - 2\Lambda^2 \left(A^{\pi\pi}p^2 + B^{\pi\pi}\right)^{-1}_{kl}. \quad (45)
\end{aligned}$$

When taking the approximate expressions (20), (29) for matrices $\hat{A}, \hat{B}$ one derives the inverse propagators:

$$\begin{aligned}
\left(A^{\sigma\sigma}p^2 + B^{\sigma\sigma}\right)^{-1}_{kl} &\simeq \left[6 \ln \frac{\Lambda^2}{m_t^2} (p^2 + m_\sigma^2)(p^2 + m_{\sigma'}^2)\right]^{-1} \\
&\times \begin{pmatrix} \frac{3}{2}p^2 - 2\Delta_{22} & \frac{\sqrt{3}}{2}p^2 + 2\Delta_{12} \\ \frac{\sqrt{3}}{2}p^2 + 2\Delta_{12} & 4p^2 \ln \frac{\Lambda^2}{m_t^2} + \frac{4\Delta_{11}\Delta_{22} - 6\Delta_{12}^2}{\Delta_{22}} \end{pmatrix}; \\
\left(A^{\pi\pi}p^2 + B^{\pi\pi}\right)^{-1}_{kl} &\simeq \left[6 \ln \frac{\Lambda^2}{m_t^2} p^2 (p^2 + m_{\pi'}^2)\right]^{-1} \\
&\times \begin{pmatrix} \frac{3}{2}p^2 - 2\Delta_{22} & \frac{\sqrt{3}}{2}p^2 + 2\Delta_{12} \\ \frac{\sqrt{3}}{2}p^2 + 2\Delta_{12} & 4p^2 \ln \frac{\Lambda^2}{m_t^2} - \frac{2\Delta_{12}^2}{\Delta_{22}} \end{pmatrix}. \qquad (46)
\end{aligned}$$

In particular, the strictly local quark density can be presented as a superposition of two currents:

$$\bar{t}t = \frac{1}{2}(\bar{t}f_1 t - \sqrt{3}\bar{t}f_2 t), \tag{47}$$

and respectively in the scalar channel the correlator (32) (for $N_f = 2$) reads,

$$\Pi_\sigma(p^2) = -\frac{N_c\Lambda^2}{16\pi^2}\left[\Gamma^{(\sigma)}_{11} + 3\Gamma^{(\sigma)}_{22} - 2\sqrt{3}\Gamma^{(\sigma)}_{12}\right] = C^\sigma + \frac{Z^\sigma}{p^2 + m_\sigma^2} + \frac{Z^{\sigma'}}{p^2 + m_{\sigma'}^2};$$

$$
\begin{aligned}
C^{\sigma} &= -\frac{N_c\Lambda^2}{2\pi^2} \simeq \frac{2 < \bar{q}q >}{M_0}; \\
Z^{\sigma} &= -\frac{N_c\Lambda^4}{48\pi^2 \ln\frac{\Lambda^2}{m_t^2}\,(m_{\sigma'}^2 - m_{\sigma}^2)} \\
&\quad \times \left[2\Delta_{22} + 4\sqrt{3}\Delta_{12} - 3\frac{4\Delta_{11}\Delta_{22} - 6\Delta_{12}^2}{\Delta_{22}} + \left(12\ln\frac{\Lambda^2}{m_t^2} - \frac{3}{2}\right) m_{\sigma}^2\right]; \\
Z^{\sigma'} &= \frac{N_c\Lambda^4}{48\pi^2 \ln\frac{\Lambda^2}{m_t^2}\,(m_{\sigma'}^2 - m_{\sigma}^2)} \\
&\quad \times \left[2\Delta_{22} + 4\sqrt{3}\Delta_{12} - 3\frac{4\Delta_{11}\Delta_{22} - 6\Delta_{12}^2}{\Delta_{22}} + \left(12\ln\frac{\Lambda^2}{m_t^2} - \frac{3}{2}\right) m_{\sigma'}^2\right]; \qquad (48)
\end{aligned}
$$

and similarly in the pseudoscalar channel,

$$
\begin{aligned}
\Pi_{\pi}(p^2) &= -\frac{N_c\Lambda^2}{16\pi^2}\left[\Gamma_{11}^{(\pi)} + 3\Gamma_{22}^{(\pi)} - 2\sqrt{3}\Gamma_{12}^{(\pi)}\right] = C^{\pi} + \frac{Z^{\pi}}{p^2} + \frac{Z^{\pi'}}{p^2 + m_{\pi'}^2}; \\
C^{\pi} &= C^{\sigma} = -\frac{N_c\Lambda^2}{2\pi^2} \simeq \frac{2 < \bar{q}q >}{M_0}; \\
Z^{\pi} &= -\frac{N_c\Lambda^4}{48\pi^2 \ln\frac{\Lambda^2}{m_t^2}\, m_{\pi'}^2}\left[2\Delta_{22} + 4\sqrt{3}\Delta_{12} + \frac{6\Delta_{12}^2}{\Delta_{22}}\right]; \\
Z^{\pi'} &= \frac{N_c\Lambda^4}{48\pi^2 \ln\frac{\Lambda^2}{m_t^2}\, m_{\pi'}^2}\left[2\Delta_{22} + 4\sqrt{3}\Delta_{12} + \frac{6\Delta_{12}^2}{\Delta_{22}} + \left(12\ln\frac{\Lambda^2}{m_t^2} - \frac{3}{2}\right) m_{\pi'}^2\right]. \qquad (49)
\end{aligned}
$$

Now we are able to impose and check the CSR constraints (37). First of all one can see that the chiral symmetry is not broken in leading asymptotics, $C^{\pi} = C^{\sigma}$. Next we check the subleading asymptotics which represents the generalized σ-model relation:

$$
Z^{\sigma} + Z^{\sigma'} = Z^{\pi} + Z^{\pi'} \simeq \frac{N_c\Lambda^4}{2\pi^2}, \qquad (50)
$$

which is fulfilled, in fact, at a higher precision including subleading $1/\ln\Lambda^2$ terms.

The last, approximate constraint in (37) is satisfied in the leading-log approximation because from (48) (49) : $Z_{\sigma'} \simeq Z_{\pi'} \gg Z_{\sigma}$, and from (30), (31) : $m_{\sigma'}^2 \simeq m_{\pi'}^2$. Thus:

$$
Z^{\sigma} m_{\sigma}^2 + Z^{\sigma'} m_{\sigma'}^2 \simeq Z^{\sigma'} m_{\sigma'}^2 \simeq Z^{\pi} m_{\pi'}^2. \qquad (51)
$$

However in the next-to-leading order this relation is not automatically satisfied for a variety of coupling constants Δ_{ij} and becomes also sensitive to the value of four-quark condensate Δ_{SP} in (34), *i.e.* which, in turn, is model dependent. We postpone the analysis of this constraint at subleading order to a next paper.

5 Conclusions

1. We have proved that CSR at high energies indeed takes place in the EQQM of type I for Two-Higgs doublets in the Gross-Neveu phase. Thereby these models in the GN phase can be regarded as effective models originating from a QCD-like underlying theory. More realistic predictions for masses and coupling constants should be based, of course, on the full SM action including vector bosons and on the RG improved calculations of low-energy parameters both in Higgs and in fermion sectors.

2. One can also show that in other phases explored in two-channel models in [16, 18], such as the anomalous or special phase, the last CSR constraint cannot be fulfilled for *any* choice of parameters. It means that such phases cannot be realized in effective models motivated by underlying QCD-like fundamental theory. But the question remains open about what kind of underlying theory could induce such phases at lower energies.

3. The CSR requirement to the leading order prescribes the rather large gap between masses of the lightest scalar Higgs boson and Higgs bosons in the second doublet. Meantime, all heavy masses are predicted to be nearly equal $m_h \sim m_A \sim m_{H^\pm}$. Their relative splitting is defined by the next-to-leading order which requires to draw a particular model for dim-6 terms in OPE.

4. It is remarkable that the above pattern for heavy Higgs particles corresponds fairly well to the predictions found with the help of fine-tuning hypothesis [26].

5. There exists another way to build multi-channel (nonlocal but separable) quark models [25] and it is of certain interest to develop the similar CSR analysis in that approach.

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References

[1] J.F.Gunion, H.E.Haber, G.Kane and S.Dawson, *The Higgs Hunter's Guide* Addison-Wesley, 1990.

[2] Y.Nambu, G.Jona-Lasinio, Phys.Rev. 122 (1961) 345.

[3] V. A. Miransky, M. Tanabashi and K. Yamawaki, *Mod. Phys. Lett.* **A4**, 1043 (1989); *Phys. Lett.* **B221**, 177 (1989);
W. J. Marciano, *Phys. Rev. Lett.* **62**, 2793 (1989); *Phys. Rev.* **D41**, 219 (1990);
W.A.Bardeen, C.T.Hill and M.Lindner. Phys. Rev. D41 (1990) 1647.

[4] A. A. Andrianov, V. A. Andrianov. Preprint hep-ph/9705364;
Zapiski Nauch. Sem. PDMI (Proc. Steklov Math. Inst., St. Petersburg Dept., Russia), 245, No.14 (1996) 5.

[5] S. L. Glashow, S. Weinberg, Phys.Rev. D15 (1977) 1958.

[6] T.D.Lee, Phys. Rev. D8 (1973) 1226.

[7] G. Kreyerhoff and R. Rodenberg, Phys. Lett. B226 (1989) 323; J. Freund, G. Kreyerhoff and R. Rodenberg, Phys. Lett. B280 (1992) 267.

[8] M. Lindner and D. Lüst, Phys. Lett. B272 (1991) 91;
M. Lindner, Int. J. Mod. Phys. A8 (1993) 2167.

[9] A. A. Andrianov, V. A. Andrianov, V. L. Yudichev, R. Rodenberg. hep-ph/9709475, 36 p. ; Int. J. Mod. Phys. A14, No.3 (1999) 323.

[10] K. G. Wilson, J. B. Kogut. Phys. Rep. C12 (1974) 75.

[11] T. Appelquist, J. Terning, L. C. R. Wijewardhana. Phys.Rev.Lett. 79 (1997) 2767 and references therein.

[12] R. S. Chivukula, B. A. Dobrescu, H. Georgi, C. T. Hill. Preprint hep-ph/9809470 and references therein.

[13] A. A. Andrianov, D. Espriu, R. Tarrach. Nucl.Phys. B533 (1998) 429.

[14] M. A. Shifman, A. I. Vainstein and V. I. Zakharov. Nucl. Phys. B147 (1979) 385, 448;
L. J. Reinders, H. Rubinstein and S. Yazaki, Phys. Rept. 127 (1985) 1 and references therein;
M. Jamin and M. Münz, Z. Phys. C66 (1995) 633.

[15] G. Cvetic, Preprint hep-ph/9702381 and references therein.

[16] A.A.Andrianov and V.A.Andrianov, Int. J. Mod. Phys. A8 No.11 (1993) 1981; Theor. Math. Phys., 93 (1992) 1126; *ibid.* 94 (1993) 3; Proc. School-Sem. "Hadrons and nuclei from QCD" (Tsuruga/Vladivostok/Sapporo 1993) WSPC (1994) 341, hep-ph/9309297; Zap. Nauch. Sem. POMI (Proc. St.Petersburg Math.Inst.), 209, No.12 (1994) 3; Nucl. Phys B (Proc.Suppl.) 39B,C (1995) 257.

[17] R. Rodenberg, Int. J. Mod. Phys. A11 (1996) 4779 and references therein.

[18] A.A.Andrianov, V.A.Andrianov and V.L.Yudichev, Proc. X Int. Worksh. on HEP and QFT (Zvenigorod, Russia, Sept. 1995) MSU Publ. (1996) 211, hep-ph/9512256; Proc. XI Int. Worksh. on HEP and QFT (St.Petersburg, Sept. 1996), MSU Publ. (1997); hep-ph/9610376;
Theor. Math. Phys., 108 (1996) 1069.

[19] G. Cvetic, C. S. Kim, S. S. Hwang. Phys. Rev. D58 (1998) 116003.

[20] G. t'Hooft, Nucl. Phys. B72 (1974) 461;
E.Witten, Nucl. Phys. B160 (1979) 57.

[21] N. V. Krasnikov and R. Rodenberg, Nuovo Cim. A105 (1992) 1601.

[22] J. Gasser, H. Leutwyler. Nucl. Phys. B250 (1985) 465

[23] J. Bijnens, E. de Rafael, H.Zheng, Z. Phys. C 62 (1994) 437;
J. Bijnens, Phys. Rept. 265 (1996) 369.

[24] A. Dobado, D. Espriu, M. J. Herrero, Phys. Lett. B255 (1991) 405;
P. Ciafaloni , D. Espriu , Phys. Rev. D56 (1997) 1752.

[25] M.K.Volkov, C.Weiss, Phys.Rev. D 56 (1997) 221;
M.K.Volkov, D.Ebert, M.Nagy, Preprint hep-ph/9705334.

[26] A.A.Andrianov, R. Rodenberg and N. V. Romanenko, Nuovo Cim. 108A (5) (1995) 577.