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Quantum field theory and particle masses

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Quantum field theory and particle masses.

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It is shown that in consistent quantum gauge field theories the interacting fermion and boson cannot both have mass zero. The electron as well as any charged fermion which can interact with the mass zero photon must have a finite mass in order that the gauge field theory shall be consistent. And the quarks interacting with the zero mass gluon must be massive. In the same way a zero mass neutrino can in a consistent weak interaction gauge field theory interact only with a massive gauge boson.

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Introduction.

In quantum gauge field theories describing the interaction between a fermion and a boson a prerequisite for the acceptability of the theory as a physical theory, is that the infrared divergence which occurs when the boson has mass zero, is removed as a non–physical phenomenon. We shall here analyse the infrared problem and show that non–consistent results are obtained for a gauge field theory in which a zero mass fermion couples to a zero mass boson. We show that for an acceptable theory, at least one of the fields, the fermion or bosons fields, must be described by a non–zero mass particle field.

Finite quark mass.

The process we discuss here specifically is $e^+e^- \to q\bar{q}g$ to first order in the strong coupling parameter. It is clear that from the cross sections obtained, results for similar processes not involving quarks and gluons, e.g. $e^+e^- \to \mu^+\mu^-\gamma$ and $e^+e^- \to \nu\bar{\nu}Z_0$, may be derived by appropriate changes of constants. The cross section for $e^+e^- \to q\bar{q}$ to first order in the strong interactions is for high electron energy E in the center of mass system [1]

$$\sigma(e^+e^- \to q\bar{q}) = \sigma_0\left[1+\frac{4\alpha_s}{3\pi}\left\{-\frac{1}{2}\ln^2\eta_q-\frac{3}{2}\ln\eta_q-2+\frac{2\pi^2}{3}+(\ln\eta_q+1)\ln\frac{\eta_q}{\eta_g}\right\}\right], \quad (1)$$

where $\sigma_0 = (4\pi/3)(\alpha^2/s)h_f$, $\eta_q = m_q^2/s$ and $\eta_g = m_g^2/s$ with m_q the quark mass and m_g the usual gluon mass parameter, and $s = 4E^2$. The coupling parameter h_f takes into account the exchange of Z_0 and is given by

$$h_f = Q_f^2 - 2Q_f \mathrm{Re} f(s) v v_f + 2|f(s)|^2(v^2+a^2)(v_f^2+a_f^2)$$

for quarks of flavour f with charge Q_f and weak coupling parameters v_f and a_f, and $f(s)=(4\sin^2\theta_W)^{-1} s/(s-M_Z^2 + iM_Z\Gamma)$. v and a are the corresponding electron weak coupling prameters. For simplicity we present our results in terms of cross sections integrated over angles.

We now describe an experiment in which all gluons of energies less than a specified value $E_{g\,max}$ are recorded, corresponding to the scaled energy $X_g = E_{gmax}/E$. The cross section which includes the elastic cross section Eq.(1) is infrared divergence free and given by [2]

$$\sigma(e^+e^- \to q\bar{q}g, x_g \leq X_g) = \sigma_0\Big[1 + \frac{4\alpha_s}{3\pi}\Big\{\ln\Big|\frac{1-X_g}{\eta_q} - 3\Big| - 1\Big]\Big[2\ln X_g + \tfrac{1}{2}(3-X_g)(1-X_g)\Big] + \tfrac{3}{4}[1-(1-X_g)^2] - \tfrac{1}{2}(1-X_g^2) - L_2(1-X_g)\Big\}\Big]. \quad (2)$$

with $L_2(x)$ the Euler dilogarithm function. It is easy to see that for $X_g=1$, the cross section is given by the simple well known expression, the total cross section to order α_s

$$\sigma(e^+e^- \to q\bar{q}g) = \sigma_0(1+\frac{\alpha_s}{\pi}). \quad (3)$$

Clearly this result is in agreement with the Kinoshita–Lee–Nauenberg theorem [3]: The summation over all gluon final states gives a result free of quark mass singularities. The physical cross section Eq.(2) does however contain $\ln\eta_q$, the quark mass singularity.

For an experiment in which quarks are recorded down to a scaled quark energy $X = E_{qmin}/E$,which includes the cross section Eq.(1), the infrared divergence free cross section, corresponding to Eq.(2) is

$$\sigma(e^+e^- \to q\bar{q}\bar{g}, x \geq X) = \sigma_0\Big[1 + \frac{\alpha_s}{3\pi}\Big\{\frac{1}{2}\ln(X^2/\eta_q)[2\ln(1-X)+X+\frac{1}{2}X^2] - \ln(1-X)[1+X-\frac{1}{4}X^2] + \frac{3}{4} - 2X + \frac{1}{2}\ln^2(1-X) + L_2(X)\Big\}\Big]. \quad (4)$$

Again the total cross section is (for X=0) given by Eq.(3).

Mass zero quarks.

We now discuss the same process corresponding to Eqs.(2) and (4), for mass zero quarks. The cross section for $m_q = 0$ corresponding to Eq.(1) was first given by Sterman and Weinberg [4]

$$\sigma(e^+e^- \to q\bar{q},\, m_q = 0) = \sigma_0\left[1 + \frac{4\alpha_s}{3\pi}\left\{-\frac{1}{2}\ln^2\eta_g - \frac{3}{2}\ln\eta_g - \frac{7}{4} + \frac{\pi^2}{6}\right\}\right], \tag{5}$$

where the infrared gluon parameter η_g has to be set equal to zero in the final result of the calculation of the cross section. We give some details of the calculation of the cross section involving the emission of gluons with $x_g \leq X_g$ corresponding to Eq.(2) for finite quark mass. The cross section is found to be given by [5]

$$\sigma(e^+e^- \to q\bar{q}g,\, x_g \leq X_g m_q = 0) =$$

$$= \sigma_0 \frac{2\alpha_s}{3\pi}\int dx_g \int dx \left[\frac{x^2 + \bar{x}^2}{(1-x)(1-\bar{x})} - \frac{\eta_g}{(1-x)^2} - \frac{\eta_g}{(1-\bar{x})^2}\right], \tag{6}$$

with integration limits $(1-x)_{max,min} = \frac{1}{2}(x_g \pm \sqrt{x_g^2 - \bar{m}_g^2}$ and $\bar{m}_g \leq x_g \leq X_g$, with $\bar{m}_g = m_g/E$ the scaled gluon mass. The result is

$$\sigma(e^+e^- \to q\bar{q}g,\, x_g \leq X_g,\, m_q = 0) = \sigma_0 \frac{4\alpha_s}{3\pi}\left\{\frac{1}{2}\ln^2\eta_g + \frac{3}{2}\ln\eta_g + \frac{5}{2} - \frac{\pi^2}{6} - \right.$$
$$\left. - \ln\eta_g\, H(X_g) + f_1(X_g)\right\}, \tag{7}$$

which gives the cross section, adding Eqs.(5) and (7)

$$\sigma(x_g \leq X_g, m_q = 0) = \sigma_0\left[1 + \frac{4\alpha_s}{3\pi}\left\{\frac{3}{4} - \ln\eta_g\, H(X_g) + f_1(X_g)\right\}\right], \qquad (8)$$

where we have defined the functions

$$H(X_g) = 2\ln X_g + (1-X_g) + \tfrac{1}{2}(1-X_g)^2 \qquad (9)$$

and

$$f_1(X_g) = 2\ln X_g[\ln X_g - 2X_g + \tfrac{1}{2}X_g^2] + 3X_g - \tfrac{1}{2}X_g - \tfrac{5}{2}. \qquad (10)$$

In the same way, the cross section for $x \geq X$ for massless quarks, corresponding to Eq.(4) for massiv quarks is found to be given by

$$\sigma(x \geq X, m_q = 0) = \sigma_0[1 + \frac{4\alpha_s}{3\pi}\left\{\frac{3}{4} - \ln\eta_g H(1-X) + g_1(X)\right\}\Big], \qquad (11)$$

where $H(1-X)$ is the same function as in Eq.(9) with $1-X$ substituted for X_g

$$H(1-X) = 2\ln(1-X) + X + \tfrac{1}{2}X^2,$$

and $g_1(X)$ is given by

$$g_1(X) = \tfrac{1}{2}\ln X\, H(1-X) + \tfrac{1}{2}\ln^2(1-X) - \tfrac{1}{2}\ln(1-X)[3 - X - \tfrac{1}{2}X^2] - \\ - \tfrac{3}{2}X - \tfrac{3}{8}X^2 + L_2(X). \qquad (12)$$

Clearly Eqs. (8) and (11) are meaningless since the infrared divergences have not cancelled: the cross sections are (negative) infinite for $\eta_g = 0$. Moreover for the particular values $X_g = 1$ in Eq.(7) and $X = 0$ in Eq.(11) the total cross section is finite and given by Eq.(3). Clearly the cross sections Eqs. (7) and (11) are inconsistent for $m_g = m_q = 0$.

Finite quark mass in the limit $m_q = 0$.

We now turn to Eqs. (2) and (4) taking the limit $m_q \to 0$, which gives

$$\sigma(x_g \leq X_g, m_q \to 0) = \sigma_0 \left[1 + \frac{4\alpha_s}{3\pi} \left\{ \frac{3}{4} - \ln\eta_g \, H(X_g) + f(X_g) \right\} \right], \tag{13}$$

with $H(X_g)$ the same function as in Eq.(9) and

$$f_2(X_g) = H(X_g)[\ln(1-X_g)-1] - \frac{3}{4}(1-X_g)^2 - \frac{1}{2}(1-X_g^2) + L_2(1-X_g). \tag{14}$$

Correspondingly Eq.(4) may be written in the form

$$\sigma(x \geq X, m_q \to 0) = \sigma_0 [1 + \frac{4\alpha_s}{3\pi} \left\{ \frac{3}{4} - \ln\eta_g \, H(1-X) + g_2(X) \right\}] \tag{15}$$

with

$$g_2(X) = \ln(1-X)[2\ln X - 1 - X + \frac{1}{4} X^2] - \ln^2(1-X) + 2L_2(X) - 2X . \tag{16}$$

As for the case of massless quarks we find logarithmic singularities for $m_g \to 0$, here represented by $\ln\eta_g$ multiplied with the same functions $H(X_g)$, respectively $H(1-X)$. In contrast to this the total cross section, i.e. for the particular values $X_g=1$, respectively $X = 0$, is finite and given by Eq.(3).

Inconsistencies.

Clearly Eqs.(8) and (11) are finite for $m_q = 0$ but $m_g \neq 0$ and Eqs.(13) and (15) are finite for $m_g = 0$ but $m_q \neq 0$: Consistent cross section are obtained if at least one of the fermion or boson masses is different from zero. On the other hand a consistent theory cannot be obtained if both masses are zero.

It should be noted that the functions $f_1(X_g)$ and $f_2(X_g)$ and similarly $g_1(X)$ and $g_2(X)$ are not identical, which is a simple consequence of the fact that the limiting procedures for m_g and m_q are different for the two cases $m_q=0$, $m_g \to 0$ and $m_g \to 0$, $m_q \to 0$ in this order.

It would not be unreasonable to assume that an exponentiation like "Schwinger's conjecture" [1,6] may be obtained taking higher orders of α_s into account, replacing e.g. $1-\text{const.}\ln\eta_g\, H(x_g)$ by $\exp(-\text{const.}\ln\eta_g H(X_g))$ [7]. This would lead to the result that $\sigma(x_g \leq X_g) = 0$, for $\eta_g = 0$ and $\eta_q = 0$, for all values of X_g , except for $X_g = 1$, where the cross sextion would be finite and given to first order by Eq.(3). This would mean that there is no interaction between a zero mass gluon and a zero mass quark for $X_g < 1$, but the total cross section would be finite, which is clearly an inconsistent result.

The consequences of our results are that a zero mass fermion cannot consistently interact with the zero mass photon or gluon, and in general any charged particles or a particle carrying color must have a mass. To be more exact, turning the argument around,the theory of a mass zero fermion interacting with the photon or gluon cannot be incorporated in present quantum gauge field theories. On the other hand, the massless standard model neutrino can in a consistent theory interact only with massive gauge bosons. It also follows that the electro–weak theory before symmetry breaking is not a consistent physical theory.

The results do not say anything about the size of the mass of the fermion interacting with a photon or gluon, only that it is non–zero. Likewise it does not explain why the Z_0 and W gauge bosons are so heavy.

References

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[4] Georg Sterman and Steven Weinberg, Phys.Rev.Lett. 30. 1436 (1977).

[5] The cross section including the important η_g terms is the same as for finite η_q given in reference [2] except that η_g is replaced by $2\eta_q$.

[6] For the case of exponentiation for finite mass electrons scattered in an atomic field see e.g. Relativistic Quantum Theory, part 2 by E.M. Lifshitz and L.P. Pitaevskii, Pergamon Press, Oxford, 1974.

[7] Note that $\ln\eta_g \, H(X_g)$ is positive for all values of $X_g < 1$.