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Rotational dependence of Coulomb energy differences

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Abstract

The rotational dependence of Coulomb energy differences is investigated throughout the $f_{7/2}$ shell using a cranked shell model framework. Predictions are made for the change in Coulomb energy differences at the first band crossing of odd- and even-mass mirror nuclei. It is found that this change is smaller in even-mass mirror nuclei where it depends critically on the relative shift in the bandcrossing frequencies of the two mirror nuclei. These results account for the recently observed behaviour in the $A = 47$ and 49 mirror nuclei.

With the steady increase in sensitivity achieved in γ-ray spectroscopy techniques over recent years, it has now become feasible to study mirror nuclei in the $f_{7/2}$ shell to very high angular momenta and hence to investigate, for the first time, the angular momentum dependence of Coulomb energy differences (CEDs) through and above the bandcrossing region. The first measurements of this type [1–4] revealed a striking discontinuity in the behaviour of the CED in the neighbourhood of the bandcrossing which could be interpreted [5] in terms of the alignment of particles, taking into account the fact that only protons contribute to the Coulomb energy. Indeed, this seemingly incongruous relationship between rotational alignment and CED can be understood by recalling that at the bandcrossing there is a transition from a fully paired $J = 0$ configuration to an aligned configuration with predominant contribution from the $J = M_x = 6$ state (in the $f_{7/2}$ shell). The $J = 0$ paired state has the largest overlap between the proton orbits and hence the maximum Coulomb energy, while the aligned state has the smallest value so that, at a bandcrossing due to the alignment of protons, the Coulomb energy will exhibit a transitional behaviour. In spite of the fact that a reliable evaluation of the CED requires a string of correction terms [6], it was shown that the observed change in CED follows the pattern expected from the simple alignment mechanism, decreasing

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at the crossing point and apparently reaching the expected saturation value after the onset of the alignment.

Very recently, new experimental studies [7,8] have extended knowledge of the heaviest mirror nuclei with $A = 47$ and 49 up to the maximum spin achievable in a pure $f_{7/2}$ shell and hence well beyond the first bandcrossing. However, these data reveal an unexpected *decrease* in the CED after the first alignment. It is therefore the purpose of the present work to explain these new observations by providing a more complete study of the angular momentum dependence of the CED in the $f_{7/2}$ shell. Moreover, these results will provide predictions for, as yet, unmeasured CEDs in other pairs of mirror nuclei in this region, including those involving even-mass nuclei.

The Coulomb energy of two protons in a harmonic oscillator orbit (n, l, j) coupled to angular momentum J is given by

$$V_c(J) = \sum_{SL} (2j+1)^2 (2S+1)(2L+1) \begin{Bmatrix} 1/2 & l & j \\ 1/2 & l & j \\ S & L & J \end{Bmatrix}^2$$
$$\times \sum_{n'l'N\Lambda} (n'l'N\Lambda | nlnl; L)^2 \int_0^\infty |R_{n'l'}(r)|^2 V_c(r) r^2 dr, \qquad (1)$$

where the symbol $(\ldots|\ldots)$ is the Talmi-Moshinsky bracket [9,10], R_{nl} is the harmonic oscillator radial wave function in the relative coordinate and V_c is the Coulomb potential. The calculated Coulomb energies for protons in the $f_{7/2}$ shell are given in Table 1 where they are compared with the values obtained from the empirical binding energies and the level energies of $A = 42$ nuclei, following the procedure as outlined in [5]. As is apparent from the table, the calculated Coulomb energy is in good agreement with the empirically estimated value for the $J = 0$ state but deviates for higher angular momentum states. This deviation, as pointed out in [5], can be attributed to the use of the harmonic oscillator potential. In the present study, we use the empirically obtained Coulomb matrix elements in order to incorporate a first-order description of the omitted correction terms [6] necessary to describe systems with more than two particles in the $f_{7/2}$ shell.

The rotational dependence of the Coulomb energies is investigated in a cranked shell model framework consisting of a cranked deformed one-body term and a scalar two-body term which is taken as the delta interaction [11]

$$H' = h_{def} - g\delta(\vec{r}_1 - \vec{r}_2) - \omega J_x. \qquad (2)$$

Here h_{def} is the quadrupole-deformed mean field,

$$h_{def} = -4\kappa\sqrt{\frac{4\pi}{5}}\sum_i Y_{20}(\hat{r}_i), \tag{3}$$

and κ is the deformation energy, which is related to the usual deformation parameter β by

$$\kappa \simeq 0.16\hbar\omega_0(N+3/2)\beta, \tag{4}$$

where $\hbar\omega_0$ is the harmonic oscillator frequency of the deformed potential and N the major-shell quantum number. Energies are expressed in units of $G = g\int |R_{nl}(r)|^4 r^2 dr$.

The model hamiltonian (2) is solved for particles in the $f_{7/2}$ shell with $\kappa = 1.5$ MeV which approximately corresponds to the observed deformation ($\beta \sim 0.2$) in the $f_{7/2}$ region. The results for the CED between odd-mass mirror partners are plotted in Fig. 1 against the rotational frequency, the latter being given in units of the pairing strength G which takes a value of $\sim$ 1 MeV in this region. It is to be noted that in the absence of an explicit neutron-proton (np) interaction one may perform the calculations for protons only. The np interaction will somewhat modify the results, as discussed later. Thus, for example, the CED between the systems of four-protons+three-neutrons (4p+3n) and 3p+4n are shown as 4p-3p. It can be seen from Fig. 1 that, at low rotational frequencies, the CED is almost constant but at $\hbar\omega = 0.6 \sim 0.8G$, we observe a steady rise or fall in this quantity. In fact, the overall behaviour of the CED as a function of the rotational frequency can be understood by considering the accompanying pattern of rotational alignment as discussed below.

The alignments in the $f_{7/2}$ shell for various proton numbers are plotted in Fig. 2 as a function of the rotational frequency. In the case of two protons (2p), we note a rapid increase in i_x at around $\hbar\omega = 0.65G$ which is due to the alignment of two protons. For the 3p case, the (AB)-crossing is blocked due to the odd particle. The first crossing for this odd-particle number is observed at $\hbar\omega = 1.0G$. This (BC)-crossing, as is well known, is very broad compared to the (AB)-crossing and the accompanying gain in alignment is also lower. In the case of 4p, the (AB)-crossing is seen at around $\hbar\omega = 0.7G$ and is slightly higher than in the 2p case. The gain in alignment continues to increase after the (AB)-crossing has taken place due to the alignment of two more protons.

The CEDs of Fig. 1 can now be understood by noting that, at any crossing representing an alignment of protons, the Coulomb energy will drop due to the transition from the paired $J = M = 0$ state to the two-particle aligned $J = M_x = 6$ state. Thus, taking the (4p-3p) case as an example, the differences in the Coulomb energy of the two mirror nuclei will show a fall at the frequency at which the (AB)-crossing begins in the four-particle case, since there is no alignment in the 3p nucleus at these frequencies. In the (5p-4p) case, the result is the inverse, with a rise in CED taking place at the 4p crossing frequency. These examples simulate the mirror partners ^{47}Cr/^{47}V and ^{49}Mn/^{49}Cr, respectively and the results can be compared with the recently measured CEDs for these nuclei [8,7]. The calculated change is approximately 150 keV in each case, which is in reasonable agreement with the experimental value of about 120 keV observed for the $A = 49$ pair, with the $A = 47$ empirical shift of ~ 90 keV being somewhat less. However, it is also to be noted that the experimental rise is somewhat sharper than the calculated one. This may indicate that the deformation used in the present work is somewhat larger than the actual deformation for these mirror nuclei.

Earlier treatments of these effects [2,5] anticipated that the CED would saturate after the alignment was complete and the first empirical results seemed to confirm this behaviour. However, it was pointed out earlier that the new data [7] for CEDs above the bandcrossing show a marked decrease, in contrast to the previous expectations. It can be seen in Fig. 1 that the calculated CEDs successfully reproduce this behaviour, saturating a little after the first alignment but then showing a trend towards a reduction in the effect at higher rotational frequencies. Again, taking the (4p-3p) case as the example, and referring again to Fig. 2, this can be understood from the fact that the current calculation accounts for the effects of alignment in *both* members of the mirror pair. Hence, the 3p crossing, which is delayed to a higher rotational frequency, gives rise to a decrease in the Coulomb energy for the 3p nucleus resulting in a matching increase in the CED of the (4p-3p) pair at around $\hbar\omega = 1.1G$. The inverse behaviour can be expected in the case of the (5p-4p) mirror nuclei.

A similar procedure can be followed for the pairs of even-mass mirror nuclei in the $f_{7/2}$ shell and the results are plotted in Fig. 3. The CED for the (4p-2p) case, for example, corresponds to the mirror pair ^{46}Cr and ^{46}Ti. The CED in this case is again constant at low rotational frequency but then shows an increase at around $\hbar\omega = 0.65G$. This corresponds to the (AB)-crossing in the 2p nucleus. Since the (AB)-crossing in the 4p nucleus occurs at a somewhat later frequency, we observe this increase but it is cut off prematurely by the onset of the alignment in the 4p nucleus. If the (AB)-crossings in the 2p and the 4p case had occurred at the same rotational frequency, then we would not have predicted any increase at all in the calculated CED. The drop in CED after $\hbar\omega = 0.65G$ is due to the continuing influence of the 4p (AB)-crossing. The Coulomb energy continues to drop at higher rotational frequencies with the alignment of a further two particles.

Another result of the competition between the first alignments in the two even-mass nuclei is that the change in the CED for the even-mass mirror partners at the alignment can be significantly lower than the corresponding change obtained for the odd-mass nuclei. More generally, the frequency and magnitude of the changes in CEDs of the even-mass mirror nuclei will be particularly sensitive to the relative shift in bandcrossing frequency in each partner and will therefore show more variation across the shell than for the odd-mass systems.

We would like to add here that earlier studies [5] show that the effects on CEDs predicted above may be somewhat attenuated by the inclusion of the np interaction since the alignments in the presence of the np interaction are linear superpositions of neutron and proton angular momenta [12].

To summarise, we have accounted for the behaviour of the CED of yrast states recently observed in the $A = 49$ and 47 mirror nuclei in the framework of a simple cranked shell model with the empirically estimated Coulomb matrix elements. In the case of odd-mass mirror nuclei the (AB)-crossing is blocked for the odd particles and the initial change in the Coulomb energy is due to the (AB)-crossing in the even-proton nucleus. The later onset of the (BC)-crossing in the odd-proton nucleus gives rise to the observed decrease in the effect at higher frequency. For the even-mass mirror nuclei the situation is different: the (AB)-crossing is either blocked or not blocked in both partners and the magnitude of the change in the CED depends on the relative positions of the two crossings which are of the same type, (AB) or (BC). The result is that this change is reduced, relative to that in the odd-mass mirror nuclei, and depends critically on the shift in frequency between the first bandcrossings in the two mirror nuclei. In the event the crossings are simultaneous, no change in the CED will be observed.

These results therefore provide a set of predictions which can be tested against further measurements of CEDs of the yrast states in $f_{7/2}$-shell nuclei. They illustrate that such differences may provide a sensitive probe of the changes in the spatial distribution of the protons in the nucleus with increasing excitation energy.

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Table 1
Experimental and calculated Coulomb matrix elements (in keV) as a function of angular momentum

	$J = 0$	$J = 2$	$J = 4$	$J = 6$
Expt.	578(6)	486(6)	374(7)	330(9)
Calc.	591	505	458	449

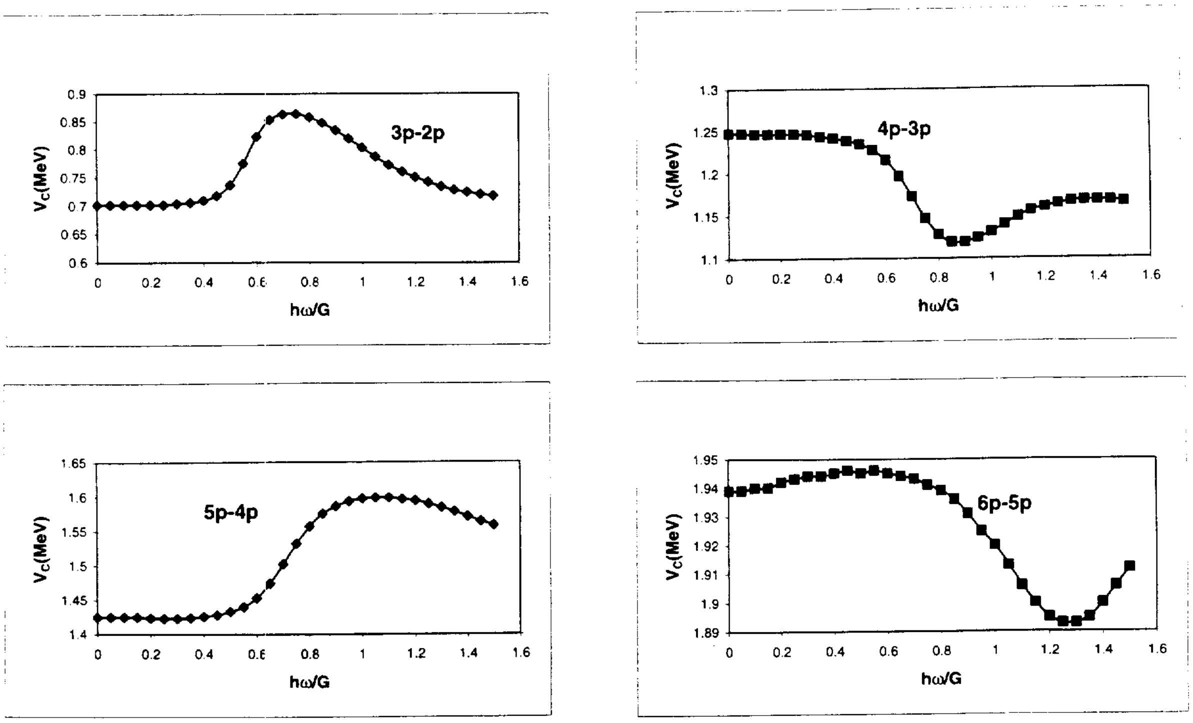

Fig. 1. Predicted Coulomb energy differences for pairs of odd-mass mirror nuclei, labeled by the proton numbers of each partner.

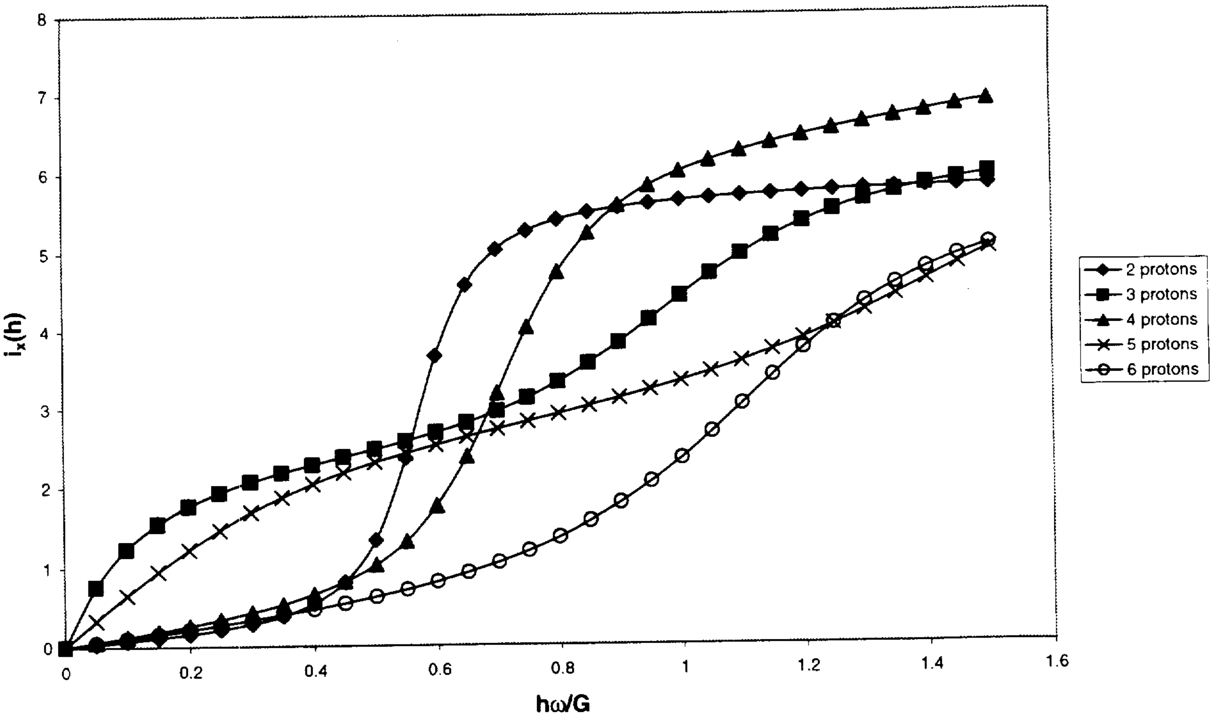

Fig. 2. The aligned proton angular momentum i_x as a function of rotational frequency for various proton numbers in the $f_{7/2}$ shell.

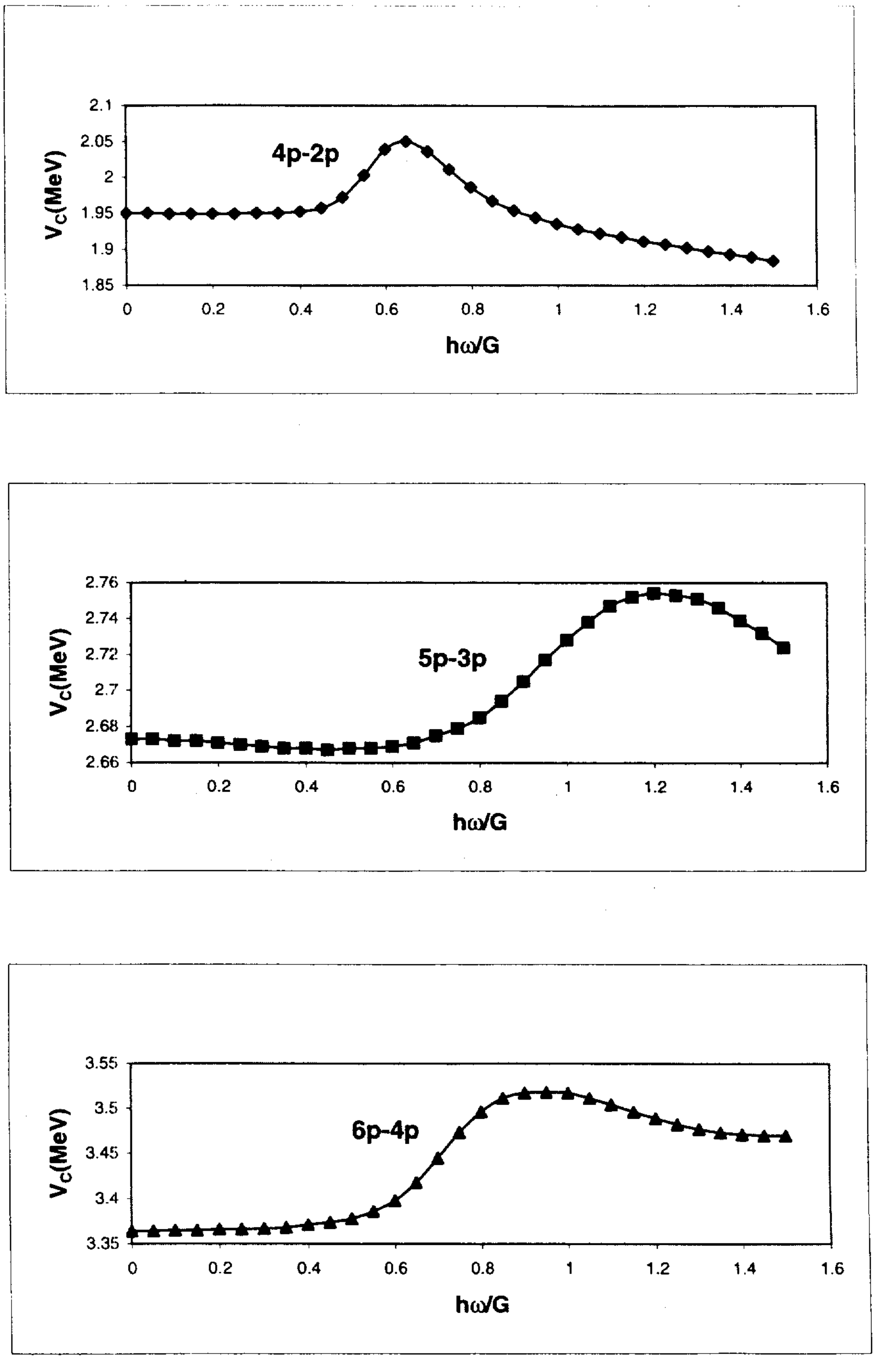

Fig. 3. Predicted Coulomb energy differences for pairs of even-mass mirror nuclei, labeled by the proton numbers of each partner.