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High Energy Electron-Photon Showers in Single Crystals

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Abstract

The characteristics of an electromagnetic shower in axially aligned single crystals are investigated numerically for energy of initial electrons and photons from tens to hundreds GeV. The energy and angular distributions of created particles at different thicknesses are compared for Ge and W crystals.

1 Introduction

It is well known that the mechanisms of photon emission [1] and pair production [2] processes in single crystals depend on the angle of incidence of an initial particle ϑ_0 with respect to crystal axes or planes. At sufficiently high particle energy these coherent effects appear to be especially strong for $\vartheta_0 \ll V_0/m$ (V_0 is the scale of corresponding potential) at axial alignment, being one to two order of magnitude larger than in corresponding amorphous medium. Along with spectra of created particles and photons are also very different from those providing by Bethe-Heitler (BH) mechanism, it naturally leads to many distinctive features (see [4]) of the specific electron-photon cascade in oriented single crystal as compared to that in amorphous medium.

Let us remind that the pair production probability in the field of crystal axes falls off exponentially for photon energy below ω_t, revealing some kind of threshold behavior. We determine ω_t more definitely as follows

$$W_e(\omega_t) = W_e^{BH}, \tag{1}$$

where $W_e(\omega)$ is the probability of pair creation by a photon in the field of axis and W_e^{BH} is the pair production probability in the corresponding amorphous medium. When initial particle energy ε_0 $(\omega_0) \gg \omega_t$

and lower boundary of recorded particle energy $\varepsilon_f \geq \omega_t$, coherent mechanism dominate in both pair production and photon emission processes during cascade development. This development is described by a set kinetic equations (see, e.g. [3]) in terms of averaged number of charged particles (electrons and positrons) $N_e(\varepsilon, t)$ and photons $N_\gamma(\omega, t)$ with given energies ε and ω at some fixed depth t. We call it ***hard cascade***, which properties were investigated in [4], where under some simplifying assumptions analytical solutions of kinetic equations were obtained. This solution shows that the number of particles and photons in the developed shower (after many emission and pair creation events) grows exponentially just as in an amorphous medium, than (for particles with energy $\varepsilon \geq \varepsilon_f$) it attains a maximum at the thickness $t = t_{op}$:

$$t_{op} = \frac{2}{5} \int_{\varepsilon_f}^{\varepsilon_0} \frac{d\varepsilon}{\varepsilon} \left(L_{ch}(\varepsilon) + L_{ph}(\varepsilon) \right), \tag{2}$$

$L_{ch}(\varepsilon)$ is the characteristic radiation length depending on energy, crystal type and the axis under consideration $L_{ch}^{-1}(\varepsilon) = -\dfrac{1}{\varepsilon}\dfrac{d\varepsilon}{dt}$ and $L_{ph}^{-1}(\varepsilon) = W_e(\varepsilon)$. The corresponding estimate in an amorphous medium where both characteristic lengths are independent of energy is

$$t_{op} \simeq L_{rad} \ln \frac{\varepsilon_0}{\varepsilon_f}. \tag{3}$$

When $\varepsilon_0 < \omega_t$ and $\varepsilon_f \ll \omega_t$ (we call it *soft cascade*) the pair production occurs owing to BH mechanism, while photon emission is still dominated by the coherent one. Some properties of a *soft cascade* were investigated in [5], where, in particular, a model spectrum of radiation at axial alignment was suggested used in the present paper too. And, finally, we call *mixed cascade* the situation when ε_0 $(\omega_0) > \omega_t$ (or ε_0 $(\omega_0) \gg \omega_t$), while $\varepsilon_f \ll \omega_t$ and both coherent and BH mechanisms contribute at different stages of the cascade development.

At relatively small thickness one can use decomposition over number of generations ([7]):

$$N_e(\varepsilon, t) = \sum_{n=0}^{\infty} N_e^{(n)}(\varepsilon, t), \tag{4}$$

where the first term of the series ($n = 0$) allows for the evolution of the initial distribution due to the photon emission only. The second one ($n = 1$) takes into account creation of pairs of the first generation by photons emitted by the initial particle etc. An analysis made for an amorphous medium shows that first two terms in Eq.(4) give satisfactory description of a shower (within 10% accuracy) only for $t < L_{rad}$.

The present paper is mainly dedicated to the comparison of the *mixed cascade* properties in different crystals. We perform numerical calculations (Monte-Carlo simulation) for $< 110 >$ axis of germanium and $< 100 >$ axis of tungsten single crystals obtaining dependence of cascade characteristics on initial particle type, crystal thickness (up to two radiation lengths in corresponding amorphous medium), and initial particle energy in the range from a few tens to a few hundreds of GeV.

The development of an electron-photon cascade at the axial alignment has been observed and investigated for the first time in [6]. We have compared in [3] our results (see also Fig.1b in [7]) to those of [6] and have found a quite good agreement with experimental data.

2 Discussion of results

It is well known that in a thin crystal as a consequence of a flux redistribution determined by initial conditions electrons may radiate more strongly than positrons. However, the full-scale cascade can develop only in thick crystals where the distribution of both electrons and positrons in the transverse phase space becomes uniform, resulting in the same radiation. Assuming that the probabilities of the emission of radiation from electrons and positrons are the same, we have studied electromagnetic shower properties by means of Monte-Carlo simulation for particles incident along $< 110 >$ axis of Ge and along $< 100 >$

axis of W single crystals with energies from a few tens to a few hundreds of GeV. The incoherent contributions to probabilities $W_\gamma(\varepsilon,\omega)$ and $W_e(\omega,\varepsilon)$; ionization energy losses and multiple scattering were described by formulae given in [5]. The coherent contribution to the photon emission probability was calculated according to eq.(2) of [5] giving model description of radiation spectrum at axial channeling. For high energies it turns into intensity spectrum calculated in constant field approximation (CFA). The coherent contribution to the electron-positron pair production probability was calculated in CFA also.

All the results obtained describe the charged particles (photons) created within a crystal (the initial particle was not recorded) and going out from a crystal. Initial particles momenta were assumed to be aligned exactly along the axis chosen. Usually particles with energies satisfying the inequality $\varepsilon > \varepsilon_f = 5\ MeV$ were recorded.

The total number of charged particles with $\varepsilon > 5\ MeV$ is shown in Fig.1 vs crystal thickness L measured in units of the corresponding radiation length: $L_{rad} = 2.3\ cm$ in Ge and $L_{rad} = 0.35\ cm$ in W; Fig.1(a) is for initial electrons with energies indicated and Fig.1(b) is for initial photons. Comparing Fig.1(a) to Fig.1(b), we see that the curves presenting numbers of created charged particles are quite similar in shape for the same initial energy of electrons and photons. Since simulation was performed for two different crystals it's interesting to compare their efficiency. Let us remind that shower characteristics are identical for any amorphous media if thickness is measured in corresponding radiation lengths in the so called approximation A when bremsstrahlung and pair creation are taken into account only (see [8], sect. 65). In crystals situation is different because all the probabilities are specific functions of the energy for each crystal. For very small thickness ($L \ll L_{rad}$ in our case) the number of created charged particles N_{ch} can be estimated (see eq.(27) in [4] for initial electrons) as $N_{ch} \simeq W_\gamma(\varepsilon_0)W_e(\omega_c(\varepsilon_0))L^2$ for initial electrons and $N_{ch} \simeq 2W_e(\omega_0)L$ for initial photons. Using actual probabilities $W_\gamma(\varepsilon)$ and $W_e(\omega)$ we find for initial electrons that $N_{ch}^{Ge} > N_{ch}^{W}$ for the given value of L/L_{rad} for energies up to 300 GeV. Fig.1(a) shows that this relation holds up

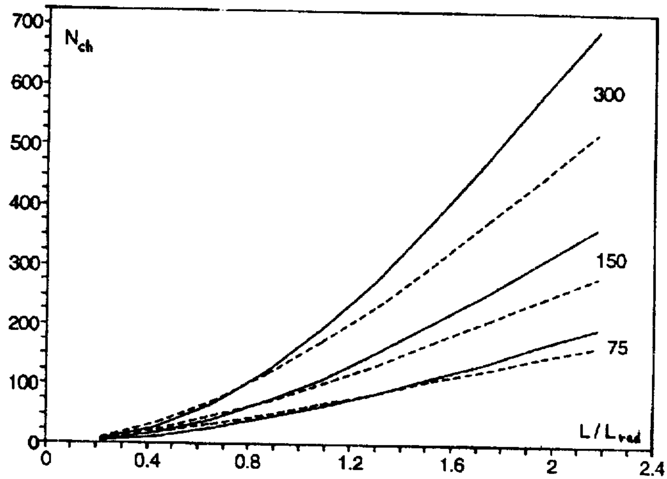

Fig 1(a). The number of created charged particles N_{ch} with energy $\varepsilon > 5\ MeV$ vs crystal thickness in tungsten W (axis $< 100 >$), solid curves and germanium Ge (axis $< 110 >$), dashed curves. The initial particle is an electron with the energy indicated.

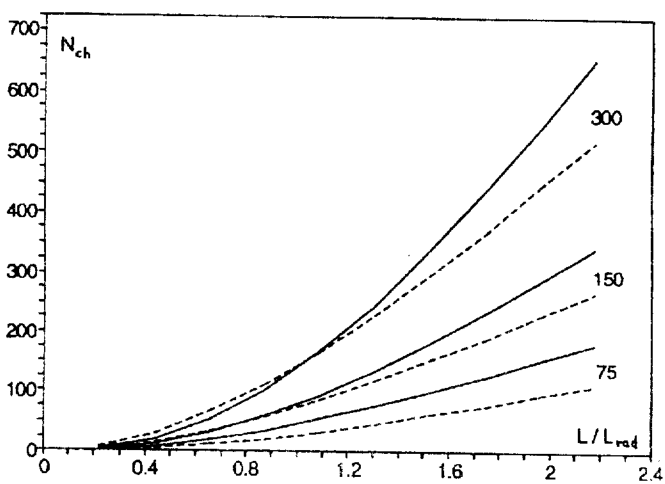

Fig 1(b). The same as in Fig.1(a) but for initial photons with the energy indicated.

to $L/L_{rad} \sim 1$. However, at larger thickness there is an opposite situation and gain in W is quite sizable. The intersection point $N_{ch}^{Ge} = N_{ch}^{W}$ shifts towards smaller relative thicknesses when the energy increases.

For initial photons the estimate gives $N_{ch}^{W} > N_{ch}^{Ge}$ for $\omega_0 < 270\,GeV$. Correspondingly, the relation between N_{ch}^{W} and N_{ch}^{Ge} for $\omega_0 = 300\,GeV$ is qualitatively different as compared to lower energies what can be seen in Fig.1(b).

In Fig.2 the number of created charged particles vs the energy of the initial particle (an electron or a photon) is given for the crystal thickness $L/L_{rad} = 1$ in Ge and W.

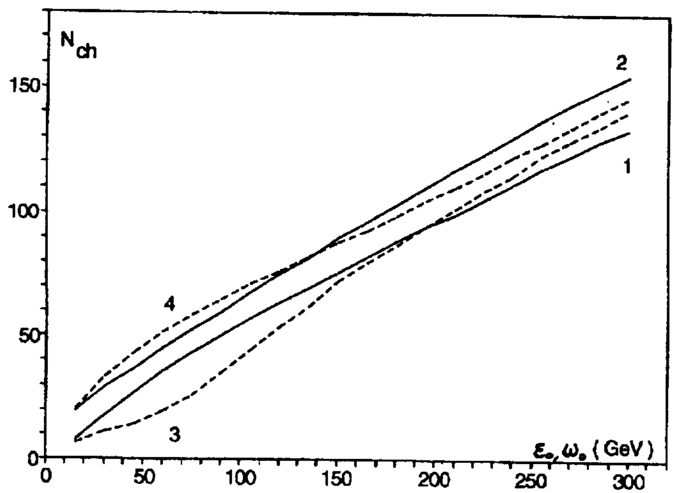

Fig. 2. The number of created charged particles N_{ch} with energy $\varepsilon > 5\ MeV$ outgoing from a crystal of thickness $L/L_{rad} = 1$ vs the initial electron (curves 2 and 4) or photon (curves 1 and 3) energy for W (axis < 100 >), solid curves and for Ge (axis < 110 >), dashed curves.

At lowest energy considered the curves for initial electrons in Ge and W are close to each other since crystal effects are still relatively weak and for amorphous media they should coincide as it was said above. The same is true for initial photons. When the energy increases the difference between numbers of charged particles created in the same crystal by initial electrons or photons first increases. Then starting with energies $\varepsilon_0(\omega_0) \sim \omega_t$ it decreases and from energies $\varepsilon_0(\omega_0) \sim (2 \div 3)\omega_t$ the behavior changes once more to the gradual increase. It is seen in Fig.2 that the closing of the corresponding curves stops at energies $(50 \div 60)\ GeV$ for W while for Ge additional simulation performed up to the energy $550\ GeV$ shows that the closing mentioned stops at energies $\sim 300\ GeV$. The behavior observed is inherent to any crystal with different energy scale depending on values of characteristic energies of emitted photons ω_c (ratio of these energies in W and Ge is $\sim 4 \div 5$) and on values of the threshold energies ω_t ($\omega_t^{Ge}/\omega_t^{W} \sim 3$).

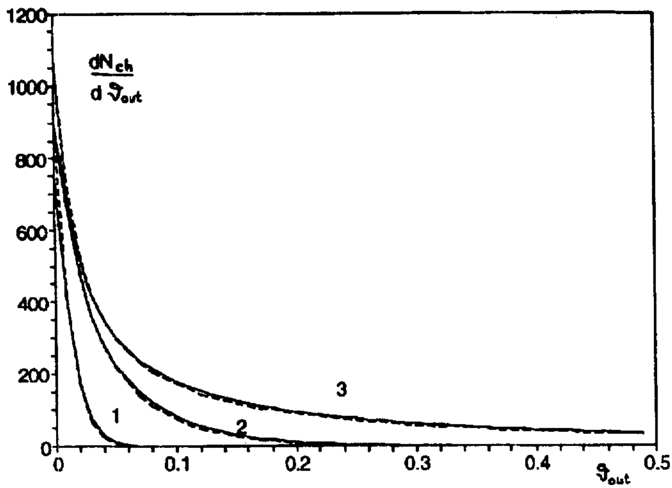

Fig. 3. The distribution of charged particles over the angle ϑ_{out} for the Ge (solid) and W (dashed) crystals of thickness $L/L_{rad} = 1$ at the initial energy $\varepsilon_0 = 150\ GeV$. Curves 1 are for particles with energies $\varepsilon > 500\ MeV$, curves 2 are for $\varepsilon > 100\ MeV$, curves 3 are for $\varepsilon > 5\ MeV$.

The angular distributions of the created charged particles in Ge and W crystals are presented in Fig.3, where ϑ_{out} is the angle of the outgoing particle momentum with respect to the axis. The harder are the particles the narrower is the angular distribution. This is evident though from the very beginning since multiple scattering becomes less effective and natural angular spread inherent to QED-processes $\vartheta \sim m/\varepsilon$ is narrowing with the increase of the energy. We have found also that when the initial energy increases (cf. Fig.3 in [7] for $\varepsilon_0 = 50GeV$ in 2.5 cm thick Ge crystal) the angular distribution narrows what is less pronounced for larger boundary energies ε_f. Let us stress out that in used collinear approximation the angular distribution is entirely due to the multiple scattering of arising charged particles. As it is seen in Fig.1(a) the total number of charged particles at $L/L_{rad} = 1$ for Ge and W crystal is almost the same. The corresponding curves in Fig.3 also almost coincide. The small difference between them means that charged particles in W are somewhat harder than in Ge.

The spectral distributions of charged particles in Ge and W crystals are shown in Fig.4(a) and (b) respectively for all the particles and those having outgoing angle $\vartheta_{out} \leq 10^{-2}$ at different energies of incident electrons and crystal thicknesses $L/L_{rad} = 2$. The most of the particles are created with relatively low energy and continue to lose it as far as moving within a crystal. As a result, the spectral distributions of charged particles in Fig. 4(a) (as well as of photons: see Fig.4(b) in [7]) have the maximum at the boundary energy ($\varepsilon_f = 5\ MeV$ in our case) and decrease fast while energy of outgoing particles increases. But owing to the mentioned angular-energy correlation the maximum in Fig.4(b) is shifted to higher energies. Note that for photons under the same conditions ($\vartheta_{out} \leq 10^{-2}$) the maximum retains the position at the boundary energy.

The standard set of kinetic equations (see, e.g. Eq.(2) in [7]) describe the cascade process in terms of the mean values giving averaged characteristics of the cascade, and do not provide any information about fluctuations in the stochastic process under consideration. These fluctuations were discussed for electromagnetic showers in amorphous media in many papers (see, e.g. [8], sect. 81). Monte-Carlo simulation procedure adequately describes all details of the cascade development but to obtain reliable averaged characteristics, sufficiently high statistics is needed. An important illustration of the scale of fluctuations

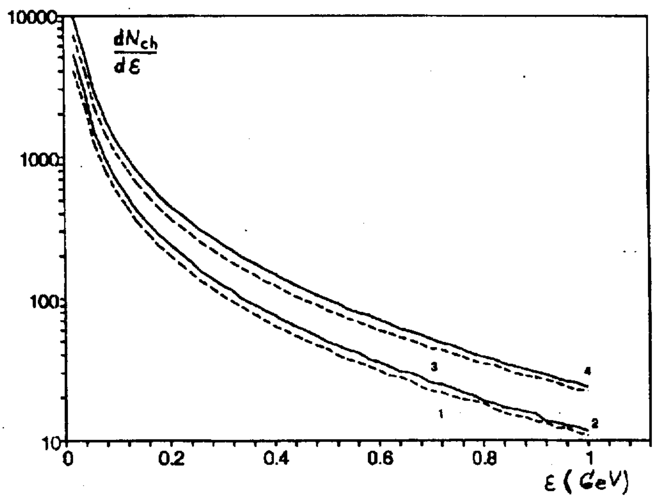

Fig. 4(a). The distribution over the energy ε of charged particles for the Ge (curves 1 and 3) and W (curves 2 and 4) crystals of thickness $L/L_{rad} = 2$. The energy of initial electrons $\varepsilon_0 = 150\ GeV$ (curves 1 and 2) and $\varepsilon_0 = 300\ GeV$ (curves 3 and 4).

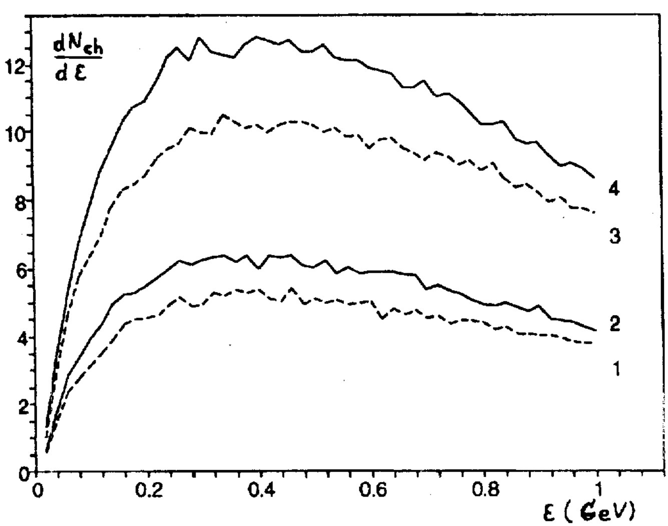

Fig. 4(b). The same as in Fig.4(a) but for $\vartheta_{out} \leq 10^{-2}$.

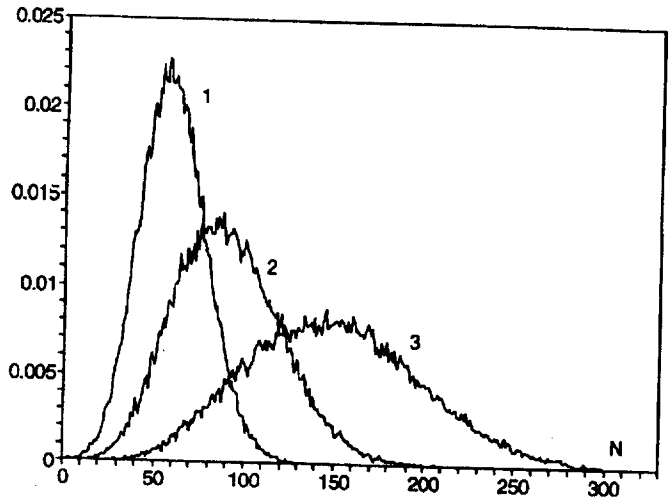

Fig. 5(a). The distribution over the number of charged particles created in the Ge crystal (axis < 110 >) of thickness $L/L_{rad} = 1$. Curve 1 is for the initial electron energy $\varepsilon_0 = 75\ GeV$, curve 2 is for $\varepsilon_0 = 150\ GeV$ and curve 3 is for $\varepsilon_0 = 300\ GeV$.

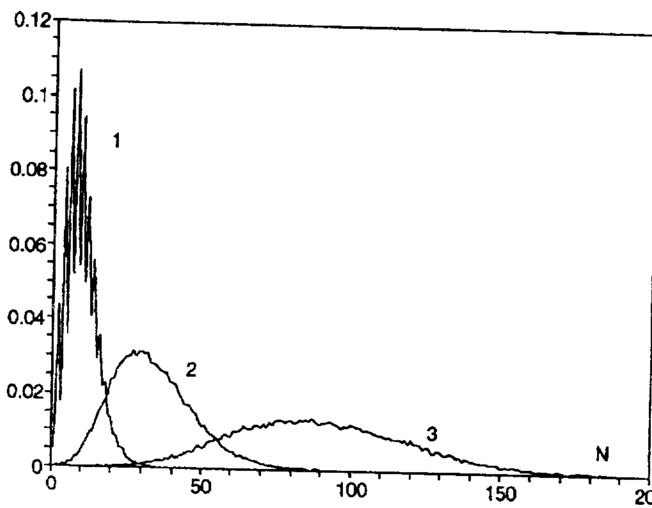

Fig. 5(b). The same for Ge crystal (axis < 110 >) but for the given electron energy $\varepsilon_0 = 150\ GeV$ at different thicknesses $L/L_{rad} = 0.25$, curve 1, $L/L_{rad} = 0.5$, curve 2, $L/L_{rad} = 1$, curve 3.

is given in Fig.5(a,b) where the distribution over the multiplicity of created charged particles is shown. All the curves are normalized to unity. Our calculations show that FWHH of obtained distributions is roughly proportional to the mean multiplicity. Let us stress out that from the solution of kinetic equations we have instead of each curve in Fig.5(a,b) one quantity only: the mean number of created charged particles. Just these averaged characteristics are given in Figs.1-4.

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