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HIGH ENERGY NEUTRINO-NUCLEON SCATTERING, CURRENT ALGEBRA AND PARTONS

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A B S T R A C T

The structure functions F_i which describe high energy neutrino-nucleon scattering are discussed. The analogue of the Callan-Gross result for electroproduction is derived, viz.

$$\lim_{q^2 \to -\infty} F_1 = \lim_{q^2 \to -\infty} F_3 = 0$$

with the algebra of field commutators and

$$\lim_{q^2 \to -\infty} (F_2 - \omega F_1) = 0$$

with the quark model commutators. A sum rule for F_3 is derived assuming that F_3 satisfies an unsubtracted dispersion relation, as suggested by Regge theory. The breakdown of these current algebra results in perturbation theory is discussed. The F_i are also discussed in the "parton" model, in which it is shown that F_3 is a measure of the average baryon number of the partons. In the "quark-parton" model of Bjorken and Paschos F_3 is small and our sum rule valid. In the "field theory parton" model of Drell, Levy and Yan F_3 is large, in contradiction to Regge theory, and our sum rule invalid. This model predicts that anti-neutrino scattering vanishes in the backward direction.

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1. INTRODUCTION

Recently there has been much interest in the study of inelastic lepton-hadron scattering at high energies and large momentum transfers. Stimulated by the SLAC experiments on high energy elastic electron-proton scattering [1], various models have been proposed to correlate and explain the data. These include current algebra [2)-4)], the "parton" model of a point-like structure within the nucleon [5)-7)], Regge poles [8),9)] and vector dominance [10)].

We discuss here high energy neutrino-nucleon scattering and in particular the vector-axial vector interference term (VAIT), which is especially model dependent.

In Section 2 we discuss the kinematics of neutrino-proton scattering. In Section 3 we show that the (VAIT) is excluded in the limit of large momentum transfer if one assumes the commutation relations of the algebra of fields. For the quark model we derive a sum rule, relating the space-space commutators of the weak current to the VAIT.

Section 4 is devoted to the "parton" model. We argue that the size of VAIT in this model is a measure of the average baryon number of the constituent partons. Therefore, a model which assumes a large cloud of quark-anti-quark pairs [6)] will lead to a small VAIT. This is compared with Regge theory and current algebra.

In Section 5 we examine a field-theoretic model which attempts to justify the parton model [7)]. In this model the contribution of anti-particles is suppressed dynamically and therefore the VAIT is large. This is in contradiction to the predictions of Regge theory, since the vacuum trajectory cannot contribute to the VAIT. This model also predicts that the cross-section for the scattering of anti-neutrinos off nuclei should vanish in the backward direction, and allows one to relate the structure functions of neutrino scattering to those of electron scattering. A comparison is made with recent CERN data.

2. KINEMATICS

The inelastic scattering cross-section of a neutrino from an unpolarized nucleon may be written as [2),11)]

$$\frac{\pi}{EE'}\frac{d\sigma^{(\nu,\bar\nu)}}{d\Omega\, dE'} = \frac{M\, d\sigma^{(\nu,\bar\nu)}}{dq^2\, d\nu} =$$

$$= \frac{E'}{E}\frac{G^2}{2\pi}\left[\cos^2\frac{\theta}{2} W_2^{(\nu,\bar\nu)} + 2\sin^2\frac{\theta}{2} W_1^{(\nu,\bar\nu)} \mp \frac{E+E'}{M}\sin^2\frac{\theta}{2} W_3^{(\nu,\bar\nu)}\right] \tag{1}$$

with E (E') being the energy of the incident (scattered) neutrino (muon or electron) in the lab. frame, θ the scattering angle of the lepton, $q^2 = -4EE'\sin^2\theta/2$ the momentum transfer squared to the nucleon, $\nu = q\cdot P = (E-E')M$ the energy transfer to the nucleon, and P (q) the momentum carried by the target nucleon (the weak current). The structure functions $W_i^{(\nu,\bar\nu)}$ are functions of q^2 and ν, and are given by:

$$\frac{1}{2\pi}\int d^4x\, e^{iq\cdot x}\langle \underset{\sim}{P}|\left[J_\mu^{\mp}(x), J_\nu^{\pm}(0)\right]|\underset{\sim}{P}\rangle = -\left(g_{\mu\nu} - q_\mu q_\nu/q^2\right)W_1^{\binom{\nu}{\bar\nu}}(q^2,\nu)$$

$$+\frac{1}{M^2}\left(P_\mu - \frac{\nu}{q^2}q_\mu\right)\left(P_\nu - \frac{\nu}{q^2}q_\nu\right)W_2^{\binom{\nu}{\bar\nu}}(q^2,\nu) - i\,\varepsilon_{\mu\nu\alpha\beta}\frac{P_\alpha q_\beta}{2M^2}W_3^{\binom{\nu}{\bar\nu}}(q^2,\nu) + \cdots \tag{2}$$

The terms deleted in (2) are proportional to $P_\mu q_\nu + q_\mu P_\nu$ and $q_\mu q_\nu$ and do not contribute to the cross-section when one neglects, as we do, the lepton masses. Our normalization is that the weak current is

$$J_\mu^{+} = (J_\mu^{-})^{\dagger} = \bar{p}'\gamma_\mu(1-\gamma_5)(n'\cos\theta_c + \lambda'\sin\theta_c)$$

and

$$\langle \underset{\sim}{P}|\underset{\sim}{P}'\rangle = (2\pi)^3\,\frac{E}{M}\,\delta^3(\underset{\sim}{P}-\underset{\sim}{P}').$$

All matrix elements will be averaged over nucleon spins.

It is convenient to use the dimensionless variables [2]:

$$x = \nu / ME \tag{3}$$

$$\omega = -q^2/\nu \tag{4}$$

and to define,

$$\begin{aligned} F_1^{(\nu/\bar\nu)}(\omega,q^2) &= M W_1^{(\nu/\bar\nu)}(\nu,q^2) \\ F_2^{(\nu/\bar\nu)}(\omega,q^2) &= \nu/M\, W_2^{(\nu/\bar\nu)}(\nu,q^2) \\ F_3^{(\nu/\bar\nu)}(\omega,q^2) &= \nu/M\, W_3^{(\nu/\bar\nu)}(\nu,q^2) \end{aligned} \tag{5}$$

In terms of these variables we have

$$\frac{d^2\sigma^{(\nu/\bar\nu)}}{dx\,d\omega} = \frac{G^2ME}{2\pi}\left[\left(1-x-\frac{Mx\omega}{4E}\right)F_2^{(\nu/\bar\nu)} + \frac{x^2}{2}\omega F_1^{(\nu/\bar\nu)} \mp \frac{x}{2}\left(1-\frac{x}{2}\right)\omega F_3^{(\nu/\bar\nu)}\right] \tag{6}$$

and for large E, $0 \leq x \leq 1$, $0 \leq \omega \leq 2$.

The expression (2) is a positive semi-definite form. The W_i's therefore satisfy the inequalities

$$0 \leq \frac{1}{2M^2}\sqrt{\nu^2 - M^2q^2}\,|W_3^{(\nu/\bar\nu)}| \leq W_1^{(\nu/\bar\nu)} \leq \left(1-\frac{\nu^2}{M^2q^2}\right)W_2^{(\nu/\bar\nu)}. \tag{7}$$

We are interested in the energy region where $q^2 \to -\infty$, and ω is kept fixed. In this limit Bjorken has argued that the functions $F_i(\omega, q^2)$ approach finite limits [2], $F_i^{(\nu,\bar\nu)}(\omega)$ ("scale invariance"). A sufficient condition that guarantees the existence of these finite limits is that

$$\lim_{q^2\to-\infty}(-q^2)\int_{-q^2/2}^{\infty} W_2^{(\nu/\bar\nu)}(q^2,\nu)\frac{d\nu}{\nu} < \infty.$$

The corresponding condition in the case of electron-proton scattering seems to be well satisfied [1], and the indications are that the same holds for neutrino-nucleon scattering [12].

We then have the inequalities

$$0 \leq \frac{1}{2} \left| F_3^{(\nu,\bar{\nu})}(\omega) \right| \leq F_1^{(\nu,\bar{\nu})}(\omega) \leq \frac{1}{\omega} F_2^{(\nu,\bar{\nu})}(\omega) , \tag{8}$$

which contain the positivity requirements on the transverse (F_1) and longitudinal $(F_2 - \omega F_1)$ cross-sections of the virtual weak current.

Finally we note that under crossing

$$W_i^{\nu}(q^2, \nu) = - W_i^{\bar{\nu}}(q^2, -\nu) . \tag{9}$$

3. CURRENT ALGEBRA SUM RULES

Sum rules for inelastic neutrino-nucleon scattering have been derived by Adler [11] and Bjorken [13]. There are sum rules for the combinations $W_i^{\nu}(q^2,\nu)-W_i^{\bar{\nu}}(q^2,\nu)$; i.e., they correspond to the isotopic spin odd part of the amplitude. However one can also derive sum rules for the crossing even combinations

$$W_i^{\bar{\nu}}(q^2,\nu)+W_i^{\nu}(q^2,\nu)\equiv 2W_i(q^2,\nu) \tag{10}$$

These are of particular interest, since in experiments that scatter neutrinos off heavy nuclei the average cross-section per nucleon is given in terms of the W_i's (if we neglect terms proportional to $\sin^2\theta_c$).

The sum rules for W_1 and W_2 are equivalent to those derived by Callan and Gross, Ref. 4), for electron scattering. They are derived by writing a fixed q^2 dispersion relation for the T product of the currents, and equating the large $|q_0|$ limit of this dispersion relation to that obtained from the Bjorken limit in terms of equal time commutators (ETCR). Thus, we derive, analogously to Ref. 4), that:

$$\int_0^2 d\omega\left[\left(F_2(\omega)-\omega F_1(\omega)\right)\hat{P}_i\hat{P}_j+\omega F_1(\omega)\left(\hat{P}_i\hat{P}_j-\delta_{ij}\right)\right]=C_{ij}(\underset{\sim}{P}), \tag{11}$$

$$C_{ij}(\underset{\sim}{P})=\lim_{|\underset{\sim}{P}|\to\infty}\frac{iM}{2|\underset{\sim}{P}|^2}\int d^3\underset{\sim}{x}\,\langle \underset{\sim}{P}|\left[\partial_0 J_i^+(\underset{\sim}{x},0),\,J_j^-(0)\right]|\underset{\sim}{P}\rangle_{AVE.}, \tag{12}$$

where the average is over neutron and proton states. The value of the equal time commutator in (12) is unknown. However, one can derive its tensorial structure given the structure of the currents. As in Ref. 4) one can easily show that if the weak current satisfies the ETCR of the algebra of fields, or is constructed from spin zero fields, then

$$C_{ij}(\underset{\sim}{P}) = C\delta_{ij}\,. \qquad \text{(algebra of fields, spin zero)} \tag{13}$$

Similarly if the weak current is bilinear in spin $\frac{1}{2}$ fields only (as in the quark model) then

$$C_{ij}(\underset{\sim}{P}) = C(\hat{P}_i\hat{P}_j - \delta_{ij})\,. \qquad \text{(quark model, spin } \tfrac{1}{2} \text{ constituents)} \tag{14}$$

Since $F_2 - \omega F_1$ and F_1 are positive definite, one immediately concludes that

$$\lim_{q^2\to-\infty} F_1(\omega,q^2) = \lim_{q^2\to-\infty} F_3(\omega,q^2) = 0 \qquad \text{(algebra of fields, spin zero constituents)} \tag{15}$$

$$\lim_{q^2\to-\infty} \left[F_2(\omega,q^2) - \omega F_1(\omega,q^2)\right] = 0 \qquad \text{(quark model, spin } \tfrac{1}{2} \text{ constituents)} \tag{16}$$

The vanishing of F_3 in the algebra of fields is guaranteed by the inequality (8).

One can also derive a sum rule for W_3 in terms of ETCR of the space components of the weak current with itself. Using the method outlined above one derives that

$$\int d^3\underset{\sim}{x} \, \langle \underset{\sim}{P} | [J_i^+(\underset{\sim}{x},0), J_j(0)] | \underset{\sim}{P} \rangle_{\text{AVE.}} = \lim_{q^2\to-\infty} \frac{i}{M} \varepsilon_{ijk} P_k \int_0^2 d\omega \, F_3(\omega,q^2), \tag{17}$$

where again the average is over proton and neutron states.

This commutator vanishes for the algebra of fields, consistent with (14). In the quark model the commutator in (17), averaged over an isotopic spin multiplet, is given by matrix elements of the hypercharge and baryon currents. The resulting sum rule is then

$$\int_0^2 d\omega\, F_3(\omega) = \frac{1}{2}\int_0^2 d\omega \left[F_3^{\nu}(\omega) + F_3^{\bar{\nu}}(\omega)\right] = 4B + Y(2 - 3\sin^2\theta_c). \tag{18}$$

The right-hand side of (18) is surprisingly large (≈ 6 in the case of nucleons), especially when compared to the experimental size of F_2^{EM}, measured in the SLAC experiments [1]. There one has

$$\int_0^2 d\omega\, F_2^{EM}(\omega) \simeq 0.32 \qquad \text{(experiment)}.$$

One should note that if the inequalities (8) are saturated then $F_3 \sim \frac{1}{\omega} F_2$. Furthermore, if $F_2 \rightarrow$ const. as $\omega \rightarrow 0$, as would be expected from either Pomeranchuk dominance, the analogy to F_2^{EM} or from the experiments themselves [14], then the integral in (18) is logarithmically divergent. This is due to the fact that if $F_3(\omega) \rightarrow \frac{1}{\omega}$ as $\omega \rightarrow 0$, then $W_3(q^2, \nu) \rightarrow$ const. as $\nu \rightarrow \infty$, and one can no longer derive a sum rule for W_3, since the derivation rests on the assumption of unsubtracted dispersion relations. If, however, W_3 vanishes for large ν, as Regge pole dominance suggests (W_3 has the quantum numbers of ω, ϕ in the t channel) then the sum rule should converge rapidly.

A serious objection has been raised to the use of the Bjorken limit to derive sum rules such as (11) and (17) by Adler and Tung [14], and by Jackiw and Preparata [15]. They show that the Bjorken limit breaks down, and the resulting sum rules are untrue in second order perturbation theory. However, there is no reason to assume that perturbation theory is relevant to the discussion of high energy behaviour. In particular, in second order perturbation theory all the limits, $\lim_{q^2 \rightarrow \infty} F_i(q^2, \omega)$ are infinite, due to logarithmic factors

$\log q^2/m^2$. Even when these are removed the sum rules given by (11) diverge. In the case of electron-nucleon scattering, this contradicts experiment. Thus, the real world seems to be less divergent than perturbation theory indicates.

Alternatively, one can say that the sum rules are certainly correct (and trivial) to lowest order in perturbation theory (no interaction). If, as is suggested by the "parton" model, leptons, at high energies and large momentum transfer, interact with hadrons as if the latter were bare particles, then the sum rules could be valid. This is certainly the case in the parton model for the relations (15) and (16).

4. THE PARTON MODEL

The basic idea of the model [5),6)] is that in a frame in which the proton has infinite momentum (to which the neutrino-proton centre of mass frame is a good approximation at high energies) the time of interaction may be short compared to the lifetime of the virtual states of the proton (which are slowed by time dilation). For large $(-q^2)$ the neutrino is supposed to scatter incoherently off the virtual constituents, or "partons", which are supposed to behave like free particles.

In electroproduction the longitudinal cross-section vanishes if partons have spin $\frac{1}{2}$ and the transverse cross-section vanishes if they have spin 0 [6)]. The latter case seems to be excluded experimentally [16),17)] so we will consider spin $\frac{1}{2}$ partons here, indicating finally how the results are modified for an admixture of spin 0 partons.

Assuming that the partons have small mass and negligible transverse momentum, compared to $\sqrt{-q^2}$, then the i^{th} parton has momentum $P^i_\mu \simeq x_i P_\mu$ where P_μ is the proton momentum and $\sum_i x_i = 1$. The i^{th} particle contributes

$$W_1^{\binom{\nu}{\bar\nu}} = \frac{2\nu}{M} \left| J_i^{\binom{\nu}{\bar\nu}} \right|^2 \delta(q^2 + 2x_i\, q\cdot P) ,$$

$$W_2^{\binom{\nu}{\bar\nu}} = 4 x_i M \left| J_i^{\binom{\nu}{\bar\nu}} \right|^2 \delta(q^2 + 2x_i\, q\cdot P) ,$$

$$W_3^{\binom{\nu}{\bar\nu}} = 4M \xi_i \left| J_i^{\binom{\nu}{\bar\nu}} \right|^2 \delta(q^2 + 2x_i\, q\cdot P) ,$$

$$\left| J_i^{\binom{\nu}{\bar\nu}} \right|^2 = \sum_j \left[|\langle j| I^\pm |i\rangle|^2 \cos^2\theta_c + |\langle j| U^\pm |i\rangle|^2 \sin^2\theta_c \right] , \tag{19}$$

$\xi_i = -$ (+) if the i^{th} particle is a baryon (anti-baryon), $I^{\pm}$ ($U^{\pm}$) are the I (U) spin raising (lowering) operators and we have assumed $g_A/g_V = -1$ for partons. Inserting these relations in Eq. (1) we see that anti-neutrino (neutrino) scattering off a baryon (anti-baryon) vanishes in the backward direction. This can be simply understood in terms of helicities since the $1-\gamma_5$ projects out definite helicity states. Thus angular momentum cannot be conserved when $\theta = \pi$ for anti-neutrinos (neutrinos) incident on baryons (anti-baryons), as indicated in Fig. 1. (We are indebted to Professor J.S. Bell for this observation.) Hence, if partons are all baryons the model is very simply tested.

Adding up the contributions from all partons gives:

$$W_1^{(\nu)} = \frac{2\nu}{M} \sum_N P_N \left\langle \sum_i |J_i|^2 \right\rangle_N \int_0^1 f_N(x)\, \delta(q^2 + 2\nu x)\, dx =$$

$$= \frac{1}{M} \sum_N P_N \left\langle \sum_i |J_i|^2 \right\rangle_N f_N(y),$$

$$W_2^{(\nu)} = 4M \sum_N P_N \left\langle \sum_i |J_i|^2 \right\rangle_N \int_0^1 x f_N(x)\, \delta(q^2 + 2\nu x)\, dx =$$

$$= \frac{2M}{\nu} \sum_N P_N \left\langle \sum_i |J_i|^2 \right\rangle_N y\, f_N(y),$$

$$W_3^{(\nu)} = -4M \sum_N P_N \left\langle \sum_i \xi_i |J_i|^2 \right\rangle_N \int_0^1 f_N(x)\, \delta(q^2 + 2\nu x)\, dx =$$

$$= -\frac{2M}{\nu} \sum_N P_N \left\langle \sum_i \xi_i |J_i|^2 \right\rangle_N f_N(y), \tag{20}$$

where

$P(N)$ = probability of finding a configuration of N partons;

$\langle\ \rangle_N$ indicates the average value for a configuration of N partons;

$f_N(y)$ = probability of finding a parton with a fraction y of the proton's longitudinal momentum;

$y = -q^2/2\nu = +\frac{1}{2}\omega$.

Similar relations hold for $W_{1,2}^{\text{electromagnetic}}$ with $2\langle\sum_i |J_i|^2\rangle_N$ replaced by $\langle\sum_i Q_i^2\rangle_N$ [6]. In that case it is known experimentally [1] that $\nu W_2 \to \text{const.}$ as $y \to 0$; hence it is clear that $\sum_N P_N \langle\sum_i Q_i^2\rangle_N$ must diverge and configurations with an infinite number of partons are required.

The value of $W_3/W_{1,2}$ compared to the value for a free baryon is clearly a measure of the ratio of the average effective baryon number to the average effective value of J^2. Since configurations with large N are required [and presumably contain $1+\frac{(N-1)}{2}$ baryons and $\frac{(N-1)}{2}$ anti-baryons] it might seem naïvely that this ratio would be small.

In Section 5 we discuss a model in which this is not the case (the anti-baryon contributions being dynamically suppressed). Here, however, it is instructive to consider the model discussed by Bjorken and Paschos [6] in which the naïve expectation is realized. The nucleon is envisaged as consisting of three quarks and a statistical distribution of quarks and anti-quarks with:

$$P_N = \frac{1}{(1-\ln 2)\,N(N-1)}$$

$$f_N(y) = (N-1)(1-y)^{N-2} . \tag{21}$$

$f_N(y)$ is derived by assuming that the joint distribution function of the x_i's for all the partons is a constant and P_N is a simple function with a form which ensures that $\nu W_2 \to$ const. as $y \to 0$. [This model gives a reasonable fit to the shape, but not the magnitude, of $F_2^{\text{electromagnetic}}(y)$]. We have

$$\langle \sum_i |J_i^{\nu}|^2 \rangle_p = \cos^2\theta_c + N/3 - 1 \;,\quad \langle \sum_i |J_i^{\bar\nu}|^2 \rangle_p = 1 + N/3$$

$$\langle \sum_i |J_i^{\nu}|^2 \rangle_n = 2\cos^2\theta_c + N/3 - 1 \;,\quad \langle \sum_i |J_i^{\bar\nu}|^2 \rangle_n = N/3$$

$$\langle \sum_i \xi_i |J_i^{\nu}|^2 \rangle_p = \cos^2\theta_c \qquad \langle \sum_i \xi_i |J_i^{\bar\nu}|^2 \rangle_p = 2$$

$$\langle \sum_i \xi_i |J_i^{\nu}|^2 \rangle_n = 2\cos^2\theta_c \qquad \langle \sum_i \xi_i |J_i^{\bar\nu}|^2 \rangle_n = 1 \tag{22}$$

Summing over N we obtain:

$$W_1^{(\nu)}(\text{proton}) = \frac{1}{M(1-\ln 2)}\left[\frac{1-y}{3(2y-y^2)} + \frac{(\cos^2\theta_c - 1)}{2(1-y)^2}\left(\ln\left(\frac{2-y}{y}\right) - 2(1-y)\right)\right] =$$

$$= \frac{\nu}{y} W_2^{(\nu)}(\text{proton}) \,,$$

$$\nu W_3^{(\nu)}(\text{proton}) = -\frac{M\cos^2\theta_c}{(1-\ln 2)(1-y)^2}\left\{\ln\left(\frac{2-y}{y}\right) - 2(1-y)\right\}, \ldots \text{etc.} \tag{23}$$

W_1 and W_2 have the behaviour expected from Pomeranchuk dominance as $y \to 0$; $\nu W_3 \sim \log y$ while Regge behaviour is $\nu W_3 \sim y^{-\alpha(0)}$ with $\alpha(0) \simeq 0.5$. The model is therefore in qualitative agreement with Regge behaviour and does not predict any striking difference between ν and $\bar{\nu}$ scattering; asymptotically $(y \to 0)$ ν and $\bar{\nu}$ scattering off neutrons or protons all become the same. In this model $W_3 \to 0$ $(y \to 0)$ and the sum rule of Section 3 should hold.

If one adds spin 0 partons then σ_L no longer vanishes. However, these partons do not contribute to either W_1 or W_3 whose ratio remains unchanged.

5. THE DRELL-LEVY-YAN MODEL

Drell, Levy and Yan (DLY) [7] have studied W_1 and W_2 for electroproduction in a canonical field theory of pions and nucleons with the additional assumption of a cut-off $(k_{\perp\,max})$ in transverse momentum. By analyzing all possible diagrams as ν and $q^2 \to \infty$ they "derive" the parton model. DLY argue further that if the limit is taken in such a way that $\omega \to 0$ then the time ordered ladder diagrams in which the current couples only to the proton dominate to each order in the strong coupling constant g.

DLY's arguments do not depend on the nature of the current; in the case of the weak current their equations give immediately

$$\nu W_2^{(\nu/\bar\nu)} = M^2 \omega W_1^{(\nu/\bar\nu)} = -\tfrac{1}{2}\omega\nu W_3^{(\nu/\bar\nu)} \approx \omega^{-\zeta+1}$$

$$\zeta = \frac{3}{4\pi}\,\frac{g^2}{4\pi}\,\ln\left[1+\frac{K_\perp^2}{M^2}\right]. \tag{24}$$

These results are supposed to be true for q^2 large enough for scale invariance to hold and $\nu \gg q^2$, so that they correspond to the following asymptotic behaviour in ν for large fixed q^2:

$$W_1 \sim \nu^{\zeta}$$

$$W_2 \sim \nu^{\zeta-2}$$

$$W_3 \sim \nu^{\zeta-1} \tag{25}$$

As DLY remark, with $\zeta = 1$ (corresponding to $k_{max} \simeq 500$ MeV) this behaviour is in agreement with the SLAC data for W_2 and corresponds to the usual assumption that W_1 and W_2 are dominated

Pomeranchuk exchange; furthermore $k_{\perp max}$ is in reasonable agreement with the cut-off observed in high energy diffraction scattering.

We notice, however, that in this case W_3 also behaves as if it were dominated by the Pomeranchuk - which cannot be the case as negative G parity is exchanged in the crossed channel.

It is therefore hard to accept the statement by DLY that they have derived the ladder model and that the asymptotic behaviour of their structure functions is dominated by Regge poles. The fact that $k_{\perp max}$ is consistent with strong interaction data may be fortuitous.

In this limit the model predicts that the nucleon interactions like a bare nucleon so that according to our helicity argument (or the explicit relations among the W's derived above, which also hold for a free nucleon and saturate the inequalities) $\bar{\nu}$ scattering will vanish when $\theta = \pi$. This is in striking contrast to models with Regge behaviour and provides a test of the model. In fact in this case, Eq. (6) gives

$$\frac{d\sigma^{(\nu)}}{d\omega\, dx} \underset{E\to\infty}{=} \frac{G^2 ME}{2\pi} F_2(\omega)$$

$$\frac{d\sigma^{\bar{\nu}}}{d\omega\, dx} \underset{E\to\infty}{=} \frac{G^2 ME}{2\pi} F_2(\omega)\,(1-x)^2. \tag{26}$$

In the CERN neutrino scattering experiments a relatively flat x distribution is observed [12], consistent with (26). Anti-neutrino scattering experiments at large energies have not yet been performed.

In order to relate W_i^{weak} to $W_i^{electromagnetic}$ it is more realistic to generalize the model to the whole octet of baryons and mesons (this clearly leaves the results above unchanged). The dominant contribution comes from the ladder graphs with n rungs as $n \to \infty$. We require the probability $P_i^n(k)$ of finding a baryon i after n interactions when the initial baryon state was k. If P_{ij} is the probability of a transition from a baryon i to a baryon j, then:

$$P_i^n(k) = \sum_j P_{ij} P_j^{n-1}(k)$$

$$P_{ij} = P_{ji}$$

$$\sum_j P_{ij} = 1$$

$$P_{ij} \geq 0 \tag{27}$$

As $n \to \infty$ a state of equilibrium is reached and

$$P_i^n(k) \Rightarrow P_i^{n-1}(k) \Rightarrow \lambda_i(k),$$

$$\sum_j P_{ij} \lambda_j(k) = \lambda_i(k).$$

Clearly a solution of this equation is that all the λ_i's are equal and provided P_{ij} cannot be brought into block diagonal form by reordering rows and columns it is easy to show that this is the only solution. Therefore as $n \to \infty$ the probability is the same for all the baryons and independent of the initial state, provided some sets of baryon do not decouple from others (as is the case), - which is, perhaps, obvious.

Adding up the contributions from all baryons, and taking into account the fact that the weak current $(\sim J^V J^V + J^A J^A)$ gains a factor of two compared to the electromagnetic current $(\sim J^V J^V)$, we find (independent of the Cabibbo angle)

$$W_1^{\text{WEAK}} = 3\, W_1^{\text{EM.}}$$

$$W_2^{\text{WEAK}} = 3\, W_2^{\text{EM.}}$$

The SLAC data give $W_2^{\text{e.m.}} \simeq 0.35$ while a fit to the CERN neutrino data [12] with the inequalities saturated gives $W_2^{\text{weak}} \simeq 0.5 \pm 0.2$. This is not really a stringent test of the model. It is easily altered (e.g.) by adding the contribution of decuplet states which give $W_1^{\text{weak}} = 2W_1^{\text{e.m.}}$ etc. Adding this with some dynamical weighting factor, gives $3W_1^{\text{e.m.}} \gtrsim W_1^{\text{weak}} \gtrsim 2W_1^{\text{e.m.}}$.

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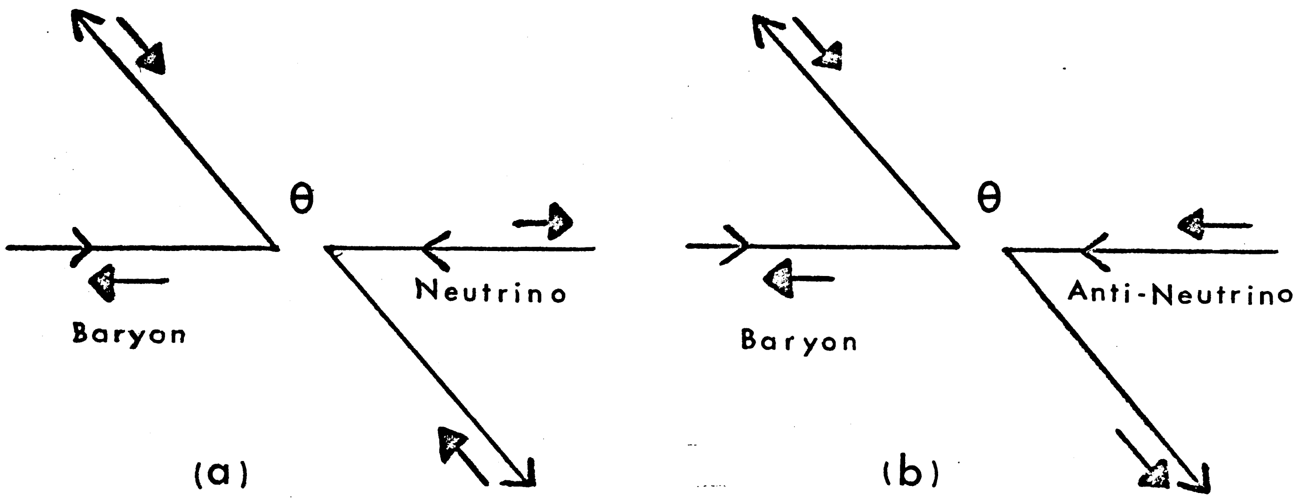

The arrows (→) indicate the direction of the helicities. In the case (b) angular momentum cannot be conserved in the backward direction, when $\theta = \pi$.