SYMMETRY BREAKING CORRECTIONS TO WEAK VECTOR CURRENTS

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ABSTRACT

A general method for calculating the renormalization of partially conserved vector currents due to symmetry breaking is discussed. As an application, the renormalization of the strangeness changing weak vector current is calculated in the pole approximation and its effect on universality is discussed.

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INTRODUCTION

Much attention has been devoted in the past few years to the question of universality of the weak interactions [1]. Until recently this was taken to mean that the unrenormalized coupling constants of the μ decay and the nuclear β decay interaction Hamiltonians were identical. The conserved vector current hypothesis [2] then allows us to state the equality of the two constants also in the presence of strong interactions.

Cabibbo [3], within the framework of SU(3), has proposed a slightly different version of universality, whereby the ratio of these two constants is equal to $\cos\theta$ rather than 1. Furthermore the vector strangeness changing coupling constant is in the ratio $\mathrm{tg}\theta$ to the vector strangeness conserving one. Therefore it is of critical importance for the verification of universality to have an accurate determination of the Cabibbo angle θ, in the presence of interactions which break SU(3) symmetry as the latter cause the strangeness changing vector currents to be no longer conserved.

A first estimate of the renormalization of the $|\Delta S| = 1$ coupling constant, or equivalently of the angle θ, was made by Sakurai [4] by comparing the decay rates of $K^* \to K\pi$ and $\rho \to \pi\pi$. He obtained for the renormalization ratio $(Z_2^{\frac{1}{2}}(K)Z_2^{\frac{1}{2}}(\pi))/(Z_1(K\pi))$ a deviation from unity (the unitary limit) of nearly 25%. It was later shown, on general symmetry grounds [5], that this renormalization was actually of second order in the symmetry breaking which leads one to believe that Sakurai's estimate was probably too large. The primary purpose of this paper is to give a direct evaluation of the renormalization effects for the coupling constant due to the violation of SU(3). A suitable technique for calculating these effects for partially conserved currents has recently been proposed [6].

In Section I we briefly recall the method of Ref. [6] in its most general formulation and apply it to the recently investigated SU(6) symmetry. In Section II we show how the contribution of two particle sates to the renormalization can be evaluated and briefly describe a method for studying the contribution of many particle intermediate states. Finally, in Section III, we give the numerical results of a model calculation of the renormalization of the $|\Delta S| = 1$ current for $K\pi$ transition and discuss how this affects universality.

I. It was shown in Ref. [6] that by an appropriate use of the commutators of the space integrated current densities, defining the algebra of a group, it was possible to study the deviation of the renormalized coupling constants from the bare ones, as a consequence of the presence of a symmetry breaking interaction.

Assuming the underlying symmetry group to be a (semi-simple) Lie group, the infinitesimal generators can be written, in Racah notation [7], as E_α and H_i, the latter being the mutually commuting ones. The integrated current densities can then be put in the form

$$\begin{aligned} Q_\alpha &= \int J_0^{(\alpha)} d\vec{x} = G_0^{(\alpha)} E_\alpha \\ Q_i &= \int J_0^{(i)} d\vec{x} = G_0^{(i)} H_i \end{aligned} \tag{1}$$

where we have explicitly introduced the unrenormalized coupling constants $G_0^{(\alpha)}$ which are present in the Hamiltonian (some of them can eventually be equal).

In a given representation, the physical states are simultaneous eigenstates of the H_i operators ; in the following it is convenient to consider matrix elements only between states corresponding to the highest weight M of the involved representation. We recall that the E_α 's have non-vanishing matrix elements only between $|M\rangle$ and $|M-\alpha\rangle$ states such that

$$\langle M | E_\alpha | M-\alpha \rangle = \sqrt{M_i \alpha^i} \tag{2}$$

α^i being the components of the root α . If the symmetry group is broken and the Q_α are no longer exactly conserved, a manifestation of this fact is represented by the introduction of renormalized quantities $G^{(\alpha)}(p)$ which are defined by

$$\langle M, \vec{p}' | Q_\alpha | M-\alpha, \vec{p} \rangle = \sqrt{M_i \alpha^i}\; G^{(\alpha)}(p)\; \delta(\vec{p}-\vec{p}') \tag{3}$$

The deviation of the $G^{(\alpha)}$'s from their unrenormalized value $G_0^{(\alpha)}$ is a measure of the action of the symmetry breaking interaction ; our aim is to express $(G_0^{(\alpha)}/G^{(\alpha)}(p))^2$ as a function of this symmetry breaking.

To this end we examine the commutator of "opposite charges"

$$\begin{aligned} [Q_\alpha , Q_{-\alpha}] &= G_0^{(\alpha)2}\, \alpha^i H_i \\ Q_{-\alpha} &= Q_\alpha^{+} \end{aligned} \tag{4}$$

and take the expectation value of this quantity between the physical state corresponding to the highest weight of the given representation

$$\langle M, p | [Q_\alpha , Q_{-\alpha}] | M, p' \rangle = G_0^{(\alpha)2}\, \alpha^i M_i\; \delta(\vec{p}-\vec{p}') \tag{5}$$

We now insert a complete set of physical states and obtain, using Eq. (3),

$$G_0^{(\alpha)^2}\delta(\vec{p}-\vec{p}\,') = G^{(\alpha)^2}(p)\,\delta(\vec{p}-\vec{p}\,') + \delta G_\alpha^2(p)\,\delta(\vec{p}-\vec{p}\,') \qquad (6)$$

where

$$\delta G_\alpha^2(p)\,\delta(\vec{p}-\vec{p}\,') = \frac{1}{\alpha^i M_i}\left\{\sum_{n\neq M-\alpha}\langle M,p'|Q_\alpha|n\rangle\langle n|Q_{-\alpha}|M,p\rangle \right. \qquad (7)$$

$$\left. - \sum_{n'}\langle M,p'|Q_{-\alpha}|n'\rangle\langle n'|Q_\alpha|M\,p\rangle\right\}$$

When the Q_α's are no longer conserved, they of course do not commute with the symmetry breaking part H_B of the Hamiltonaian, and we thus write

$$[Q_{\pm\alpha}, H] = [Q_{\pm\alpha}, H_B] = C_{\pm\alpha\,B}^{\pm\alpha}\,H_{\pm\alpha} \qquad (8)$$

where

$$H_{\pm\alpha} = i\int D_{\pm\alpha}\,d\vec{x} \qquad (9)$$

and $C_{\alpha\,B}^{\alpha}$ is a structure constant depending on the transformation properties of H_B under the considered symmetry group. This then allows us to express δG_α^2 as

$$\delta G_\alpha^2 = (2\pi)^6 \sum_{n\neq M-\alpha}(C_{\alpha\,B}^{\alpha})^2\,G_0^{(\alpha)^2}\frac{\langle M|D_\alpha|n\rangle\langle n|D_{-\alpha}|M\rangle}{(E_M-E_n)^2}\,\delta(\vec{p}_n-\vec{p})$$

$$-\ \text{crossed term} \qquad (10)$$

As explained in Ref. [6], Eq. (6) gives a continuous set of sum rules corresponding to the different values of $|\vec{p}|$, the best of which, i.e., the one for which the renormalization is smallest, is the obtained as $|\vec{p}| \to \infty$. This limit corresponds to having the momentum transfer equal to zero and therefore $G(\infty)$ is what one usually refers to as "the renormalized coupling constant".

We shall now apply these methods to an examination of the symmetry group SU(6) which has recently attracted much attention [8]. In the limit of exact symmetry, there are 27 additional constants of the motion [9] other than the usual

$$Q^{(k)} = G_0^{(k)} \int \bar{B} \gamma_0 F^k B \, d\vec{x} \tag{11}$$

where we only write down the baryon contribution. These other 27 conserved charges are

$$\begin{aligned} Q_\ell &= G_{0\ell} \int \bar{B} \gamma_\ell \gamma_5 B \, d\vec{x} \\ Q_\ell^{(k)} &= G_{0\ell}^{(k)} \int \bar{B} \gamma_\ell \gamma_5 F^k B \, d\vec{x} \end{aligned} \tag{12}$$

where the F^k $(k = 1....8)$ are the usual SU(3) matrices and the fields are assumed to be the unrenormalized ones. Let us treat, as an illustration, the simple case in which

$$Q_\ell^{(k)} \to Q_3^{(+)} = G_0^A \int \bar{N} \tau^+ \gamma_3 \gamma_5 N \, d\vec{x} \tag{13}$$

and the renormalized quantity is taken, e.g., between states of spin-up

$$\langle P, p', \uparrow | Q_3^+ | N, p, \uparrow \rangle = G^A(p) \, \delta(\vec{p} - \vec{p}\,') \tag{14}$$

Then, following our previous considerations, we take the expectation value of $[Q_3^{(+)}, Q_3^{(-)}]_- = G_0^{A^2} Q_3^{(o)}$ between physical spin-up proton states, introduce a complete set of intermediate states, and selecting neutron spin-up states, we obtain

$$G_0^{A^2}\delta(\vec{p}-\vec{p}\,') = G^{A^2}\delta(\vec{p}-\vec{p}\,') + \delta G^{A^2}\delta(\vec{p}-\vec{p}\,') \tag{15}$$

with

$$\delta G^{A^2}(p) = (2\pi)^6 \sum_{\alpha \neq N(1)} G_0^{A^2} \frac{\langle P,\uparrow | D_3^{(+)} | \alpha \rangle \langle \alpha | D_3^{(-)} | P,\uparrow \rangle \, \delta(\vec{p}_\alpha - \vec{p})}{(E_P - E_\alpha)^2} - \text{crossed term} \tag{16}$$

Writing the weak axial current as being of the form

$$\bar{N}\,(F^i + a^i D^i)\,\sigma_k\, N$$

in the state limit, where the D^i now represent the other eight-dimensional representation of SU(3) we can state that the F part of the current is not renormalized to first order in the breaking of SU(6).

II. We now wish to turn to the question of calculating the renormalization $\delta G_\alpha^2(p)/G_0(\alpha)^2$, as given in Eq. (10), which is, as we have previously emphasized, of second order in the symmetry breaking. Let us begin by focusing our attention on the contribution of two particle intermediate states. This can be written as

$$\delta G_\alpha^2(p) = \int F \frac{d^4p_1\, d^4p_2\, \delta(p_1^2 - m_1^2)\,\delta(p_2^2 - m_2^2)\,\theta(p_{1_0})\,\theta(p_{2_0})}{2\,\mathcal{E}\,(\mathcal{E}_1 + \mathcal{E}_2 - \mathcal{E})^2} \tag{17}$$

where

$$\mathcal{E}_i = \sqrt{|\vec{p}_i|^2 + m_i^2}$$

and F is a Lorentz invariant function. Introducing new variables

$$P = p_1 + p_2 \qquad q = p_1 - p_2$$

and labelling $P^2 = s$ and $(P-p)^2 = \Delta^2$, we arrive at the following expression for (17)

$$\delta G_\alpha^2 (p) = \frac{1}{4\mathcal{E}} \int_{(m_1+m_2)^2}^{\infty} \frac{ds}{\sqrt{|\vec{p}|^2 + s}} \frac{I(\Delta^2, s)}{\left(\sqrt{|\vec{p}|^2+s} - \sqrt{|\vec{p}|^2 + m^2}\right)^2} \quad (18)$$

$$I(\Delta^2, s) = \int d^4q \, \delta\left((P+q)^2 - 4m_1^2\right) \delta\left((P-q)^2 - 4m_2^2\right)$$
$$\theta(P_0 - q_0)\, \theta(P_0 + q_0) \; F \quad (19)$$

F being a function of s, Δ^2 and $p \cdot q$.

As $|\vec{p}| \to \infty$, $\Delta^2 = \Delta_0^2 \to 0$ and the best sum rule is then distinguished by

$$\lim_{|\vec{p}| \to \infty} I(s, \Delta^2) = I(s, 0)$$

Making use of the following integral relation - p is the four momentum of the particle of mass m -

$$\int ds = \frac{2m^2}{\pi} \int \frac{\delta\left[(P-p)^2\right]}{P^2 - m^2} d^4P$$

(17) can be recast in the form

$$\delta G_\alpha^2(\infty) = \frac{2m^2}{\pi} \int \frac{d^4p_1\, d^4p_2}{\left[(p_1+p_2)^2-m^2\right]^3}\, \delta(p_1^2-m_1^2)\, \delta(p_2^2-m_2^2) \tag{20}$$

$$\theta(p_{10})\theta(p_{20})\,\delta\left[(p_1+p_2-p)^2\right] F\left[(p_1+p_2)^2, (p_1+p_2-p)^2, p\cdot(p_1-p_2)\right]$$

which can readily be generalized to many particle intermediate states. The problem then is to determine the invariant function F which, aside from some kinematical factors, is given by

$$\langle p|D_\alpha|p_1p_2\rangle\langle p_1p_2|D_{-\alpha}|p\rangle - \langle p|D_{-\alpha}|p_1p_2\rangle\langle p_1p_2|D_\alpha|p\rangle \tag{21}$$

where the intermediate states $|p_1p_2\rangle$ are physical ones, that is eigenstates of the total Hamiltonian H.

The most elegant treatment would probably be a dispersion theoretic one, eventually for the matrix element of $J_{o\alpha}(0)$, related to D_α by

$$\begin{aligned}\langle a|D_\alpha(0)|b\rangle &= -\frac{i}{G_o^\alpha}\langle a|\left[J_{o\alpha}(0), H\right]|b\rangle \\ &= -\frac{i}{G_o^\alpha}(E_b-E_a)\langle a|J_{o\alpha}(0)|b\rangle\end{aligned} \tag{22}$$

keeping only pole terms as a first approximation and making use of known data whenever possible.

What we have chosen to do however is a relatively simple perturbation theory calculation, evaluating the Feynman diagrams given in the figure and taking the effective symmetry breaking Hamiltonian as proportional to δm^2 *) where δm^2 is given by the mass formula [10]

$$m_i^2 = m^2 + \delta m^2 C_{ii}^8 \tag{23}$$

and then making use of the Wigner-Eckart theorem, which is consistent with a purely second order in the symmetry breaking calculation.

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*) In fact, in the limit $|\vec{p}| \to \infty$, $\Delta E \sim \delta m^2$.

III. Let us apply our considerations to the specific problem of obtaining an evaluation of the renormalization of the strangeness changing vector current due to the breaking of SU(3) symmetry, focusing our attention on K-π transitions. In this particular case, our sum rule reads

$$\frac{\left(G_0^s\right)^2}{G_{K\pi}^2} = \frac{Z_1^2(K\pi)}{Z_2(K)\,Z_2(\pi)} = 1 + \frac{\delta G_{K\pi}^2(\infty)}{G_{K\pi}^2} \tag{24}$$

where G_0^s is the bare coupling constant which is in the fixed ratio tgθ to the bare coupling constant of β decay. In the expression corresponding to (10) for $\delta G_{K\pi}^2$, if we take for example as initial state $|m\rangle = |\pi^+\rangle$ we thus see that the intermediate states in the direct term must have charge zero and strangeness minus one while those in the crossed term must have charge two and strangeness plus one. Taking only into consideration the two particle intermediate states, with the lowest mass that are allowed, i.e., one pseudoscalar meson and one vector meson, we shall limit our calculation to estimating this contribution, neglecting other states, e.g., two vector meson or nucleon antinucleon states.

The contribution to the invariant function F of the graphs of the figure where now, p (e.g., π^+) and p_1 are pseudoscalar mesons, and p_2 a vector meson is

$$\frac{\left(G_0^s\right)^2}{(2\pi)^3}\, g^2 \sum_{r} \left[A_1 \frac{2\,\varepsilon_\mu^{(r)} p_1^\mu}{(p_1+p_2)^2-m^2} + A_2 \frac{2\,\varepsilon_\mu^{(r)} p^\mu}{(p-p_2)^2-m^2} + \frac{A_3\,\varepsilon_\mu^{(r)} (p+p_1)^\mu}{(p-p_1)^2-M^2}\right]^2 \tag{25}$$

where m and M are the pseudoscalar and vector meson masses respectively, g a common ps-ps-v coupling constant, $\varepsilon_\mu^{(r)}$ the vector meson's polarization, A_1, A_2, products of appropriate Clebsch-Gordan coefficients times δm^2 and A_3 a product of Clebsch-Gordan coefficients, times δM^2 †). In our calculations we have used the following values *) of our parameters [11]

$$m^2 = (410\ \mathrm{Mev})^2$$

$$M^2 = (848\ \mathrm{Mev})^2$$

$$(\dagger\dagger)\quad \delta m^2 = 12.74\ 10^4\ \mathrm{Mev}^2 \qquad (26)$$

$$\delta M^2 = 12.78\ 10^4\ \mathrm{Mev}^2$$

$$(**)\quad \frac{g^2}{4\pi} \simeq 0.7$$

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†) The square of the term in which one breaks the symmetry on the vector line is logarithmically divergent. We have evaluated it by introducing a cut-off at $s \approx 3(\text{baryon masses})^2$. Moreover this contribution is multiplied by a rather small Clebsch-Gordan coefficient and in any case far from being the leading term in δG_α^2.

*) We must also add that our results are not very sensitive to the masses introduced in the calculations of the graphs of the figure, either physical ones or mean values for mass δM, as has been verified in some particular case. The correction δG_α^2 might be changed by an amount of a few percent.

††) This corresponds to Clebsch-Gordan coefficients such that

$$m_\pi^2 = m^2 - \frac{2}{\sqrt{3}}\delta m^2$$

$$m_K^2 = m^2 + \frac{1}{\sqrt{3}}\delta m^2$$

$$m_\eta^2 = m^2 + \frac{2}{\sqrt{3}}\delta m^2$$

$$\delta m^2 = \sqrt{3}\,\frac{(m_\eta^2 + 2m_K^2 - 3m_\pi^2)}{10} \qquad \delta M^2 = \frac{M_{K^*}^2 - M_\rho^2}{\sqrt{3}}$$

**) This is a mean value calculated from the widths of K^* and ρ decaying according to the Hamiltonians

$$ig\, K_-^{*\mu}(\pi_0 \partial_\mu K_+ - K_0 \partial_\mu \pi_+$$

$$2ig\, \rho_-^\mu (\pi_0 \partial_\mu \pi_+ - \pi_+ \partial_\mu \pi_0)$$

With these values we finally obtain

$$\frac{\left(Z_{1\,K\pi}\right)^2}{Z_{2K}\,Z_{2\pi}} = 1.067 \tag{27}$$

which is slightly greater than one as opposed to Sakurai's value [4] of 0.66. Taking our value for the renormalization and the best present experimental value for θ_V, which is 0.218±0.015 rad. [12], we find

$$\begin{aligned} \sin\theta &= (\sin\theta)_{exp}\sqrt{1.067} = 0.223 \\ \cos\theta &= 0.975 \pm 0.003 \end{aligned} \tag{28}$$

which finally implies that the comparison between the muon weak charge and the vector β decay weak charge is

$$\begin{aligned} G_\mu &= (1.0225 \pm 0.0030) \times (0.975 \pm 0.003) \\ &= (0.997 \pm 0.006)\, G_\beta \end{aligned} \tag{29}$$

Our evaluation has of course been a rough one, containing several assumptions, the main one probably being to neglect all intermediate states other than those containing one pseudoscalar meson and one vector meson. The value of $G_\mu = G_\beta$, namely universality, lies within the range of the experimental errors. We might add in conclusion that the effect of the renormalization calculated by us is to make the agreement with universality slightly worse as it changes $\cos\theta$ by -0.001.

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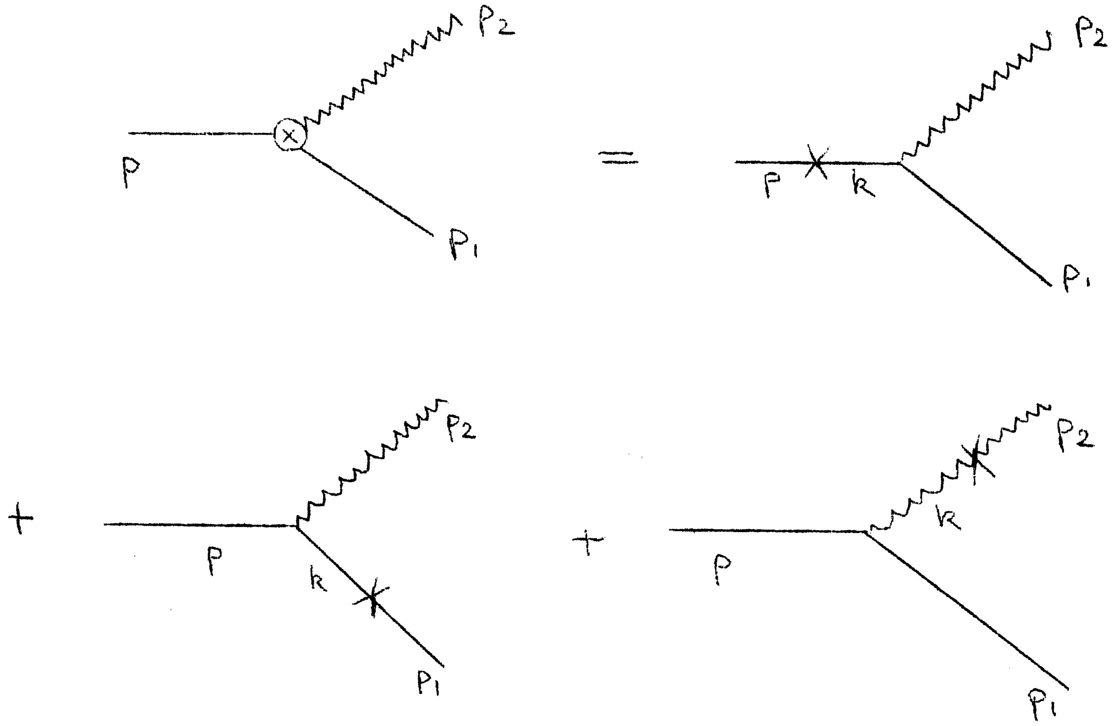

FIGURE CAPTION

The cross indicates the breaking of the SU_3 symmetry.