NOTE ON HIGH-ENERGY PROTON-PROTON SCATTERING

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Cocconi et al. [1] have measured at Brookhaven the scattering of protons by protons up to very high momentum transfer, $-t = 24\ (\mathrm{GeV/c})^2$. Orear [2] has pointed out that the cross-section at fairly large angles ($\gtrsim 30^\circ$) can be represented rather well by a universal function of the transverse momentum of the scattered proton, $p_\perp$, thus :

$$\frac{d\sigma}{d\omega} = 10^{-25.47}\, e^{-p_\perp/0.151} \qquad (1)$$

where $d\sigma/d\omega$ is in $\mathrm{cm}^2/\mathrm{ster}$, and $p_\perp$ in GeV/c. The diffraction peak at small angles can be represented by

$$\frac{d\sigma}{d\omega} = \left(\frac{k\sigma_{tot}}{4\pi}\right)^2 e^{9t} \qquad (\theta < 20^\circ) \qquad (2)$$

where $-t = 2p^2(1-\cos\theta)$ is measured in $(\mathrm{GeV/c})^2$. From 65° to 90° the cross-section does not change much with angle but depends strongly on energy; Cocconi [3] finds that this can be written as

$$\frac{d\sigma}{d\omega} = 10^{-24.60}\, e^{-3.38w} \qquad (3)$$

where w is the centre-of-mass energy in GeV. He points out that this cross-section is approximately inversely proportional to the number of final states accessible at energy w, as calculated by Fast and Hagedorn [4],

$$N = e^{3.17(w-1.80)} \; . \qquad (4)$$

Then the large-angle processes may be interpreted as formation of a compound state, with a cross-section of about 1 mb, plus subsequent statistical disintegration of the compound state into any of the N available final states with equal probability. If (4) is an overestimate, as it well may be, but if the energy dependence of (4) is still approximately correct, the interpretation could still be valid, but with a smaller cross-section for forming the compound nucleus.

If this statistical interpretation of the large angle scattering is accepted, it is tempting to interpret the scattering at smaller angles as a sum of a statistical and a diffraction contribution, and to assume that the latter (apart from exchange effects) depends just on t. In fact, Minami [5] has made a specific assumption on the t dependence, viz. that the diffraction part of the scattered amplitude is the sum of two exponentials in t which interfere constructively, and that to the diffraction cross-section there is added a statistical contribution depending only on s, thus :

$$\frac{d\sigma}{d\omega} = (A\,e^{\alpha t} + B\,e^{\beta t})^2 + C\,e^{-\gamma w} . \qquad (5)$$

We wish to point out that Minami's assumption is not tenable in view of the experimental data, and that in fact the cross-section cannot be written in the more general form

$$\frac{d\sigma}{d\omega} = f^2(t) + g(s) . \qquad (6)$$

Before comparing this expression with experimental data, we want to show that there is no appreciable contribution from exchange. If $f(t)$ is the amplitude for direct scattering with four-momentum change t, then $f(u)$ will be the amplitude for exchange scattering. The maximum possible interference contribution to $d\sigma/d\omega$ is then

$$\mathrm{Int} = 2\,|f(t)\,f(u)| . \qquad (7)$$

Now to give the hypothesis (6) the most favourable treatment, we shall assume that $f^2(t)$ is, for moderate scattering angles (say 30–50°), as close as possible to expression (1) which is known to represent the data very well for $\theta > 30°$. Now for moderate θ,

$$p_{\perp} = p \sin\theta \approx 2p \sin\tfrac{1}{2}\theta = \sqrt{-t} \qquad (8)$$

so that the best approximation for $f(t)$ is (in this region)

$$f(t) = 10^{-12.73} \exp(-3.3(-t)^{\frac{1}{2}}) . \tag{9}$$

Now we assume that (9) is also valid at sufficiently large argument to obtain the exchange contribution $f(u)$. We therefore set, using (7)

$$\begin{aligned} \text{Int} &= 2 \cdot 10^{-25.47} \exp(-3.3[(-t)^{\frac{1}{2}}+(-u)^{\frac{1}{2}}]) \\ &= 10^{-25.17} \exp(-6.6p\ (\sin\tfrac{1}{2}\theta + \cos\tfrac{1}{2}\theta)) . \end{aligned} \tag{10}$$

Now for small θ, say $\theta < 45^o$, Int is obviously negligible compared with $f^2(t)$. But for $\theta > 45^o$, we have $\sin\frac{1}{2}\theta + \cos\frac{1}{2}\theta > 1.305$ and therefore

$$\text{Int} < 10^{-25.17}\ e^{-8.64\,p} \approx 10^{-25.17}\ e^{-4.2\,w} \tag{11}$$

using Cocconi's approximation $w = 2.1p$ which is valid on the average. It is seen that (11) is negligible compared with (3). This means that <u>if</u> the direct diffraction amplitude depends only on t (and not also on s), <u>then</u> the interference term is always negligible compared to the statistical term at large angles ($\theta > 45^o$). Thus we can use (6) without an added interference term.

The experimental data [1] are known to show, for any given t, a strong decrease with increasing s. To fit these as well as possible by a formula of type (6), we should use the largest possible $g(s)$ which is just Cocconi's "statistical" expression (3), derived from 90^o scattering. In Table 1 we have used the data for $-t = 6.0$ and 10.0, the two t values for which the largest number of observations (4) exist. We give the observed $d\sigma/d\omega$ in cm^2/ster, the statistical contribution $g(s)$ calculated from (3), and the difference. This difference should be constant, $f^2(t)$, if (6) were valid. It is obvious that this is not the case. For $-t = 6.0$, even after subtracting the "statistical" contribution, the cross-section decreases by a factor of 4 as w increases. Indeed, the correction

$g(s)$ is negligible already for the second line, $p_0 = 15.7$, so that $f^2(t)$ should be very nearly equal to the directly observed cross-section. That the latter decreases strongly with increasing energy is evident [1]. The decrease is far outside the experimental error of about 25%. For $-t = 10.0$, the first line is zero, simply because this line refers to 90° scattering and was therefore used to deduce the statistical $g(s)$. To obtain a smooth behaviour of the difference, less should have been subtracted; i.e., only part of the 90° cross-section should be attributed to the statistical process. This, of course, would make the energy dependence of the "difference" even stronger. If the first line of $-t = 10.0$ is disregarded, the other lines still show strong energy dependence, and at least for the last two entries ($p_0 = 20.9$ and 28.7) this conclusion is well outside the experimental error. We thus conclude that (6) is not valid.

If we examine Minami's own analysis which is only at one energy ($p_0 = 16.7$ GeV/c) we find that his fit is also not very good. His theoretical curve (Case II) is about a factor 1.8 too low around $60-70^\circ$, and nearly by the same factor too high around 40°, both outside experimental error. If the 40° region had been used for a better adjustment of the parameters, the discrepancy around $60-70^\circ$ would have been even larger, at least a factor of 2. We have made the same observation at other energies : if the parameters in $f(t)$ are adjusted to fit small and medium angle data, and $g(s)$ to fit the 90° data, then the theory gives too low cross-sections around 60°. This is because the theoretical $f(t)$ falls too fast with Θ so that essentially only $g(s)$ is left at 60°; but the observed cross-section at 60° is substantially larger than that at 90° from which $g(s)$ is deduced - about a factor of 6 according to the accumulated data curve of reference [3], Fig. 2.

We must therefore accept the fact that the cross-section is not just a function of t alone but depends also on s, even if a statistical contribution has first been subtracted. Eq. (1) is a possible form, noting that

$$p_\perp = p \sin\Theta = (-t)^{\frac{1}{2}} \left(1 + \frac{t}{s-4m^2}\right)^{\frac{1}{2}} \qquad (12)$$

in which $4m^2$ may be neglected. The formula may of course be modified to be linear for small t (which were not used in deriving (1)), e.g., by substituting

$$p_{\perp} \rightarrow (t_1-t)^{\frac{1}{2}} (1+t/s)^{\frac{1}{2}} \tag{13}$$

where t_1 can still be chosen to improve the fit. However, the complete curve cannot be fitted by any choice of t_1 since the coefficient in (1), $10^{-25.47}$, is too small to fit the forward cross-section, and the second exponential in (1) can only decrease the cross-section.

A behaviour, for large t, as

$$d\sigma/d\omega \sim \exp(-\alpha(-t)^{\frac{1}{2}}) \tag{14}$$

rather than as

$$\exp \beta t \tag{15}$$

is reasonable. As Kinoshita [6] has shown, a formula like (14) should be expected if the scattering amplitude for large real s is sufficiently bounded in the complex plane of $\cos\theta$ (i.e., of t).

Since the cross-section now depends on s for a given t, we may obtain a Regge type representation. For $-t \ll s$, (13) and (1) give

$$\ln(d\sigma/d\omega)_{exp} = A(t)+3.3(-t)^{\frac{3}{2}} s^{-1} \quad . \tag{16}$$

In the Regge theory

$$\ln(d\sigma/d\omega)_{Reg} = \beta(t) + [\alpha(t)-1] \ln s \ . \tag{17}$$

The behaviour is not identical but only a limited range of s was used in the experiments, and only the dependence on s matters. Equating the derivatives of (16) and (17) with respect to s, and using the definitions for s and t, we get

$$\alpha(t) = 1 - \frac{3.3}{2}(1-\cos\theta)(-t)^{\frac{1}{2}} \quad . \tag{18}$$

In Table 1, we have used data around 50° for $-t = 6$, and around 60° for $-t = 10$. Inserting these values

$$\alpha = -0.5 \quad \text{for} \quad -t = 6$$

$$\alpha = -1.6 \quad \text{for} \quad -t = 10 \quad .$$

Not much meaning can be given to these results, especially because α for the leading trajectory may not be able to become < -1.

We have shown that the assumption of a statistical contribution to the cross-section does not significantly simplify the remainder of the cross-section, in that this remainder still depends on s as well as t. The question then is whether there is any evidence for the statistical part. The $d\sigma/d\omega$ is about constant from 70° to 90° [reference [3], Fig. 2] which means that the 90° cross-section is about three times higher than expected from the smooth function (1). Part of this may still be due to some constructive interference effect but probably not all. Thus there is some, rather slight, evidence for another phenomenon near 90°. It would be interesting to see whether this is really statistical. If so, then excited states of the nucleon should be formed with a probability proportional to their statistical weight. It should be possible to observe the energy spectrum of the nucleons emitted at a given, large angle θ_{lab}; this should indicate the formation of excited states in the recoil nucleon. The continuous background in the energy spectrum, arising from the disintegration of excited states emitted in the direction θ_{lab}, should not be too disturbing, at least for the first 1 GeV down from the energy of the elastically scattered nucleon.

R E F E R E N C E S

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$-t$	p_o	Θ_{cm}	w	$d\sigma/d\omega$	$g(s)$	diff.
$(GeV/c)^2$	GeV/c	degree	GeV/c	obs.	calc.	
6.0	11.1	68.3	4.77	$4.0 \cdot 10^{-32}$	$2.5 \cdot 10^{-32}$	$1.5 \cdot 10^{-32}$
6.0	15.7	55.4	5.60	$1.29 \cdot 10^{-32}$	$1.6 \cdot 10^{-33}$	$1.13 \cdot 10^{-32}$
6.0	21.7	46.2	6.53	$6.0 \cdot 10^{-33}$	$6.8 \cdot 10^{-35}$	$5.9 \cdot 10^{-33}$
6.0	31.5	37.7	7.83	$3.5 \cdot 10^{-33}$	$0.9 \cdot 10^{-36}$	$3.5 \cdot 10^{-33}$
9.9	11.4	90.0	4.82	$2.2 \cdot 10^{-32}$	$2.2 \cdot 10^{-32}$	0
10.0	14.2	78.5	5.34	$5.1 \cdot 10^{-33}$	$3.9 \cdot 10^{-33}$	$1.2 \cdot 10^{-33}$
10.0	20.9	62.1	6.41	$4.8 \cdot 10^{-34}$	$1.0 \cdot 10^{-34}$	$3.8 \cdot 10^{-34}$
10.0	28.7	52.0	7.47	$1.47 \cdot 10^{-34}$	$2.8 \cdot 10^{-36}$	$1.4 \cdot 10^{-34}$

TABLE 1

Cross-sections

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