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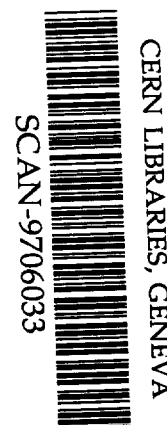
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**High-Frequency Electric  
Field Measurement Using  
a Toroidal Antenna**

Ki Ha Lee  
**Earth Sciences Division**

January 1997



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## **High-Frequency Electric Field Measurement Using a Toroidal Antenna**

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# High-Frequency Electric Field Measurement Using a Toroidal Antenna

Ki Ha Lee  
Earth Sciences Division

## Introduction

In this paper I describe an innovative method of measuring high-frequency electric fields using a toroid. For typical geophysical applications the new sensor will detect electric fields for a wide range of spectrum starting from 1.0 MHz. This window, in particular the lower frequency range between 1.0 to 100 MHz, has not been used for existing electromagnetic or radar systems to detect small objects in the upper few meters of the ground. Ground penetrating radar (GPR) can be used successfully in this depth range if the ground is resistive but most soils are, in fact, conductive (0.01 to 1.0 S/m) rendering GPR inefficient. For example, in a soil of 0.2 S/m the maximum range for a typical GPR is only 17 cm. Other factors controlling the resolution of GPR system for small objects is the spatial averaging inherent in the electric dipole antenna and the scattering caused by soil inhomogeneities of dimensions comparable to the wavelength (and antenna size). For maximum resolution it is desirable to use the highest frequencies but the scattering is large and target identification is poor.

Time-varying magnetic fields induce an emf (voltage) in a toroid. The electric field at the center of the toroid is shown to be linearly related to this induced voltage. By measuring the voltage across a toroid one can easily and accurately determine the electric field. The new sensor will greatly simplify the cumbersome procedure involved with GPR measurements with its center frequency less than 100 MHz. The overall size of the toroidal sensor can be as small as a few inches. It is this size advantage that will not only allow easy fabrication and deployment of multi-component devices either on the surface or in a borehole, but it will render greatly improved resolution over conventional systems.

The toroid can be used as a transmitter if current is forced to flow through the winding (Wait, 1995). Generated within the toroid is a strong azimuthal magnetic field, which in turn can be considered equivalent to that of an electric dipole.

## Theoretical basis

In a source-free region, Maxwell's equations in the frequency domain with an  $e^{i\omega t}$  time dependence are

$$\nabla \times E = -i\omega\mu H, \quad \text{and} \quad (1)$$

$$\nabla \times H = (\sigma + i\omega\epsilon)E. \quad (2)$$

From equation (2) one can obtain the electric field as

$$E = \frac{\nabla \times H}{(\sigma + i\omega\epsilon)}, \quad (3)$$

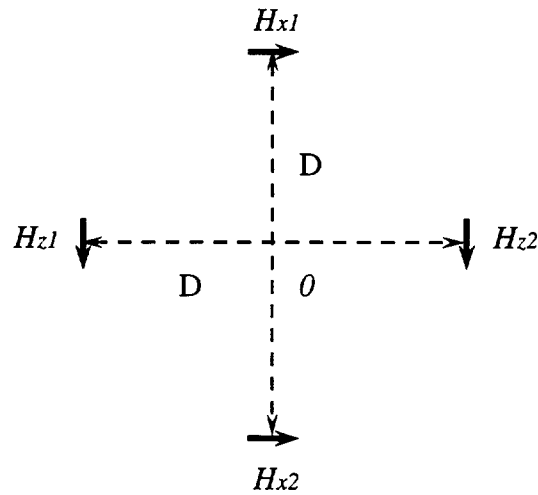
from which each component of the electric field can be written as

$$E_x = \frac{1}{(\sigma + i\omega\epsilon)} \left( \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \right), \quad (4)$$

$$E_y = \frac{1}{(\sigma + i\omega\epsilon)} \left( \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} \right), \text{ and} \quad (5)$$

$$E_z = \frac{1}{(\sigma + i\omega\epsilon)} \left( \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right). \quad (6)$$

Equations (4), (5), and (6) show that electric fields can be obtained by first measuring magnetic fields and taking 'rotation' of them. The rotation (curl) operation needs to be approximate in nature because in practice it cannot be evaluated at a point in space. Let us take as an example the solution of  $E_y$ , shown by equation (5). First, magnetic fields are measured at four points in space as shown in Figure 1.



**Figure 1**

Then the electric field at the center can be evaluated as

$$E_{y0} = \frac{1}{D(\sigma + i\omega\epsilon)} (H_{x2} - H_{x1} - H_{x2} + H_{x1}), \quad (7)$$

where  $D$  is the distance used for making the difference measurement. The same distance is used for both components in this example.

In cylindrical coordinate, Figure 1 may be represented by

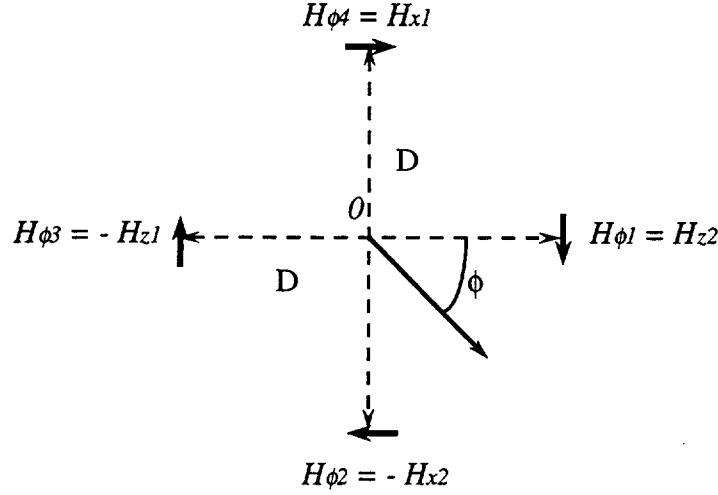


Figure 2

The electric field given by equation (7) can also be replaced by

$$E_{y0} = \frac{1}{D(\sigma + i\omega\epsilon)} (H_{\phi1} + H_{\phi2} + H_{\phi3} + H_{\phi4}), \quad (8)$$

or, equivalently

$$E_{y0} = \frac{1}{D(\sigma + i\omega\epsilon)} \sum_{j=1}^4 H_{\phi_j}. \quad (9)$$

Because of the geometrical similarity, any set of two pairs of orthogonal magnetic fields will give exactly the same electric field at the center. So, if we consider  $N$  such sets, we can write

$$E_{y0} = \frac{1}{ND(\sigma + i\omega\epsilon)} \sum_{j=1}^{4N} H_{\phi_j}. \quad (10)$$

This is a useful relationship relating the sum of azimuthal magnetic field measurements to the axial electric field at the center of such an arrangement.

### Inductive electric field measurement

Magnetic fields can be measured using a loop. In the presence of a time-varying magnetic field a small voltage ( $\Delta V$ ) is induced in a loop and is given by

$$\Delta V = -i\omega\mu \int H \cdot ds \cong -i\omega\mu AH, \quad (11)$$

where  $A$  is the area of the loop, and it is assumed that the magnetic field is normal to the loop. The magnetic field supporting this voltage can be estimated by

$$H = \frac{\Delta V}{-i\omega\mu A}. \quad (12)$$

Consider a four-loop system as shown in Fig. 3.

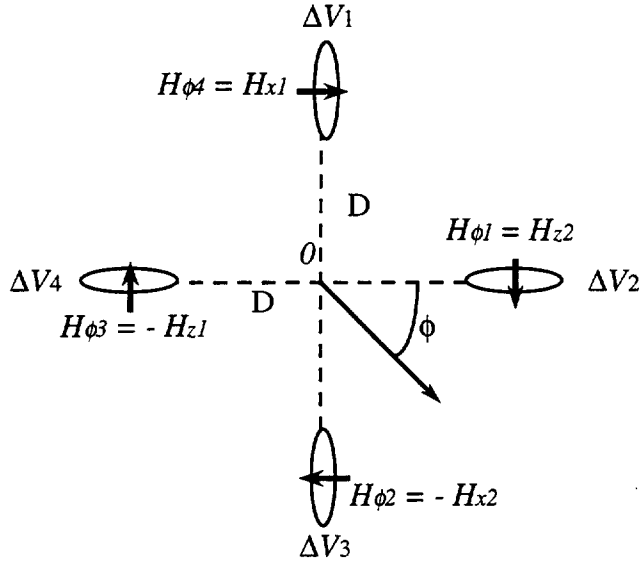


Figure 3

By substituting equation (12) into (9), we obtain

$$E_{y_0} = \frac{1}{D(\sigma + i\omega\epsilon)} \frac{1}{(-i\omega\mu A)} \sum_{j=1}^4 \Delta V_j = \frac{1}{DAk^2} \sum_{j=1}^4 \Delta V_j, \quad (13)$$

where  $k$  is the propagation constant. We now consider a measurement scheme using a toroid consisting of  $4N$  continuously wound loops with  $N$  equal number of loops in each quadrant. In view of equations (10) and (13), we can write

$$E_{y_0} = \frac{1}{NDAk^2} \sum_{j=1}^{4N} \Delta V_j. \quad (14)$$

Because all loops are wound continuously, the summation can be replaced by a total voltage induced in a toroid consisting of  $4N$  loops. The final expression for the electric field then becomes

$$E_{y_0} = \frac{V}{NDAk^2}. \quad (15)$$

The electric field measured this way may be called the 'inductive' measurement as opposed to the 'capacitive' one common to most of the electric field measurement schemes using antennae.

### Practical consideration - Sensitivity analysis

The 'inductive' method of measuring the electric field is based on the voltage measurement using a toroid, hence we need to evaluate the amplitude of the expected emf

induced within a typical toroid and see if we can measure it. From equation (15), the voltage sum induced in a toroid is found to be

$$V = k^2 N D A E_{y_0}. \quad (16)$$

Consider a vertical magnetic dipole source of unit moment, 10 m away from the point of measurement on the surface of a 100 ohm-m half space. Specifications of the toroid used for the demonstration are (Fig. 4); the toroid diameter  $D=2''$ , loop diameter  $d=1''$ , and the number of turns in one quadrant  $N=25$  (total number of turns is therefore 100). The overall size of the toroid is 3''.

The electric field on the right hand side of equation (15) is obtained using the EM1D code over the half-space shown in Fig. 4. Fig. 5 shows the induced voltage as a function of frequency. As a reference, this illustration also shows the electronic noise level of a commercial amplifier. As can be seen the voltage induced in the toroid is greater than the noise limit as the frequency is increased above 1 MHz. So, the 3'' toroid has enough sensitivity to cover a range of frequencies above 1 MHz. The smallness of the sensor is a great advantage over the conventional 'capacitive' linear antenna. At 30 MHz, for example, the antenna length will be about 17', 68 times longer than the 3'' toroid. Furthermore, the size of the toroid stays the same for all frequencies because tuning is of much less concern for inductive measurements.

### Acknowledgment

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Wait, J.R., (1995), Excitation of a conducting half-space by a toroidal coil, IEEE Antennas and Propagation Magazine, Vol.37, No. 4, p.72-74.



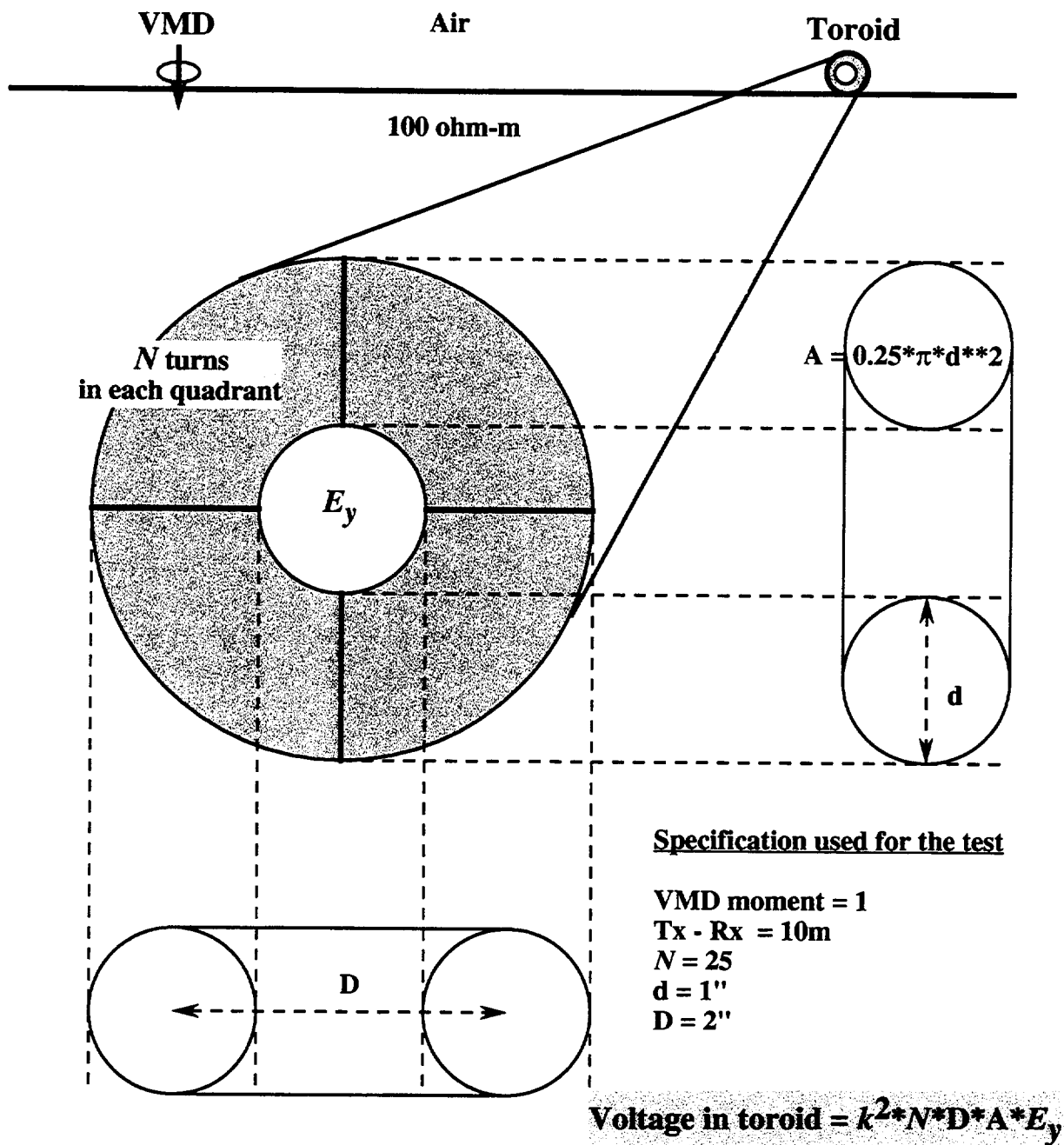


Figure 4. A toroid on a half space

Source is a VMD ( $M=1.0$ ) at (0.0, 0.0, -0.1)  
Torroid center is at (10.0, 0.0, -0.1)  
Torroid diameter: 2"; Coil diameter: 1"; Number of turns: 100  
on a 100 ohm-m half space

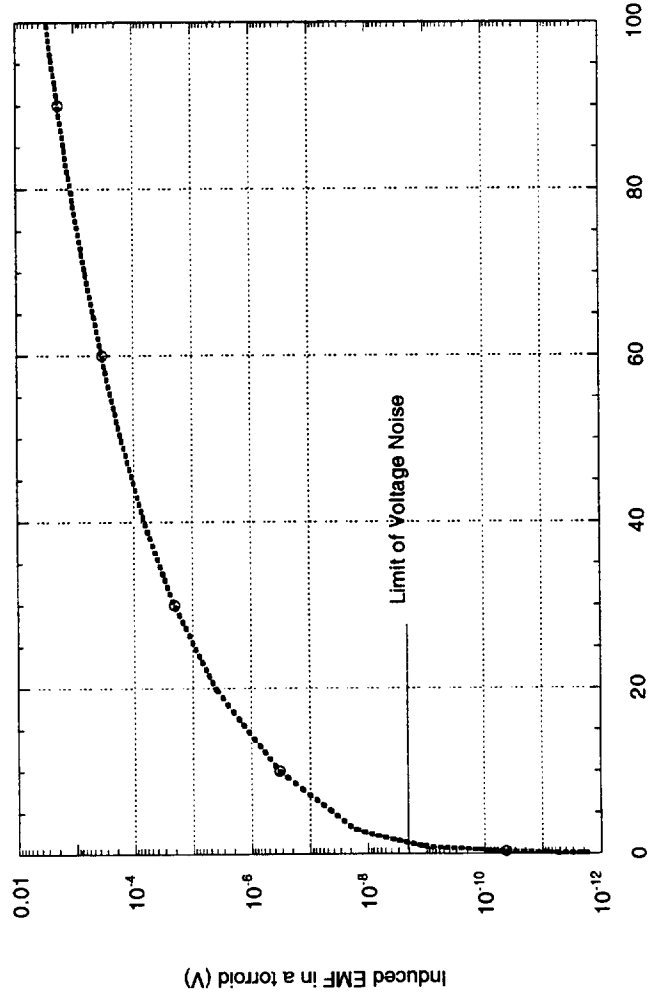


Figure 5

