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Haakon A. Olsen

Fysisk Institutt
Norges teknisk-naturvitenskapelige universitet
N–7055 Dragvoll, Norway

John B. Stav

Department of General Science
Sør-Trøndelag College
N–7005 Trondheim, Norway

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Haakon A. Olsen
Institute of Physics, University of Trondheim, N-7055 Dragvoll, Norway

John B. Stav*
Department og General Science, Sør-Trøndelag College, N-7005 Trondheim, Norway
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Abstract

The complete calculation of all final state quark-antiquark polarization effects is given.

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I. INTRODUCTION

Spin polarization effects in high energy quark-antiquark-gluon production in electron positron annihilation seem to gain interest in current experiments. On the one hand it is now possible to obtain substantial electron linear polarization at the SLAC Linear Collider [1] which improves measurements of cross sections and asymmetries. On the other hand it is to expected that observations of fianl state quark-antiquark-gluon jet polarizations may improve our understanding of the physical production process itself, as well as the mechanism of jet production.

Previously the gluon linear and circular polarization have been calculated for polarized electrons and positrons in the quark mass zero approximation [2], where it was shown that the gluon polarization is strongly influenced by beam polarizations effects.

In the present paper we present a complete calculation of all final state quark-antiquark polarization effects. The quark-antiquark polarization independent cross section has been obtained by several groups [3,4] for unpolarized leptons and [5] for polarized electrons and/or positrons.

II. THE QUARK/ANTIQUARK POLARIZATIONS

The cross section for flavour f, differential in angles and scaled quark and antiquark energies x and $\bar{x}$, respectively, is given by [6]

$$\frac{d^5\sigma_f^{q\bar{q}g}}{d\Omega d\chi dx d\bar{x}} = \frac{\alpha^2}{(2\pi)^2}\frac{\alpha_s}{s}\left\{L^{\mu\nu}_{\gamma\gamma}H^f_{\gamma\gamma\mu\nu} + 2Ref(s)L^{\mu\nu}_{\gamma Z}H^f_{\gamma Z\mu\nu} + |f(s)|^2L^{\mu\nu}_{ZZ}H^f_{ZZ\mu\nu}\right\}, \qquad (2.1)$$

where χ is the azimuthal angle of the electron momentum $\boldsymbol{p}_-$ in the coordinate system with the z-axis along the quark momentum $\boldsymbol{q}$. $L^{\mu\nu}_{\gamma\gamma}$, $L^{\mu\nu}_{\gamma Z}$ and $L^{\mu\nu}_{ZZ}$ are the lepton (e_+, e_-) tensors and $H^f_{\gamma\gamma\mu\nu}$, $H^f_{\gamma Z\mu\nu}$ and $H^f_{ZZ\mu\nu}$ the corresponding hadron tensors for photon interaction, interference of photon and Z_0 interaction, and Z_0 interaction, respectively. We include the effects of electron and positron longitudinal and transverse polarization, and the lepton tensors are given by

$$\begin{aligned}
L^{\mu\nu}_{\gamma\gamma} &= \Xi L^{\mu\nu}_1 + \xi L^{\mu\nu}_2 + L^{\mu\nu}_3,\\
L^{\mu\nu}_{\gamma Z} &= -(v\Xi - a\xi)L^{\mu\nu}_1 - (v\xi - a\Xi)L^{\mu\nu}_2 + vL^{\mu\nu}_3 + aL^{\mu\nu}_4,\\
L^{\mu\nu}_{ZZ} &= \left[(v^2+a^2)\Xi - 2va\xi\right]L^{\mu\nu}_1 + \left[(v^2+a^2)\xi - 2va\Xi\right]L^{\mu\nu}_2 - (v^2-a^2)L^{\mu\nu}_3,
\end{aligned} \qquad (2.2)$$

which include longitudinal polarization, $P^{||}_{\pm}$, effects, $\Xi = 1 - P^{||}_+P^{||}_-$ and $\xi = P^{||}_- - P^{||}_+$. Transverse polarization effects are contained in $L^{\mu\nu}_3$ and $L^{\mu\nu}_4$. The lepton tensors are [6]

$$\begin{aligned}
L^{\mu\nu}_1 &= 4(p^\mu_+p^\nu_- + p^\mu_-p^\nu_+ - g^{\mu\nu}p_+p_-),\\
L^{\mu\nu}_2 &= -4i\varepsilon^{\mu\nu}{}_{\alpha\beta}p^\alpha_+p^\beta_-,\\
L^{\mu\nu}_3 &= 4(p_+p_-)\left(P^{\perp\mu}_+P^{\perp\nu}_- + P^{\perp\mu}_-P^{\perp\nu}_+\right) + (\boldsymbol{P}^\perp_+\boldsymbol{P}^\perp_-)L^{\mu\nu}_1,\\
L^{\mu\nu}_4 &= 4i\varepsilon_{\alpha\beta\gamma\delta}\left[P^{\perp\alpha}_+p^\beta_+g^{\gamma\mu}(P^{\perp\delta}_-p^\nu_- - P^{\perp\nu}_-p^\delta_-) - (p_+, P^\perp_+ \Leftrightarrow p_-, P^\perp_-)\right].
\end{aligned} \qquad (2.3)$$

The hadronic tensors are [6]

$$
\begin{aligned}
H^f_{\gamma\gamma\mu\nu} &= \sum_{colors,S,\bar{S},e} H^f_{\mu\gamma}\overline{H}^f_{\nu\gamma} = (8s)^{-1}Q_f^2 H^f_{V\mu\nu}, \\
H^f_{\gamma Z\mu\nu} &= \sum H^f_{\mu\gamma}\overline{H}^f_{\nu Z} = (8s)^{-1}Q_f\left[v_f H^f_{V\mu\nu} - a_f H^f_{A\mu\nu}\right], \\
H^f_{ZZ\mu\nu} &= \sum H^f_{\mu Z}\overline{H}^f_{\nu Z} = (8s)^{-1}\left[(v_f^2+a_f^2)H^f_{V\mu\nu} - 2a_f v_f H^f_{A\mu\nu} + 2a_f^2 m_f^2 H^{Zf}_{V\mu\nu}\right].
\end{aligned}
\tag{2.4}
$$

In order to simplify the presentation we define the term

$$
\widetilde{H}^f_{V\mu\nu} = H^f_{V\mu\nu} + H^{Zf}_{V\mu\nu}. \tag{2.5}
$$

We extend the hadron tensors to include quark and antiquark polarizations

$$
H^f_{V\mu\nu} = H^{0,f}_{V\mu\nu} + H^{S,f}_{V\mu\nu} + H^{\bar{S},f}_{V\mu\nu} + H^{S\bar{S},f}_{V\mu\nu}, \tag{2.6}
$$

and similarly for $H^f_{A\mu\nu}$ and $H^{Zf}_{V\mu\nu}$. The hadron tensors are obtained in Appendix A:

$$
\begin{aligned}
\widetilde{H}^{0,f}_{V\mu\nu} &= -\tfrac{1}{4}\mathrm{Tr}\ \not{q}M_{\mu\alpha}\not{\bar{q}}\overline{M}^{\alpha}_{\nu}, &
\widetilde{H}^{S,f}_{V\mu\nu} &= -\tfrac{m_f}{4}\mathrm{Tr}\ \gamma_5\not{S}M_{\mu\alpha}\not{\bar{q}}\overline{M}^{\alpha}_{\nu}, \\
\widetilde{H}^{S\bar{S},f}_{V\mu\nu} &= \tfrac{m_f^2}{4}\mathrm{Tr}\ \not{S}M_{\mu\alpha}\not{\bar{S}}\,\overline{M}^{\alpha}_{\nu}, &
H^{0,f}_{A\mu\nu} &= -\tfrac{1}{4}\mathrm{Tr}\ \gamma_5\not{q}M_{\mu\alpha}\not{\bar{q}}\overline{M}^{\alpha}_{\nu}, \\
H^{S,f}_{A\mu\nu} &= -\tfrac{m_f}{4}\mathrm{Tr}\ \not{S}M_{\mu\alpha}\not{\bar{q}}\overline{M}^{\alpha}_{\nu}, &
H^{S\bar{S},f}_{A\mu\nu} &= \tfrac{m_f^2}{4}\mathrm{Tr}\ \gamma_5\not{S}M_{\mu\alpha}\not{\bar{S}}\,\overline{M}^{\alpha}_{\nu}, \\
m_f^2 H^{0,Zf}_{V\mu\nu} &= -\tfrac{m_f^2}{4}\mathrm{Tr}\ M_{\mu\alpha}\overline{M}^{\alpha}_{\nu}, &
m_f^2 H^{S,Zf}_{V\mu\nu} &= -\tfrac{m_f}{4}\mathrm{Tr}\ \gamma_5\not{S}\not{q}M_{\mu\alpha}\overline{M}^{\alpha}_{\nu}, \\
m_f^2 H^{S\bar{S},Zf}_{V\mu\nu} &= -\tfrac{1}{4}\mathrm{Tr}\ \not{S}\not{q}M_{\mu\alpha}\not{\bar{S}}\not{\bar{q}}\overline{M}^{\alpha}_{\nu}, & &
\end{aligned}
\tag{2.7}
$$

and the corresponding $\widetilde{H}^{\bar{S},f}_{V\mu\nu}$, $H^{\bar{S},f}_{A\mu\nu}$ and $H^{\bar{S},Zf}_{V\mu\nu}$ tensors are obtained by the transformation $(q \leftrightarrow \bar{q}, S \leftrightarrow \bar{S}, \mu \leftrightarrow \nu, m_f \to -m_f)$. Here S_μ and $\bar{S}_\mu$ are the polarization four vectors for the quark antiquark respectively, and the reduced matrix element $M_{\mu\alpha}$ is given by

$$
M_{\mu\alpha} = W_\alpha\gamma_\mu + \frac{1}{2qg}\gamma_\alpha\not{g}\gamma_\mu - \frac{1}{2\bar{q}g}\gamma_\mu\not{g}\gamma_\alpha, \tag{2.8}
$$

as explained in Appendix A. The quantity

$$
W_\alpha = \frac{q_\alpha}{qg} - \frac{\bar{q}_\alpha}{\bar{q}g}, \tag{2.9}
$$

is convenient for simplification of the calculations and for the presentation of the results. It satisfies $W\cdot g = 0$, thereby demonstrating explicitely the gauge invariance of the matrix element, Eq. (2.8), $M_{\mu\alpha}g^\alpha = 0$.

The quark polarization four vector $S = (S_0, \boldsymbol{S})$ is given in terms of the polarization of the quark in its rest system $\boldsymbol{\zeta}$ by

$$
\begin{aligned}
S_0 &= \zeta^{||}\frac{|\boldsymbol{q}|}{m_f}, \\
\boldsymbol{S} &= \boldsymbol{\zeta}^{\perp} + \zeta^{||}\hat{\boldsymbol{q}}\frac{E_f}{m_f}.
\end{aligned}
\tag{2.9}
$$

Here $\boldsymbol{\zeta}^{\perp}$ is the transverse and $\zeta^{||} = \boldsymbol{\zeta}\cdot\hat{\boldsymbol{q}}$ is the longitudinal polarization. The covariant polarization satisfies

$$
q\cdot S = 0, \qquad S^2 = -\boldsymbol{\zeta}^2 = -1, \tag{2.10}
$$

for the unit polarization vector $\boldsymbol{\zeta}$.

III. THE HADRON TENSORS

The structure of the hadron tensors Eq. (2.6) is such that only four trace calculations are necessary. The other hadron tensors are obtained by transformations indicated below. We choose the four tensors $\widetilde{H}^{S\bar{S},f}_{V\mu\nu}$, $H^{S\bar{S},f}_{A\mu\nu}$, $H^{S,Zf}_{V\mu\nu}$ and $H^{S\bar{S},Zf}_{V\mu\nu}$, which are explicitely given below.

The hadron tensors are

$$\widetilde{H}^{0,f}_{V\mu\nu} = -\frac{1}{m_f^2}\widetilde{H}^{S\bar{S},f}_{V\mu\nu}(S,\bar{S} \to q,\bar{q}), \tag{3.1}$$

$$\widetilde{H}^{S,f}_{V\mu\nu} = -\frac{1}{m_f}H^{S\bar{S},f}_{A\mu\nu}(\bar{S} \to \bar{q}), \tag{3.2}$$

$$\widetilde{H}^{S\bar{S},f}_{V\mu\nu} = m_f^2\left[W^2S^\alpha\bar{S}^\beta + \frac{1}{qg}(SWg^\alpha - SgW^\alpha)\bar{S}^\beta - \frac{1}{\bar{q}g}(\bar{S}Wg^\alpha - \bar{S}gW^\alpha)S^\beta \right.$$
$$\left. -\frac{1}{(qg)^2}Sgg^\alpha\bar{S}^\beta - \frac{1}{(\bar{q}g)^2}\bar{S}gg^\alpha S^\beta\right] t_{\alpha\mu\beta\nu}, \tag{3.3}$$

with

$$t_{\alpha\mu\beta\nu} = g_{\alpha\mu}g_{\beta\nu} + g_{\alpha\nu}g_{\beta\mu} - g_{\alpha\beta}g_{\mu\nu}, \tag{3.3}$$

$$H^{0,f}_{A\mu\nu} = -\frac{1}{m_f^2}H^{S\bar{S},f}_{A\mu\nu}(S,\bar{S} \to q,\bar{q}), \tag{3.5}$$

$$H^{S,f}_{A\mu\nu} = -\frac{1}{m_f}\widetilde{H}^{S\bar{S},f}_{V\mu\nu}(\bar{S} \to \bar{q}), \tag{3.6}$$

$$H^{S\bar{S},f}_{A\mu\nu} = im_f^2\left[W^2S^\alpha\bar{S}^\beta + \frac{1}{qg}(SWg^\alpha - SgW^\alpha)\bar{S}^\beta + \frac{1}{\bar{q}g}(\bar{S}Wg^\alpha - \bar{S}gW^\alpha)S^\beta \right.$$
$$\left. -\frac{1}{(qg)^2}Sgg^\alpha\bar{S}^\beta + \frac{1}{(\bar{q}g)^2}\bar{S}gg^\alpha S^\beta\right] \varepsilon_{\alpha\mu\beta\nu}. \tag{3.7}$$

Note that except for the change of sign on the third and fifth term and the replacement of $t_{\alpha\mu\beta\nu}$ by the antisymmetric tensor $i\varepsilon_{\alpha\mu\beta\nu}$, Eqs. (3.3) and (3.7) are identical.

$$H^{0,Zf}_{V\mu\nu} = \frac{1}{m_f^2}H^{S\bar{S},Zf}_{V\mu\nu}(S,\bar{S} \to q,\bar{q}) = -W^2g_{\mu\nu} + \frac{2g_\mu g_\nu}{(qg)(\bar{q}g)}, \tag{3.8}$$

$$m_f^2H^{S,Zf}_{V\mu\nu} = im_f\left[W^2S^\alpha q^\beta - \frac{1}{qg}(SWq^\alpha - qWS^\alpha)g^\beta \right.$$
$$\left. +\frac{1}{qg}(Sgq^\alpha - qgS^\alpha)\left(W^\beta - \frac{g^\beta}{\bar{q}g}\right)\right] \varepsilon_{\alpha\mu\beta\nu}. \tag{3.9}$$

The hadron tensor $H^{S\bar{S},Zf}_{V\mu\nu}$ is very complicated when written out in full. By the use of Eq. (B.3) in Appendix B we can write it in the form

$$m_f^2H^{S\bar{S},Zf}_{V\mu\nu} = -\left[\frac{1}{2}W^2S^\alpha q^\beta + \frac{1}{qg}\left\{(qWg^\beta - qgW^\beta)S^\alpha + (SWg^\alpha - SgW^\alpha)q^\beta\right\}\right]\bar{S}^\gamma\bar{q}^\delta$$

$$\times\left(\varepsilon_{\alpha\beta\mu\varepsilon}\varepsilon_{\gamma\delta\nu}{}^{\varepsilon}+t_{\alpha\beta\mu\varepsilon}t_{\gamma\delta\nu}{}^{\varepsilon}\right)$$
$$+\frac{1}{(qg)(\bar{q}g)}\left[g_\mu q^\gamma S^\delta g^\sigma - g_\mu{}^\gamma(qgS^\delta - Sgq^\delta)g^\sigma\right]\bar{S}^\alpha\bar{q}^\beta$$
$$\times\left(\varepsilon_{\alpha\beta\nu\varepsilon}\varepsilon_{\sigma\gamma\delta}{}^{\varepsilon}+t_{\alpha\beta\nu\varepsilon}t_{\sigma\gamma\delta}{}^{\varepsilon}\right)$$
$$+(q,S\Leftrightarrow\bar{q},\bar{S}). \tag{3.10}$$

The $\widetilde{H}^{\bar{S},f}_{V\mu\nu}$, $H^{\bar{S},f}_{A\mu\nu}$ and $H^{\bar{S},Zf}_{V\mu\nu}$ tensors are obtained from corresponding S dependent tensors by the transformation ($q\leftrightarrow\bar{q}, S\leftrightarrow\bar{S}, \mu\leftrightarrow\nu, m_f\rightarrow -m_f$). For completeness we give in Table 1 the hadron tensors written out in full. The calculations are checked by comparison with previously obtained results [6].

$$\begin{aligned}
H^{0,f}_{V\mu\nu} &= \frac{1}{(qg)(\bar{q}g)}\left[\left(Qq-m_f^2\frac{Qg}{\bar{q}g}\right)q^\alpha Q^\beta t_{\alpha\mu\beta\nu}\right.\\
&\quad\left.-\left(Q^2-2m_f^2\frac{Qg}{\bar{q}g}\right)q_\mu q_\nu + m_f^2\left[(Qg)g_{\mu\nu}-g_\mu g_\nu)\right]+(q\Leftrightarrow\bar{q})\right],\\
H^{0,f}_{A\mu\nu} &= \frac{-i}{(qg)(\bar{q}g)}\varepsilon_{\mu\nu\alpha\beta}\left[\left(Qq-m_f^2\frac{Qg}{\bar{q}g}\right)q^\alpha Q^\beta-(q\Leftrightarrow\bar{q})\right],\\
H^{0,Zf}_{V\mu\nu} &= \frac{1}{(qg)(\bar{q}g)}\left[\left(q\bar{q}-m_f^2\frac{qg}{\bar{q}g}\right)g_{\mu\nu}+g_\mu g_\nu+(q\Leftrightarrow\bar{q})\right].
\end{aligned} \tag{3.11}$$

In this way the calculation of $H^{S\bar{S},f}_{V\mu\nu}$, $H^{S\bar{S},f}_{A\mu\nu}$, $H^{S,f}_{V\mu\nu}$ and $H^{S,f}_{A\mu\nu}$ are checked by comparison with Eq. (3.11), by the use of the transformation indicated. Note also that the rather complicated tensor $H^{S\bar{S},Zf}_{V\mu\nu}$ equals $m_f^2 H^{0,Zf}_{V\mu\nu}$ for $S=q$ and $\bar{S}=\bar{q}$, which is a test on this calculation.

Expressed in terms of the momenta and polarizations the S and $\bar{S}$ dependent tensors are

$$\begin{aligned}
\widetilde{H}^{S,f}_{V\mu\nu} &= \frac{im_f}{(qg)(\bar{q}g)}\left[S^\alpha\left\{\left(Qq-m_f^2\frac{Qg}{\bar{q}g}\right)(Q-q)^\beta+\left(q\bar{q}-m_f^2\frac{\bar{q}g}{qg}\right)\bar{q}^\beta-\bar{q}gq^\beta\right\}\right.\\
&\quad\left.+S\bar{q}g^\alpha\bar{q}^\beta+Sg\frac{\bar{q}g}{qg}Q^\alpha\bar{q}^\beta\right]\varepsilon_{\alpha\mu\beta\nu},
\end{aligned} \tag{3.12}$$

$$\begin{aligned}
\widetilde{H}^{S\bar{S},f}_{V\mu\nu} &= -\frac{m_f^2}{(qg)(\bar{q}g)}\left[\left(q\bar{q}-m_f^2\frac{qg}{\bar{q}g}\right)\{S,\bar{S}\}_{\mu\nu}+\left(S\bar{q}-Sq\frac{\bar{q}g}{qg}\right)\{\bar{S},g\}_{\mu\nu}\right.\\
&\quad\left.+\frac{Sg}{qg}\left(\bar{q}g\{\bar{S},Q\}_{\mu\nu}-Qg\{\bar{S},\bar{q}\}_{\mu\nu}\right)+(q,S\Leftrightarrow\bar{q},\bar{S})\right],
\end{aligned} \tag{3.13}$$

where $\{a,b\}_{\mu\nu}=a^\alpha b^\beta t_{\alpha\mu\beta\nu}=a_\mu b_\nu+a_\nu b_\mu-abg_{\mu\nu}$.

$$\begin{aligned}
H^{S,f}_{A\mu\nu} &= \frac{m_f}{(qg)(\bar{q}g)}\left[\left(Qq-m_f^2\frac{Qg}{\bar{q}g}+Q\bar{q}-m_f^2\frac{Qg}{qg}\right)\{S,\bar{q}\}_{\mu\nu}+\left(\frac{Q^2}{2}-m_f^2\frac{Qg}{\bar{q}g}\right)\{S,g\}_{\mu\nu}\right.\\
&\quad\left.+\frac{Sg}{qg}\left(\bar{q}g\{Q,\bar{q}\}_{\mu\nu}-Qg\{\bar{q},\bar{q}\}_{\mu\nu}\right)+S\bar{q}\{\bar{q},g\}_{\mu\nu}-\bar{q}g\{S,Q\}_{\mu\nu}\right],
\end{aligned} \tag{3.14}$$

$$H^{S\bar{S},f}_{A\mu\nu} = -\frac{im_f^2}{(qg)(\bar{q}g)}\left[\left\{\left(q\bar{q}-m_f^2\frac{qg}{\bar{q}g}\right)S^\alpha+S\bar{q}g^\alpha\right.\right.$$

$$+\frac{Sg}{qg}\left(\bar{q}gQ^\alpha - Qg\bar{q}^\alpha\right)\Big\} \bar{S}^\beta \varepsilon_{\alpha\mu\beta\nu} - (q, S \Leftrightarrow \bar{q}, \bar{S})\Big], \tag{3.15}$$

$$m_f^2 H^{S,Zf}_{V\mu\nu} = -\frac{im_f}{(qg)(\bar{q}g)}\Big[(SgQ^\beta - S\bar{q}g^\beta)q^\alpha + \Big\{\left(q\bar{q} - m_f^2\frac{\bar{q}g}{qg}\right)(Q-\bar{q})^\beta$$
$$+\left(q\bar{q} - m_f^2\frac{qg}{\bar{q}g}\right)q^\beta - qgQ^\beta + Qgq^\beta\Big\} S^\alpha\Big]\varepsilon_{\alpha\mu\beta\nu}, \tag{3.16}$$

$$m_f^2 H^{S\bar{S},Zf}_{V\mu\nu} = -\Big\{\Big[-\frac{q\bar{q}}{(qg)(\bar{q}g)} + \frac{1}{2}\left(\frac{m_f^2}{(qg)^2} + \frac{m_f^2}{(\bar{q}g)^2}\right)\Big] S^\alpha q^\beta + \frac{1}{qg}\Big[\left(-\frac{q\bar{q}}{\bar{q}g} + \frac{m_f^2}{qg}\right)g^\beta$$
$$-\left(q^\beta - \frac{qg}{\bar{q}g}\bar{q}^\beta\right)\Big] S^\alpha + \Big[\left(\frac{Sq}{qg} - \frac{S\bar{q}}{\bar{q}g}\right)g^\alpha - Sg\left(\frac{q^\alpha}{qg} - \frac{\bar{q}^\alpha}{\bar{q}g}\right)\Big] q^\beta\Big\} \bar{S}^\gamma \bar{q}^\delta$$
$$\times\left(\varepsilon_{\alpha\beta\mu\epsilon}\varepsilon_{\gamma\delta\nu}{}^{\epsilon} + t_{\alpha\beta\mu\epsilon}t_{\gamma\delta\nu}{}^{\epsilon}\right)$$
$$+\frac{1}{(qg)(\bar{q}g)}\left[g_\mu q^\gamma S^\delta g^\sigma - g_\mu{}^\gamma(qgS^\delta - Sqq^\delta)g^\sigma\right]\bar{S}^\alpha \bar{q}^\beta$$
$$\times\left(\varepsilon_{\alpha\beta\nu\epsilon}\varepsilon_{\sigma\gamma\delta}{}^{\epsilon} + t_{\alpha\beta\nu\epsilon}t_{\sigma\gamma\delta}{}^{\epsilon}\right)$$
$$+(q, S \Leftrightarrow \bar{q}, \bar{S}). \tag{3.17}$$

IV. THE POLARIZATION DEPENDENT CROSS SECTION

We shall at the present presentation concentrate on longitudinally polarized electrons and positrons, since this is the most interesting case experimentally. The case of transversly polarization may be incorporated in the present formulation when needed.

The cross section Eq. (2.1) is conveniently written in the form

$$\frac{d^5\sigma_f^{q\bar{q}g}}{d\Omega d\chi dx d\bar{x}} = \frac{1}{4}\frac{\alpha^2}{(2\pi)^2}\frac{\alpha_s}{s}\frac{1}{(1-x)(1-\bar{x})}\Big[h_f^{(1)}(s,\xi,\Xi)\left(X_0 + X_0^{S\bar{S}}\right) + h_f^{(2)}(s,\xi,\Xi)\left(Y_0 + Y_0^{S\bar{S}}\right)$$
$$+h_f^{(5)}(s,\xi,\Xi)\left(Z_0 + Z_0^{S\bar{S}}\right) + h_f^{(7)}(s,\xi,\Xi)\left(X_0^S + X_0^{\bar{S}}\right)$$
$$+h_f^{(8)}(s,\xi,\Xi)\left(Y_0^S + Y_0^{\bar{S}}\right) + h_f^{(9)}(s,\xi,\Xi)\left(Z_0^S + Z_0^{\bar{S}}\right)\Big], \tag{4.1}$$

where

$$\begin{aligned}
X_0 + X_0^{S\bar{S}} &= \left(H_V^{0,f} + H_V^{S\bar{S},f}\right)_{\alpha\beta}\left(p_+^\alpha p_-^\beta + p_+^\beta p_-^\alpha\right), \\
Y_0 + Y_0^{S\bar{S}} &= \left(H_A^{0,f} + H_A^{S\bar{S},f}\right)_{\alpha\beta}\left(p_+^\alpha p_-^\beta - p_+^\beta p_-^\alpha\right), \\
Z_0 + Z_0^{S\bar{S}} &= \left(H_V^{0,Zf} + H_V^{S\bar{S},Zf}\right)_{\alpha\beta}\left(p_+^\alpha p_-^\beta + p_+^\beta p_-^\alpha\right), \\
X_0^S &= \left(H_V^{S,f}\right)_{\alpha\beta}\left(p_+^\alpha p_-^\beta - p_+^\beta p_-^\alpha\right), \\
Y_0^S &= \left(H_A^{S,f}\right)_{\alpha\beta}\left(p_+^\alpha p_-^\beta + p_+^\beta p_-^\alpha\right), \\
Z_0^S &= \left(H_V^{S,Zf}\right)_{\alpha\beta}\left(p_+^\alpha p_-^\beta - p_+^\beta p_-^\alpha\right),
\end{aligned} \tag{4.2}$$

with the definitions for $(H_V^{0,f})_{\alpha\beta}$ etc.

$$\begin{aligned} s^{-1}(1-x)(1-\bar{x})H^{0,f}_{V\mu\nu} &= \left(H_V^{0,f}\right)^{\alpha\beta} t_{\alpha\mu\beta\nu}, \\ s^{-1}(1-x)(1-\bar{x})H^{S,f}_{V\mu\nu} &= \left(H_V^{S,f}\right)^{\alpha\beta} i\varepsilon_{\alpha\mu\beta\nu} \\ s^{-1}(1-x)(1-\bar{x})H^{0,f}_{A\mu\nu} &= \left(H_A^{0,f}\right)^{\alpha\beta} i\varepsilon_{\alpha\mu\beta\nu}, \\ s^{-1}(1-x)(1-\bar{x})H^{S,f}_{A\mu\nu} &= \left(H_A^{S,f}\right)^{\alpha\beta} t_{\alpha\mu\beta\nu}, \end{aligned} \tag{4.3}$$

for the symmetric and antisymmetric tensors. We have used ref. [6], leaving out transversal lepton polarizations

$$\begin{aligned} L^{\mu\nu}_{\gamma\gamma} &= \Xi L_1^{\mu\nu} + \xi L_2^{\mu\nu}, \\ L^{\mu\nu}_{\gamma Z} &= -(v\Xi - a\xi)L_1^{\mu\nu} - (v\xi - a\Xi)L_2^{\mu\nu}, \\ L^{\mu\nu}_{ZZ} &= [(v^2+a^2)\Xi - 2va\xi]L_1^{\mu\nu} + [(v^2+a^2)\xi - 2va\Xi]L_2^{\mu\nu}, \end{aligned} \tag{4.4}$$

with $\Xi = 1 - P_+^{||}P_-^{||}$ and $\xi = P_-^{||} - P_+^{||}$, and $L_1^{\mu\nu}$ and $L_2^{\mu\nu}$ given in Eq. (2.3). We have further used the multiplication rules [7],

$$\begin{aligned} t_{\mu\alpha\nu\beta}t^{\mu\gamma\nu\delta} &= 2\left(g_\alpha{}^\gamma g_\beta{}^\delta + g_\alpha{}^\delta g_\beta{}^\gamma\right), \\ \varepsilon_{\mu\alpha\nu\beta}\varepsilon^{\mu\gamma\nu\delta} &= -2\left(g_\alpha{}^\gamma g_\beta{}^\delta - g_\alpha{}^\delta g_\beta{}^\gamma\right). \end{aligned} \tag{4.5}$$

The coupling functions $h_f^{(1)}$, $h_f^{(2)}$ and $h_f^{(5)}$ are given previously [6]

$$\begin{aligned} h_f^{(1)}(s,\xi,\Xi) &= Q_f^2\Xi - 2Q_f Re f(s)(v\Xi - a\xi)v_f + |f(s)|^2[(v^2+a^2)\Xi - 2va\xi](v_f^2+a_f^2), \\ h_f^{(2)}(s,\xi,\Xi) &= 2Q_f Re f(s)(v\xi - a\Xi)a_f - 2|f(s)|^2[(v^2+a^2)\xi - 2va\Xi]v_f a_f, \\ h_f^{(5)}(s,\xi,\Xi) &= 2|f(s)|^2[(v^2+a^2)\Xi - 2va\xi]a_f^2. \end{aligned} \tag{4.6}$$

The new coupling functions related to the single polarizations S or $\bar{S}$ are

$$\begin{aligned} h_f^{(7)}(s,\xi,\Xi) &= Q_f^2\xi - 2Q_f Re f(s)(v\xi - a\Xi)v_f + |f(s)|^2[(v^2+a^2)\xi - 2va\Xi](v_f^2+a_f^2), \\ h_f^{(8)}(s,\xi,\Xi) &= 2Q_f Re f(s)(v\Xi - a\xi)a_f - 2|f(s)|^2[(v^2+a^2)\Xi - 2va\xi]v_f a_f, \\ h_f^{(9)}(s,\xi,\Xi) &= 2|f(s)|^2[(v^2+a^2)\xi - 2va\Xi]a_f^2. \end{aligned} \tag{4.7}$$

Note that $h_f^{(7)}(\xi,\Xi) = h_f^{(1)}(\Xi,\xi)$, $h_f^{(8)}(\xi,\Xi) = h_f^{(2)}(\Xi,\xi)$ and $h_f^{(9)}(\xi,\Xi) = h_f^{(5)}(\Xi,\xi)$, which reflect the transformation properties of $H^{0,f}_{V\mu\nu}$ to $H^{S,f}_{V\mu\nu}$, $H^{0,f}_{A\mu\nu}$ to $H^{S,f}_{A\mu\nu}$ and $H^{0,Zf}_{V\mu\nu}$ to $H^{S,Zf}_{V\mu\nu}$. It should be noted that the coupling functions are the same for interactions with no polarization dependence (e.g. $H_V^{0,f}$) as for interactions with S, $\bar{S}$ dependence (e.g. $H_V^{S\bar{S},f}$).

The X_0, Y_0 and Z_0 functions are given previously [6]

$$\begin{aligned} X_0 &= \left(1 - \frac{\overline{m}_f^2}{2}\frac{x_g}{1-x}\right)\left[x^2(1+\beta_x^2\cos^2\theta) + \overline{m}_f^2\right] + \frac{\overline{m}_f^2}{16}\left[x_g^2(1+\cos^2\theta_g) - 8x_g\right] \\ &\quad +(x \Leftrightarrow \bar{x}, \theta \Leftrightarrow \bar{\theta}), \\ Y_0 &= 2\left\{\left(x - \frac{\overline{m}_f^2}{2}\frac{x_g}{1-x}\right)x\beta_x\cos\theta - (x \Leftrightarrow \bar{x}, \theta \Leftrightarrow \bar{\theta})\right\}, \\ Z_0 &= -\frac{\overline{m}_f^2}{4}\left\{4\left(1 - \frac{\overline{m}_f^2}{2}\frac{x_g}{1-x}\right) - x_g^2(1-\cos^2\theta_g) - 4x_g + (x \Leftrightarrow \bar{x})\right\}. \end{aligned} \tag{4.8}$$

The polarization functions are

$$\widetilde{X}_0^S = \zeta^{\|}\left\{x\bar{x}\left(1-x_g-\frac{\overline{m}_f^2}{2}\frac{x_g}{1-\bar{x}}\right)(\cos\theta-\beta_x\beta_{\bar{x}}\cos\bar{\theta})\right.$$
$$+\left[\left(x-\frac{\overline{m}_f^2}{2}\frac{x_g}{1-x}\right)(2x-\overline{m}_f^2)-\overline{m}_f^2(1-x)\right]\cos\theta$$
$$-2\left(x-\frac{\overline{m}_f^2}{2}\frac{x_g}{1-\bar{x}}\right)\frac{\bar{x}(1-x)}{x\beta_x}\beta_{\bar{x}}\cos\bar{\theta}$$
$$\left.-\frac{\bar{x}}{\beta_x}\left(1-x_g-\frac{\overline{m}_f^2}{2}\frac{x+\bar{x}}{x}\right)\left(x\beta_x\cos\theta+(2-x)\beta_{\bar{x}}\cos\bar{\theta}\right)\right\}, \tag{4.9}$$

$$Y_0^S = \zeta^{\|}\left\{x\bar{x}\left(x+\bar{x}-\frac{\overline{m}_f^2}{2}\frac{x_g^2}{(1-x)(1-\bar{x})}\right)\left(\beta_x-\beta_{\bar{x}}\cos\bar{\theta}\cos\theta\right)-2(1-x)x\beta_x\right.$$
$$+x\left(1-\frac{\overline{m}_f^2}{2}\frac{x_g}{1-x}\right)\left[x_g\beta_x+\cos\theta(x\beta_x\cos\theta+\bar{x}\beta_{\bar{x}}\cos\bar{\theta})\right]$$
$$+\frac{\bar{x}}{\beta_x}\left(1-x_g-\frac{\overline{m}_f^2}{2}\frac{x+\bar{x}}{x}\right)\left[x_g+\beta_{\bar{x}}\cos\bar{\theta}(x\beta_x\cos\theta+\bar{x}\beta_{\bar{x}}\cos\bar{\theta})\right]$$
$$\left.+\frac{2\bar{x}}{x\beta_x}\left(x-\frac{\overline{m}_f^2}{2}\frac{x_g}{1-\bar{x}}\right)\left[1-x-\frac{\bar{x}x_g}{2}(1-\beta_{\bar{x}}^2\cos^2\bar{\theta})\right]\right\}, \tag{4.10}$$

$$Z_0^S = -\overline{m}_f^2\zeta^{\|}\left\{\left[1-x_g+\frac{x}{2}(2-\bar{x})\left(\frac{x+\bar{x}}{x}-\frac{x_g}{1-\bar{x}}\right)-\frac{\overline{m}_f^2}{2}\frac{x_g}{1-x}\right]\cos\theta\right.$$
$$\left.+\left[-\frac{1-x_g}{x^2}+\frac{1}{2}\left(\frac{x+\bar{x}}{x}-\frac{x_g}{1-\bar{x}}\beta_x^2\right)\right]\frac{x\bar{x}}{\beta_x}\beta_{\bar{x}}\cos\bar{\theta}\right\}, \tag{4.11}$$

while the polarization correlation functions are given by

$$\widetilde{X}_0^{S\bar{S}} = \zeta^{\|}\bar{\zeta}^{\|}\bar{x}\left\{x\left(1-x_g-\frac{\overline{m}_f^2}{2}\frac{x_g}{1-x}\right)\left(\cos\theta\cos\bar{\theta}-\beta_x\beta_{\bar{x}}\right)\right.$$
$$-\frac{1}{\beta_x}\left(1-x_g-\frac{\overline{m}_f^2}{2}\frac{x+\bar{x}}{x}\right)\left[x_g\beta_{\bar{x}}+\cos\bar{\theta}(x\beta_x\cos\theta+\bar{x}\beta_{\bar{x}}\cos\bar{\theta})\right]$$
$$\left.-\frac{2\beta_{\bar{x}}}{x\beta_x}\left(x-\frac{\overline{m}_f^2}{2}\frac{x_g}{1-\bar{x}}\right)\left(1-x-\frac{\bar{x}x_g}{2}\sin^2\bar{\theta}\right)\right\} \tag{4.12}$$
$$+(x\Leftrightarrow\bar{x},\beta_x\Leftrightarrow\beta_{\bar{x}},\theta\Leftrightarrow\bar{\theta}),$$

$$Y_0^{S\bar{S}} = \zeta^{\|}\bar{\zeta}^{\|}\left\{x\bar{x}\left(1-x_g-\frac{\overline{m}_f^2}{2}\frac{x_g}{1-x}\right)\left(\beta_x\cos\bar{\theta}-\beta_{\bar{x}}\cos\theta\right)\right.$$
$$+\bar{x}\left(1-x_g-\frac{\overline{m}_f^2}{2}\frac{x+\bar{x}}{x}\right)\left[x_g\cos\bar{\theta}+\beta_{\bar{x}}(x\beta_x\cos\theta+\bar{x}\beta_{\bar{x}}\cos\bar{\theta})\right]$$
$$\left.+\frac{2}{x\beta_x}\left(x-\frac{\overline{m}_f^2}{2}\frac{x_g}{1-\bar{x}}\right)\left[\bar{x}(1-x)-\frac{\overline{m}_f^2}{2}x_g\right]\cos\bar{\theta}\right\} \tag{4.13}$$
$$-(x\Leftrightarrow\bar{x},\beta_x\Leftrightarrow\beta_{\bar{x}},\theta\Leftrightarrow\bar{\theta}),$$

$$Z_0^{S\bar{S}} = \overline{m}_f^2\zeta^{\|}\bar{\zeta}^{\|}\left\{\left[-\left(1-x_g-\frac{\overline{m}_f^2}{2}\frac{x_g}{1-x}\right)\cos\theta+\frac{x_g}{2(1-\bar{x})}x\beta_x\bar{x}\beta_{\bar{x}}(\cos\bar{\theta}-\cos\vartheta\cos\theta)\right.\right.$$

$$-\frac{1}{2}\Big(x\beta_x+\bar{x}\beta_{\bar{x}}\cos\vartheta\Big)\left(x\beta_x\cos\theta+\bar{x}\beta_{\bar{x}}\cos\bar{\theta}\right)\Big]\cos\bar{\theta}$$
$$+\frac{1}{4}\left[x_g^2+(x\beta_x\cos\theta+\bar{x}\beta_{\bar{x}}\cos\bar{\theta})^2\right]\cos\vartheta\Big\} \tag{4.14}$$
$$+(x\Leftrightarrow\bar{x},\beta_x\Leftrightarrow\beta_{\bar{x}},\theta\Leftrightarrow\bar{\theta}).$$

V. QUARK-ANTIQUARK POLARIZATION EFFECTS IN $q\bar{q}$ AND $q\bar{q}g$ FINAL STATES

It is of considerable use for the understanding of the polarization effects to write down the cross section for $e^+e^-\to q\bar{q}$ for longitudinally polarized electrons, positrons and quarks and antiquarks. From the same procedure as above, replacing $M_{\mu\alpha}$ by γ_μ, one readily find

$$\begin{aligned}\frac{d^2\sigma_f^{q\bar{q}}}{d\Omega}=\frac{3}{16}\frac{\alpha^2}{s}&\Big\{h^{(1)}(s)\left[(1-\zeta^{||}\bar{\zeta}^{||})(1+\cos^2\theta)+\overline{m}_f^2(1+\zeta^{||}\bar{\zeta}^{||})\sin^2\theta\right]\\&+2h^{(2)}(s)(1-\zeta^{||}\bar{\zeta}^{||})\beta\cos\theta-\frac{\overline{m}_f^2}{2}h^{(5)}(s)\left[(1-\zeta^{||}\bar{\zeta}^{||})(1+\cos^2\theta)+(1+\zeta^{||}\bar{\zeta}^{||})\sin^2\theta\right]\\&+2h^{(7)}(s)(\zeta^{||}-\bar{\zeta}^{||})\cos\theta+h^{(8)}(s)\beta(\zeta^{||}-\bar{\zeta}^{||})(1+\cos^2\theta)\\&-h^{(9)}(s)\overline{m}_f^2(\zeta^{||}-\bar{\zeta}^{||})\cos\theta\Big\},\end{aligned} \tag{5.1}$$

with $\beta=\sqrt{1-\overline{m}_f^2}$ and where the lepton longitudinal polarizations are contained in the coupling functions $h^{(i)}(s)$.

It may be instructive to write down the $q\bar{q}$-cross section at the Z_0 resonance, which shows more clearly the correlations of electron and quark-antiquark polarizations:

$$\begin{aligned}\frac{d^2\sigma_f^{q\bar{q}}(E=2M_Z)}{d\Omega}=&\frac{3}{16}\left(\frac{\alpha}{4\sin^2 2\theta_W}\right)^2\frac{1}{\Gamma^2}\left(v^2+a^2-2avP_-\right)\\&\times\Big\{(v_f^2+a_f^2)\left[(1-\zeta^{||}\bar{\zeta}^{||})(1+\cos^2\theta)+\overline{m}_f^2(1+\zeta^{||}\bar{\zeta}^{||})\sin^2\theta\right]\\&-4P_{Z_0}v_fa_f(1-\zeta^{||}\bar{\zeta}^{||})\beta\cos\theta\\&-\frac{\overline{m}_f^2}{2}a_f^2\left[(1-\zeta^{||}\bar{\zeta}^{||})(1+\cos^2\theta)+(1+\zeta^{||}\bar{\zeta}^{||})\sin^2\theta\right]\\&+4P_{Z_0}(v_f^2+a_f^2)(\zeta^{||}-\bar{\zeta}^{||})\cos\theta-2v_fa_f\beta(\zeta^{||}-\bar{\zeta}^{||})(1+\cos^2\theta)\\&-4P_{Z_0}\overline{m}_f^2a_f^2(\zeta^{||}-\bar{\zeta}^{||})\cos\theta\Big\}.\end{aligned} \tag{5.2}$$

We have here for simplicity and also because it is experimentally relevant, considered polarized electrons only, with polarization P_-, while $P_+=0$. The polarization of the created and decaying Z_0-boson is [5]

$$P_{Z_0}=\frac{-2av+(v^2+a^2)P_-}{v^2+a^2-2vaP_-}.$$

The cross section for $e^+e^-\to q\bar{q}g$ is obtained from Eq. (4.1), written out in the notation of ref. [6] with $\mathcal{F}_i(x,\bar{x})$ the polarization independent form factors, $\mathcal{F}_i^\zeta(x,\bar{x})$ the polarization dependent form factors and $\mathcal{F}_i^{\zeta\bar\zeta}(x,\bar x)$ the polarization correlation form factors. Integration over χ gives

$$\frac{d^2\sigma_f^{q\bar q g}}{d\Omega dx d\bar x} = \frac{\alpha^2}{8\pi}\frac{\alpha_s}{s}\frac{1}{(1-x)(1-\bar x)}\Big\{ h_f^{(1)}(s)\left[\mathcal{F}_1(x,\bar x)(1+\cos^2\theta)+\mathcal{F}_4(x,\bar x)\right]$$
$$+\frac{\overline{m}_f^2}{2}h_f^{(1)-}(s)\left[\mathcal{F}_2(x,\bar x)\cos^2\theta+\frac{1}{2}\mathcal{F}_5(x,\bar x)\right]+2h_f^{(2)}(s)\mathcal{F}_3(x,\bar x)\cos\theta$$
$$+h_f^{(7)}(s)\left[\zeta^{\|}\mathcal{F}_1^{\zeta}(x,\bar x)-\bar\zeta^{\|}\mathcal{F}_1^{\bar\zeta}(x,\bar x)\right]\cos\theta+h_f^{(8)}(s)\left[\zeta^{\|}\left(\mathcal{F}_2^{\zeta}(x,\bar x)(1+\cos^2\theta)+\mathcal{F}_5^{\zeta}(x,\bar x)\sin^2\theta\right)\right.$$
$$\left.-\bar\zeta^{\|}\left(\mathcal{F}_2^{\bar\zeta}(x,\bar x)(1+\cos^2\theta)+\mathcal{F}_5^{\bar\zeta}(x,\bar x)\sin^2\theta\right)\right]+h_f^{(9)}(s)\left[\zeta^{\|}\mathcal{F}_3^{\zeta}(x,\bar x)-\bar\zeta^{\|}\mathcal{F}_3^{\bar\zeta}(x,\bar x)\right]\cos\theta$$
$$+\zeta^{\|}\bar\zeta^{\|}\Big(h_f^{(1)}(s)\left[\mathcal{F}_1^{\zeta\bar\zeta}(x,\bar x)(1+\cos^2\theta)+\mathcal{F}_4^{\zeta\bar\zeta}(x,\bar x)\right]$$
$$+\frac{\overline{m}_f^2}{2}h_f^{(1)-}(s)\left[\mathcal{F}_2^{\zeta\bar\zeta}(x,\bar x)\cos^2\theta+\mathcal{F}_5^{\zeta\bar\zeta}(x,\bar x)\right]+2h_f^{(2)}(s)\mathcal{F}_3^{\zeta\bar\zeta}(x,\bar x)\cos\theta\Big)\Big\}, \tag{5.4}$$

where $h_f^{(1)-}(s)=h_f^{(1)}(s)-h_f^{(5)}(s)$.

The form factors are given by the polarization independent functions [6]

$$\mathcal{F}_1(x,\bar x)=\left(1-\frac{\overline{m}_f^2}{2}\frac{x_g}{1-x}\right)x^2\beta_x^2+\left(1-\frac{\overline{m}_f^2}{2}\frac{x_g}{1-\bar x}\right)\bar x^2\beta_{\bar x}^2(1-\frac{3}{2}\sin^2\vartheta),$$
$$\mathcal{F}_2(x,\bar x)=x_g^2-\frac{3}{2}\bar x^2\beta_{\bar x}^2\sin^2\vartheta,$$
$$\mathcal{F}_3(x,\bar x)=\left(x-\frac{\overline{m}_f^2}{2}\frac{x_g}{1-x}\right)x\beta_x-\left(\bar x-\frac{\overline{m}_f^2}{2}\frac{x_g}{1-\bar x}\right)\bar x\beta_{\bar x}\cos\vartheta, \tag{5.5}$$
$$\mathcal{F}_4(x,\bar x)=2\left(1-\frac{\overline{m}_f^2}{2}\frac{x_g}{1-\bar x}\right)\bar x^2\beta_{\bar x}^2\sin^2\vartheta+\overline{m}_f^2\left[2(1-x_g)+x_g^2\left(1-\frac{\overline{m}_f^2}{2(1-x)(1-\bar x)}\right)\right],$$
$$\mathcal{F}_5(x,\bar x)=\bar x^2\beta_{\bar x}^2\sin^2\vartheta-2(x_g^2+4x_g-4)-2\overline{m}_f^2\frac{x_g^2}{(1-x)(1-\bar x)},$$

and the functions related to polarizations are

$$\mathcal{F}_1^{\zeta}(x,\bar x)=(2x-\overline{m}_f^2)\left(x-\bar x\frac{\bar x\beta_{\bar x}}{x\beta_x}\cos\vartheta\right)+\overline{m}_f^2\left\{\frac{1}{2}\bar x^2\beta_{\bar x}^2\sin^2\vartheta-2(1-x)\right.$$
$$\left.+\bar x+\frac{1-\bar x}{1-x}x_g+\left(x+\frac{1-x}{1-\bar x}x_g\right)\frac{\bar x\beta_{\bar x}}{x\beta_x}\cos\vartheta\right\}-\mathcal{F}_3^{\zeta}(x,\bar x), \tag{5.6}$$

$$\mathcal{F}_2^{\zeta}(x,\bar x)=x^2\beta_x\left(1-\frac{\overline{m}^2}{2x}\frac{x_g}{1-x}\right)+\frac{\bar x^2}{\beta_x}-\frac{\overline{m}^2}{2x\beta_x}\bar x\left(x+\bar x+x_g\frac{1-x}{1-\bar x}\right)$$
$$-\frac{x_g}{2\beta_x}\left(1-\frac{\overline{m}^2}{2x}\frac{x_g}{1-\bar x}\right)\left(\bar x^2\beta_{\bar x}^2\sin^2\vartheta+\overline{m}^2\right)$$
$$-\frac{x\bar x}{4}(\beta_x-\beta_{\bar x}\cos\vartheta)\left(\bar x^2\beta_{\bar x}^2\sin^2\vartheta+\overline{m}^2\frac{x_g}{1-\bar x}\right), \tag{5.7}$$

$$\mathcal{F}_3^{\zeta}(x,\bar x)=\frac{\overline{m}_f^2}{2}\left(\bar x^2\beta_{\bar x}^2\sin^2\vartheta-4(1-x_g)+\frac{\overline{m}_f^2x_g^2}{(1-x)(1-\bar x)}\right), \tag{5.8}$$

$$\mathcal{F}_5^{\zeta}(x,\bar{x}) = \frac{\bar{x}^2}{\beta_x} - \frac{\overline{m}^2}{2x\beta_x}\bar{x}\left(x+\bar{x}+x_g\frac{1-x}{1-\bar{x}}\right) - \frac{x_g}{2\beta_x}\left(1-\frac{\overline{m}^2}{2x}\frac{x_g}{1-\bar{x}}\right)\left(2\bar{x}^2\beta_{\bar{x}}^2\cos^2\vartheta+\overline{m}^2\right)$$
$$-\frac{x\bar{x}}{4}(\beta_x-\beta_{\bar{x}}\cos\vartheta)\left(2\bar{x}^2\beta_{\bar{x}}^2\cos^2\vartheta+\overline{m}^2\frac{x_g}{1-\bar{x}}\right)$$
$$+\frac{\overline{m}^2}{2}x\left[\left(\frac{\bar{x}}{x}-\frac{1-x}{1-\bar{x}}\right)\bar{x}\beta_{\bar{x}}\cos\vartheta-\beta_x x_g\right]. \tag{5.9}$$

The $\mathcal{F}_i^{\bar{\zeta}}(x,\bar{x})$ functions are obtained by interchanging quark and antiquark quantities. We agree with reference [8] on the form factor $\mathcal{F}_5^{\zeta}(x,\bar{x})$ which was calculated in that paper, and with reference [9] on $\mathcal{F}_1^{\zeta}(x,\bar{x})$ and $\mathcal{F}_3^{\zeta}(x,\bar{x})$.

The polarization correlation functions are

$$\mathcal{F}_1^{\zeta\bar{\zeta}}(x,\bar{x}) = \left[x\bar{x}\left(1-x_g-\frac{\overline{m}^2}{4}\frac{x_g^2}{(1-x)(1-\bar{x})}\right)\cos\vartheta-\frac{\bar{x}\beta_{\bar{x}}}{x\beta_x}\bar{x}x_g\left(x-\frac{\overline{m}^2}{2}\frac{x_g}{1-\bar{x}}\right)\right.$$
$$\left.-\frac{\bar{x}}{\beta_x}\left(1-x_g-\frac{\overline{m}^2}{2}\frac{x+\bar{x}}{x}\right)(\bar{x}\beta_{\bar{x}}+x\beta_x\cos\vartheta)+(x\Leftrightarrow\bar{x},\beta_x\Leftrightarrow\beta_{\bar{x}})\right]$$
$$+\frac{3}{2}\sin^2\vartheta\frac{\bar{x}^2\beta_{\bar{x}}}{\beta_x}\left[1-\frac{\overline{m}^2}{2}\left(1+\frac{\bar{x}}{x}+\frac{x_g^2}{x(1-\bar{x})}\right)\right]-\frac{\overline{m}^2}{2}\mathcal{F}_2^{\zeta\bar{\zeta}}(x,\bar{x}), \tag{5.10}$$
$$\mathcal{F}_2^{\zeta\bar{\zeta}}(x,\bar{x}) = -\left((2-x_g)^2-\overline{m}_f^2\frac{x_g^2}{(1-x)(1-\bar{x})}\right)\cos\vartheta$$
$$+x\beta_x\bar{x}\beta_{\bar{x}}\left(\frac{x_g}{1-x}-\frac{1}{2}\frac{x_g}{1-\bar{x}}+\frac{3}{2}\right)\sin^2\vartheta, \tag{5.11}$$
$$\mathcal{F}_3^{\zeta\bar{\zeta}}(x,\bar{x}) = x\bar{x}\left(2(1-x_g)-\frac{\overline{m}_f^2}{2}\frac{x_g^2}{(1-x)(1-\bar{x})}\right)(\beta_x\cos\vartheta-\beta_{\bar{x}})$$
$$-x_g\left(1-x_g-\frac{\overline{m}_f^2}{2}\right)(\bar{x}\cos\vartheta-x)-\frac{\overline{m}_f^2}{2}x_g\left(\frac{\bar{x}^2}{x}\cos\vartheta-\frac{x^2}{\bar{x}}\right)$$
$$+(x\beta_x+\bar{x}\beta_{\bar{x}}\cos\vartheta)\left[\left(1-x_g-\frac{\overline{m}_f^2}{2}\right)(\bar{x}\beta_{\bar{x}}-x\beta_x)-\frac{\overline{m}_f^2}{2}\left(\frac{\bar{x}^2}{x}\beta_{\bar{x}}-\frac{x^2}{\bar{x}}\beta_x\right)\right]$$
$$+2\left(x-\frac{\overline{m}_f^2}{2}\frac{x_g}{1-\bar{x}}\right)\left(\bar{x}-\frac{\overline{m}_f^2}{2}\frac{x_g}{1-x}\right)\left(\frac{1-x}{x\beta_x}\cos\vartheta-\frac{1-\bar{x}}{\bar{x}\beta_{\bar{x}}}\right), \tag{5.12}$$
$$\mathcal{F}_4^{\zeta\bar{\zeta}}(x,\bar{x}) = \left[-x\bar{x}\left(1-x_g-\frac{\overline{m}^2}{4}\frac{x_g^2}{(1-x)(1-\bar{x})}\right)\cos\vartheta\right.$$
$$-\frac{\bar{x}}{\beta_x}\left(1-x_g-\frac{\overline{m}^2}{2}\frac{x+\bar{x}}{x}\right)(x_g\beta_{\bar{x}}-\bar{x}\beta_{\bar{x}}-x\beta_x\cos\vartheta)$$
$$\left.+2\frac{\bar{x}\beta_{\bar{x}}}{x\beta_x}(1-\bar{x})(1-x_g)\left(x-\frac{\overline{m}^2}{2}\frac{x_g}{1-\bar{x}}\right)+(x\Leftrightarrow\bar{x},\beta_x\Leftrightarrow\beta_{\bar{x}})\right]$$
$$-2\sin^2\vartheta\frac{\bar{x}^2\beta_{\bar{x}}}{\beta_x}\left[1-\frac{\overline{m}^2}{2}\left(1+\frac{\bar{x}}{x}+\frac{x_g^2}{x(1-\bar{x})}\right)\right]$$
$$+\frac{\overline{m}^2}{2}\left(\mathcal{F}_2^{\zeta\bar{\zeta}}(x,\bar{x})-\mathcal{F}_5^{\zeta\bar{\zeta}}(x,\bar{x})\right), \tag{5.13}$$

$$\mathcal{F}_5^{\zeta\bar\zeta}(x,\bar x) = x_g^2\cos\vartheta + \frac{1}{2}\frac{1-x}{1-\bar x}x\beta_x\bar x\beta_{\bar x}\sin^2\vartheta. \tag{5.14}$$

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APPENDIX A

It is convenient for the calculation to rewrite the Z-coupling matrix element

$$\overline{u}_f\left[\not{e}\frac{\not{q}+\not{g}+m_f}{2qg}\gamma_\mu(v_f-a_f\gamma_5)+\gamma_\mu(v_f-a_f\gamma_5)\frac{\bar{\not{q}}+\not{g}-m_f}{2qg}\not{e}\right]v_f,$$

in the form $\overline{u}_f M_{\mu\alpha}(v_f-a_f\gamma_5)e^\alpha v_f$ with

$$M_{\mu\alpha}=W_\alpha\gamma_\mu+\frac{\gamma_\alpha\not{g}\gamma_\mu}{2qg}-\frac{\gamma_\mu\not{g}\gamma_\alpha}{2\bar q g},\qquad W_\alpha=\frac{q_\alpha}{qg}-\frac{\bar q_\alpha}{\bar q g}. \tag{A.1}$$

By removal of the explicit appearence of the mass, $M_{\mu\alpha}$ has become an odd function in γ, which appears to simplify trace calculations. When we apply the projection operators $\frac{1}{4}(1+\gamma_5\not{S})(\not{q}+m_f)$ and $\frac{1}{4}(1+\gamma_5\bar{\not{S}})(\bar{\not{q}}-m_f)$ for quarks and antiquarks respectively, we find for the ZZ hadron tensor $H^f_{ZZ\mu\nu}$, Eq. (2.4), which is summed over gluon polarizations,

$$\begin{aligned}\frac{1}{8s}H^f_{ZZ\mu\nu} &= -\frac{1}{4}\mathrm{Tr}\ (1+\gamma_5\not{S})(\not{q}+m_f)M_{\mu\alpha}(v_f-a_f\gamma_5)(1+\gamma_5\bar{\not{S}})(\bar{\not{q}}-m_f)(v_f+a_f\gamma_5)\overline{M}^\alpha_\nu\\ &= -\frac{1}{4}\mathrm{Tr}\ \Big\{(v_f^2+a_f^2)\Big[\not{q}M_{\mu\alpha}\bar{\not{q}}\overline{M}^\alpha_\nu+m_f\gamma_5\not{S}M_{\mu\alpha}\bar{\not{q}}\overline{M}^\alpha_\nu-m_f\ \not{q}M_{\mu\alpha}\gamma_5\bar{\not{S}}\ \overline{M}^\alpha_\nu\\ &\quad -m_f^2\ \not{S}M_{\mu\alpha}\bar{\not{S}}\ \overline{M}^\alpha_\nu\Big]-2v_fa_f\Big[\gamma_5\not{q}M_{\mu\alpha}\bar{\not{q}}\overline{M}^\alpha_\nu+m_f\ \not{S}M_{\mu\alpha}\bar{\not{q}}\overline{M}^\alpha_\nu\\ &\quad -m_f\ \not{q}M_{\mu\alpha}\bar{\not{S}}\ \overline{M}^\alpha_\nu-m_f^2\gamma_5\not{S}M_{\mu\alpha}\bar{\not{S}}\ \overline{M}^\alpha_\nu\Big]-(v_f^2-a_f^2)\Big[m_f^2M_{\mu\alpha}\overline{M}^\alpha_\nu\\ &\quad +m_f\gamma_5\not{S}\not{q}M_{\mu\alpha}\overline{M}^\alpha_\nu-m_fM_{\mu\alpha}\gamma_5\bar{\not{S}}\bar{\not{q}}\overline{M}^\alpha_\nu+\ \not{S}\not{q}M_{\mu\alpha}\bar{\not{S}}\bar{\not{q}}\ \overline{M}^\alpha_\nu\Big]\Big\}.\end{aligned} \tag{A.2}$$

From this equation follow the hadron tensors listed in eq. (2.7) by comparison with Eqs. (2.4) and (2.5).

$$\begin{aligned}\widetilde{H}^{0,f}_{V\mu\nu} &= -\left[\frac{1}{2}W^2q^\alpha\bar q^\beta+(qW-1)\frac{g^\alpha\bar q^\beta}{qg}-W^\alpha\bar q^\beta+(q\Leftrightarrow\bar q)\right]t_{\alpha\mu\beta\nu},\\ \widetilde{H}^{S,f}_{V\mu\nu} &= im_f\Bigg[W^2S^\alpha\bar q^\beta+\frac{1}{qg}\left(SWg^\alpha-SgW^\alpha\right)\bar q^\beta+\frac{\bar qW}{\bar qg}g^\alpha S^\beta-W^\alpha S^\beta\\ &\quad -\frac{Sg}{(qg)^2}g^\alpha\bar q^\beta+\frac{1}{\bar qg}g^\alpha S^\beta\Bigg]\varepsilon_{\alpha\mu\beta\nu},\\ \widetilde{H}^{S\bar S,f}_{V\mu\nu} &= m_f^2\left[\frac{1}{2}W^2S^\alpha\bar S^\beta+\frac{1}{qg}\left(SWg^\alpha-SgW^\alpha\right)\bar S^\beta\right.\end{aligned}$$

$$
\begin{aligned}
&\left.-\frac{Sg}{(qg)^2}g^\alpha\bar{S}^\beta+(q,S\Leftrightarrow\bar{q},\bar{S})\right]t_{\alpha\mu\beta\nu},\\
H^{0,f}_{A\mu\nu}&=-i\left[\frac{1}{2}W^2q^\alpha\bar{q}^\beta+(qW-1)\frac{g^\alpha\bar{q}^\beta}{qg}-W^\alpha\bar{q}^\beta-(q\Leftrightarrow\bar{q})\right]\varepsilon_{\alpha\mu\beta\nu},\\
H^{S,f}_{A\mu\nu}&=-m_f\left[W^2S^\alpha\bar{q}^\beta+\frac{1}{qg}\left(SWg^\alpha-SgW^\alpha\right)\bar{q}^\beta-\frac{\bar{q}W}{\bar{q}g}g^\alpha S^\beta+W^\alpha S^\beta\right.\\
&\left.-\frac{Sg}{(qg)^2}g^\alpha\bar{q}^\beta-\frac{1}{\bar{q}g}g^\alpha S^\beta\right]t_{\alpha\mu\beta\nu},\\
H^{S\bar{S},f}_{A\mu\nu}&=im_f^2\left[\frac{1}{2}W^2S^\alpha\bar{S}^\beta+\frac{1}{qg}\left(SWg^\alpha-SgW^\alpha\right)\bar{S}^\beta\right.\\
&\left.-\frac{Sg}{(qg)^2}g^\alpha\bar{S}^\beta-(q,S\Leftrightarrow\bar{q},\bar{S})\right]\varepsilon_{\alpha\mu\beta\nu},\\
m_f^2H^{0,Zf}_{V\mu\nu}&=m_f^2\left[-\frac{1}{2}W^2g_{\mu\nu}+\frac{g_\mu g_\nu}{(qg)(\bar{q}g)}+(q\Leftrightarrow\bar{q})\right],\\
m_f^2H^{S,Zf}_{V\mu\nu}&=im_f\left[W^2S^\alpha q^\beta-\frac{1}{qg}(SWq^\alpha-qWS^\alpha)g^\beta\right.\\
&\left.+\frac{1}{qg}(Sgq^\alpha-qgS^\alpha)\left(W^\beta-\frac{g^\beta}{\bar{q}g}\right)\right]\varepsilon_{\alpha\mu\beta\nu},\\
m_f^2H^{S\bar{S},Zf}_{V\mu\nu}&=-\left[\frac{1}{2}W^2S^\alpha q^\beta+\frac{1}{qg}\left\{(qWg^\beta-qgW^\beta)S^\alpha+(SWg^\alpha-SgW^\alpha)q^\beta\right\}\right]\bar{S}^\gamma\bar{q}^\delta\\
&\times\left(\varepsilon_{\alpha\beta\mu\varepsilon}\varepsilon_{\gamma\delta\nu}{}^{\varepsilon}+t_{\alpha\beta\mu\varepsilon}t_{\gamma\delta\nu}{}^{\varepsilon}\right)\\
&+\frac{1}{(qg)(\bar{q}g)}\left[g_\mu q^\gamma S^\delta g^\sigma-g_\mu{}^\gamma(qgS^\delta-Sgq^\delta)g^\sigma\right]\bar{S}^\alpha\bar{q}^\beta\\
&\times\left(\varepsilon_{\alpha\beta\nu\varepsilon}\varepsilon_{\sigma\gamma\delta}{}^{\varepsilon}+t_{\alpha\beta\nu\varepsilon}t_{\sigma\gamma\delta}{}^{\varepsilon}\right)\\
&+(q,S\Leftrightarrow\bar{q},\bar{S}).
\end{aligned}
\tag{A.3}
$$

APPENDIX B

It is sometimes useful to note the relation [7]

$$\gamma_\mu\gamma_\nu\gamma_\alpha=i\gamma_5\varepsilon_{\mu\nu\alpha\beta}\gamma^\beta+t_{\mu\nu\alpha\beta}\gamma^\beta, \tag{B.1}$$

with $\varepsilon_{\mu\nu\alpha\beta}$ the completely antisymmetric tensor with $\varepsilon_{0123}=1$ and

$$t_{\mu\nu\alpha\beta}=g_{\mu\nu}g_{\alpha\beta}-g_{\mu\alpha}g_{\nu\beta}+g_{\mu\beta}g_{\nu\alpha}.$$

With the help of relation (B.1) one can write down in closed form the trace of any number of γ's, the well known traces

$$\mathrm{Tr}\ \gamma_\mu\gamma_\nu\gamma_\alpha\gamma_\sigma=4t_{\mu\nu\alpha\sigma},$$

$$\text{Tr } \gamma_5\gamma_\mu\gamma_\nu\gamma_\alpha\gamma_\sigma = 4i\varepsilon_{\mu\nu\alpha\sigma}, \tag{B.2}$$

and the not so well known traces

$$\text{Tr } \gamma_\mu\gamma_\nu\gamma_\alpha\gamma_\omega\gamma_\gamma\gamma_\sigma = 4\left(\varepsilon_{\mu\nu\alpha\beta}\varepsilon_{\omega\gamma\sigma}{}^{\beta} + t_{\mu\nu\alpha\beta}t_{\omega\gamma\sigma}{}^{\beta}\right),$$

$$\text{Tr } \gamma_5\gamma_\mu\gamma_\nu\gamma_\alpha\gamma_\omega\gamma_\gamma\gamma_\sigma = 4i\left(\varepsilon_{\mu\nu\alpha\beta}t_{\omega\gamma\sigma}{}^{\beta} - t_{\mu\nu\alpha\beta}\varepsilon_{\omega\gamma\sigma}{}^{\beta}\right). \tag{B.3}$$

The similarity between $\varepsilon_{\mu\nu\alpha\beta}$ and $t_{\mu\nu\alpha\beta}$ is shown in the relations

$$\varepsilon_{\mu\alpha\nu\beta}\varepsilon_{\omega}{}^{\alpha}{}_{\sigma}{}^{\beta} = -2\left(g_{\mu\omega}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\omega}\right),$$

$$t_{\mu\alpha\nu\beta}t_{\omega}{}^{\alpha}{}_{\sigma}{}^{\beta} = 2\left(g_{\mu\omega}g_{\nu\sigma} + g_{\mu\sigma}g_{\nu\omega}\right), \tag{B.4}$$

and

$$t_{\mu\nu\alpha\beta}t_{\omega\gamma\sigma}{}^{\beta} = g_{\mu\nu}t_{\omega\gamma\sigma\alpha} - g_{\mu\alpha}t_{\omega\gamma\sigma\nu} + g_{\nu\alpha}t_{\omega\gamma\sigma\mu},$$

$$t_{\mu\nu\alpha\beta}\varepsilon_{\omega\gamma\sigma}{}^{\beta} = g_{\mu\nu}\varepsilon_{\omega\gamma\sigma\alpha} - g_{\mu\alpha}\varepsilon_{\omega\gamma\sigma\nu} + g_{\nu\alpha}\varepsilon_{\omega\gamma\sigma\mu}, \tag{B.5}$$

and

$$\varepsilon_{\mu\nu\alpha\beta}\varepsilon_{\omega\gamma\sigma}{}^{\beta} = g_{\mu\omega}t_{\nu\alpha\gamma\sigma} - g_{\mu\gamma}t_{\nu\alpha\omega\sigma} + g_{\mu\sigma}t_{\nu\alpha\omega\gamma} - g_{\nu\alpha}t_{\mu\omega\gamma\sigma}. \tag{B.6}$$

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