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Eigenfrequencies and Quality Factors of Vibration of Aluminium Alloy Spherical Resonators

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Abstract

We report on the first results of an investigation on the eigenfrequencies and quality factors of vibration of Al 5056 spherical resonators at very low temperatures. In particular the characteristics of the lowest order spheroidal quadrupole mode, relevant for gravitational wave research, were measured. The resonators were suspended at the nodal point. Quality factors up to 1.2 10^8 were measured in a bulk sample down to 44 mK. Another spherical resonator was obtained by means of explosively-bonded stacks and showed a maximum quality factor of 2 10^7. The observed quadrupole modes appear to be split into frequency multiplets, because of the broken spherical symmetry, as theoretically predicted. The independence of the modes was measured within one part in 10^5 in amplitude.

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1. Introduction

A resonant gravitational wave (gw) detector consists of a carefully suspended solid body responding to the oscillating field of the gw and of a sensor that perceives its vibrations. Following General Relativity, gw's interact with those vibrational modes of the solid body that show a nonzero quadrupole mass-moment [1,2]. A fundamental detection problem of resonant detectors is that of minimising the fluctuation ΔE_{th}, during the optimal integration time Δt, of the energy shared by the mode according to the partition law of thermodynamics. If we characterise each mode with an eigenfrequency ω and a quality factor Q, Q/ω is the characteristic time for energy exchange and ΔE_{th} can be written as:

$$\Delta E_{th} = \frac{kT\,\omega\Delta t}{Q} \tag{1}$$

All the gw experiments with resonant-mass detectors make use of solid bodies made of a low-loss material cooled to very low temperatures [3]. One of the best materials for resonant gw detectors is the aluminium alloy Al 5056 (5.2% Mg, 0.1% Mn, 0.1% Cr). Q values of the order of $4\ 10^7$ at low temperature in the acoustic range of frequencies and for various sample geometries have been reported [4-6]. This is the highest cryogenic Q of any material commercially available today in a multi-ton monolithic form. Most of the present cryogenic resonant detectors consist of massive cylidrical bars made of this material [3].

Reduction of the noise term (1) to below the quantum level, i.e. $\Delta E_n < \hbar\omega$, is considered one of the conditions necessary to detect gw events from thousands of galaxies and requires reducing the ratio T/Q to below 10^{-8}K. This technical achievement has been reached by the ultracryogenic bar detector NAUTILUS, cooled to below a temperature of 0.1 K [7].

At present there is increasing interest in the design of large spherically shaped gw detectors. In a cylindrical bar, only one vibrational mode (the first longitudinal one) interacts strongly with the gw and the resulting gw cross section is sharply peaked in the direction perpendicular to the cylinder axis. Spherical gw detectors appear very promising as the next step in gw research with resonant detectors because the lowest order, five-fold degenerate, quadrupole mode has a large and omnidirectional cross section and allows the source direction and the wave polarisation to be determined [8]. The characteristics and potentialities of this new detector are under intense study [9-16].

In order to design a large spherical detector (several meters in diameters) two important points must be investigated:

i) *The quality factor of the vibrational modes of a solid sphere at very low temperatures.* Previous measurements have in fact investigated the Q of flexural modes of disks [7], longitudinal modes of bars [4,6] and torsional modes of resonators [17].

ii) *The fabrication technique to produce a large sample while preserving the high inherent Q of the material.* It has been proposed [18] to use an explosive welding method to produce large-diameter spheres out of explosively-bonded plates. This technique produces high-strength bonds between similar and dissimilar metals and appears capable of preserving the inherent attributes of each parent metal [19].

In this paper we present the first results of an experimental study on these two points.

2. Eigenmodes and eigenfrequencies of an elastic sphere

Let us summarise the basic equations for the eigenfunctions and eigenfrequencies of a perfectly homogeneous, isotropic sphere of radius R, made of a material having density ρ and Lamé coefficients λ and μ. The relative eigenfunctions were found by Jaerisch [20] and Lamb [21] in the last century and summarised by Love in his treatise in 1927 [22]. Elegant derivations, using modern notation, were reported by Ashby and Dreitlein [23], Wagoner and Paik [8], Zhou and Michelson [11] and Lobo [14]. These authors considered the interaction of a free sphere with the gw field.

A vibrating sphere has two classes of normal modes: toroidal modes, for which there are no volume changes and no radial displacements; and spheroidal modes, which consist of both transverse and radial components.

The eigenfunctions of the toroidal modes have the form:

$$\Psi_{lm}^{(t)} = C\psi_l(\kappa r)(\vec{r} \times \nabla Y_{lm}) \tag{2}$$

where C is the normalised amplitude, Y_{lm} are spherical harmonics and $\kappa^2 = \rho\omega_l^2/\mu$. $\psi_l(x)$ is a function given by

$$\psi_l(x) = \left(\frac{1}{x}\frac{d}{dx}\right)^l \left(\frac{\sin x}{x}\right) \tag{3}$$

The toroidal mode frequency is determined by the equation

$$(l-1)\psi_l(\kappa R) + \kappa R\psi_l'(\kappa R) = 0 \tag{4}$$

The spheroidal modes can be expressed as

$$\Psi_{lm}^{(s)} = [a_l(r)\vec{e}_r + b_l(r)R\nabla]Y_{lm}(\theta,\varphi) \tag{5}$$

where R is the radius of the sphere, and $a_l(r)$ and $b_l(r)$ are dimensionless radial eigenfunctions determined by the boundary conditions.

The spheroidal mode frequency is given by

$$u_l z_l - v_l \zeta_l = 0 \tag{6}$$

where

$$u_l = \frac{1}{(2l+1)h^2}\left[\kappa^2 R^2 \psi_l(hR) + 2(l-1)\psi_{l-1}(hR)\right] \tag{7}$$

$$v_l = -\frac{1}{(2l+1)}\left[\frac{\kappa^2}{h^2}\psi_l(hR) + \frac{2(l+2)}{hR}\psi'_{l-1}(hR)\right] \tag{8}$$

$$\zeta_l = \kappa^2 R^2 \psi_l(\kappa R) + 2(l-1)\psi_{l-1}(\kappa R) \tag{9}$$

$$z_l = \kappa^2 \frac{1}{l+1}\left[\psi_l(\kappa R) + \frac{2(l+2)}{\kappa R}\psi'_l(\kappa R)\right] \tag{10}$$

with $h^2 = \omega^2\rho/(\lambda + 2\mu)$.

It was shown that only the spheroidal modes with l=2 (quadrupole modes) interact with a general relativistic gw. [23,8]. They are five-fold degenerate and described by the spherical harmonics Y_{2m} with m=+2,+1,0,-1,-2.

The normal mode frequencies can be determined numerically for both toroidal and spheroidal vibrations. Each mode of order l is 2l+1 degenerate. In Table I we show the value of the κR roots for the lowest toroidal and spheroidal modes of vibration. The eigenfrequency values can be obtained by :

$$\omega_{nl} = \sqrt{\frac{\mu}{\rho}}\frac{(\kappa R)_{nl}}{R} \tag{11}$$

Table I – Values of the (kR) roots for the lowest spheroidal and toroidal modes of vibration.

l	n	$(kR)_{toroidal}$	$(kR)_{spheroidal}$
0	1	-	5.4322
	2	-	12.138
	3	-	18.492
	4	-	24.785
1	1	5.7635	3.5895
	2	9.0950	7.2306
	3	12.323	8.4906
	4	15.515	10.728
2	1	2.5011	2.6497
	2	7.1360	5.0878
	3	10.515	8.6168
	4	13.772	10.917
3	1	3.8647	3.9489
	2	8.4449	6.6959
	3	11.882	9.9720
	4	15.175	12.900
4	1	5.0946	5.0662
	2	9.7125	8.2994
	3	13.211	11.324
	4	16.544	14.467

3. The samples

The Al 5056 sampaes were obtained by a commercial extruded bar 300 mm diameter, without heat treatments, manufactured by Furumoto Kikoh Co.

Two 153 mm diameter Al 5056 spherical samples were machined, the first with a computer-controlled milling machine and the second with a computer controlled lathe. Both were machined to a sphericity better than 20 μm. No post machining surface cleaning or hetching was perfmed. The quality factor of the first sample was measured with two different suspensions. We will indicate these runs as referring to samples A and B. The second sphere (sample C) was machined from a block obtained by explosion welding 30 mm thick plates of the same parent alloy as for the first sample.

Sample A was suspended from the center using a 3.8 mm diameter, 160 mm long copper rod (ordinary non-annealed copper). The rod ended on the sphere side with a 10 mm long M5 thread (Fig 1-a), and on the other side, with a 25 mm long M12 screw which was bolted to the mixing chamber of a dilution refrigerator at the Kamerlingh Onnes Laboratory. A 8 mm diameter hole was drilled down to 5 mm above the center of the sphere and then the M5 thread was tapped to a depth of 10 mm. The last two threads of the copper rod on the sphere side were removed and the end was machined at 120 degrees, to match the hole. This suspension was chosen in an effort to provide good thermal contact since screwing the rod tightly provides a high pressure at the contact point and also breaks the Al oxide layer (the rod was silver-plated). A temperature of 44 mK was obtained in this way.

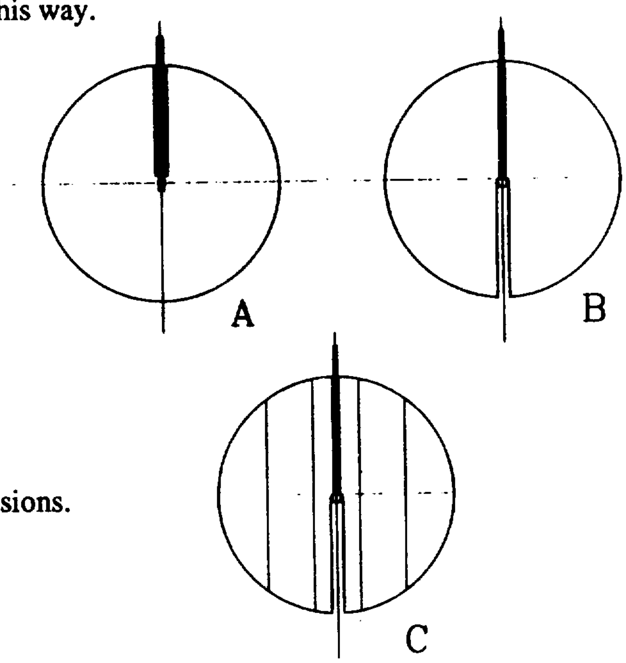

Figure 1 – Schematic drawing of suspensions.

Sample B and C were suspended by hanging them from a 2 mm annealed copper wire 160 mm long. A 6 mm diameter cylinder, 5 mm long, with one face tapered at 120 degrees, was welded on the sphere side and an M12 copper screw was welded on the refrigerator side. Both ends were silver-plated. The 8 mm diameter hole was extended to the center of the sphere and milled conic. A 4 mm diameter hole was machined on the other side of the sphere. The center of the suspension hole in sample C was symmetric respect to the two nearest band zones (~ 15 mm distance).With this suspension we gave up on good thermal contact and tried to decouple the spheres as much as possible from the suspension. These samples were cooled to about 100 mK, measured as sample A.

The suspension is the most delicate point in such experiments, because it can have a definite effect on the Q of the sample. We checked that the transversal and longitudinal eigenfrequencies of the suspension rods were far from the lowest eigenfrequencies of the spherical sample, in order to minimize the loss of the acoustic energy through the suspension.

We used as electromechanical transducers piezoelectric ceramics (PZT's) glued on small flats filed on the surface of the spheres. With sample A we used three PZT transducers. Two were round, 8 mm diameter, 1 mm thick, with silver electrodes on both faces. They were placed at 45 and 90 degrees from the suspension axis. We glued them to the sphere using silver epoxy cured overnight in an oven at 80°C. The third PZT was rectangular (3.5 x 9 x 0.3 mm) provided with the two electrodes on the upper face [25] and glued to the sphere as the round

PZT. Two 30 μm copper wires were soldered to the electrodes of each PZT and connected to a coaxial cable.

In samples B and C (and in several experiments with different alloys which are being carried out at the Kamerlingh Onnes Lab.) we used small PZT's (typically 3-4 mm^2) obtained by cutting PZT's similar to the rectangular one. In this way the mass of PZT's addenda to the sample, which could decrease its high inherent Q value, is minimised. The small PZT's were glued to the sphere using a small amount of cyanocrylate glue (instant glue) and connected to the coax cables by means of 30 μm copper wires. The ground potential is provided by the sphere via the suspension. The glue is just another capacitance in series with the ceramics, to the ground. An advantage of this technique is that the transducers can be removed by dipping them in acetone for a few hours. We could always excite and detect all resonances using two transducers placed roughly at 45 degrees from the suspension axis (on the upper hemisphere) and not too close to each other.

In order to easily identify the resonances from room temperature to 4.2 K we used a small hammer activated by a superconducting coil. A simple way of distinguishing the spheroidal modes from the toroidal modes (particularly for $l>2$ where mixing of both modes occurs) is to use some exchange gas at nitrogen or helium temperatures. The Q of the toroidal modes turned out to be insensitive to the gas since these modes have no radial displacements and hence they are not coupled to the sound modes in the gas, while the Q of the spheroidal modes are strongly affected by the gas already at a pressure of 0.1 mbar. Below 4 K we excited the resonances one by one by applying a voltage at the correct frequency to one of the transducers by means of a synthesized signal generator. The signal amplitude was detected using a low noise preamplifier and a spectrum analyser. The maximum of the resonance peak was fed to a computer and the decay time τ was extracted from a fit to an exponential decay curve. The Q was calculated from $Q=\omega\tau/2$.

The temperature was measured with a calibrated RuO_2 thin film SMD (surface mounted device) resistor, Philips 2001 Ω, glued on the surface of the sphere, with the film facing the sphere. The size of the thermometer is comparable to that of the PZT's, about 3x1.5x0.5 mm. Two 50 μm NbTi wires in CuNi matrix were used for electrical contacts between the thermometer and the measuring coaxial cables. A miniature capacitor (Philips 3.3 nF SMD capacitor) was connected in parallel with the thermometer, close to the two coaxial cables, and two additional capacitors were connected to ground (one for each cable). These capacitors have a value which strongly decreases with temperature, to about 400 pF at 4.2K and can therefore conveniently be used as thermometers between room temperature and 1 K.

The pressure of the exchange gas in the sample vacuum space was always lower than 1.5 10^{-6} mbar during the Q values measurements.

4. Experimental results

We measured the characteristics of the $l=2$ $n=1$ spheroidal modes and of the $l=2$, $n=1$ toroidal modes of the three samples. We will indicate with t the toroidal and with s the spheroidal modes, both five-fold degenerate. The s mode have the largest cross section to gw's [8,11,13,14] while the t mode do not interact with them [11,14] and could be used as a veto.

The s mode is located 6% higher in frequency than the t mode so they are easy to distinguish even without the exchange gas test.

At low temperatures, the eigenfrequencies values depend very weakly on temperature, as reported for Al 5056 samples in previous works [4-6]. The reported values refer to temperatures $T \leq 4K$.

Finite elements analysis of a sphere with the symmetry broken by an axial hole, like the one of samples B and C, shows [26] that the toroidal l=2 frequency splits into a doublet, a singlet and a doublet, while the spheroidal mode frequency splits into two doublets and a singlet. The singlet resonance frequencies are roughly equal to the theoretical degenerate values (see Table II). The s mode frequencies for samples A, B and C are shown in Fig. 2. The

Table II – Features of the Al 5056 spherical samples.

ρ (kg/m^3)	λ (N/m^2)	μ (N/m^2)	R (m)	ν_t (Hz)	ν_s (Hz)
2.7 10^3	6.3 10^{10}	3.0 10^{10}	7.65 10^{-2}	17484	18523

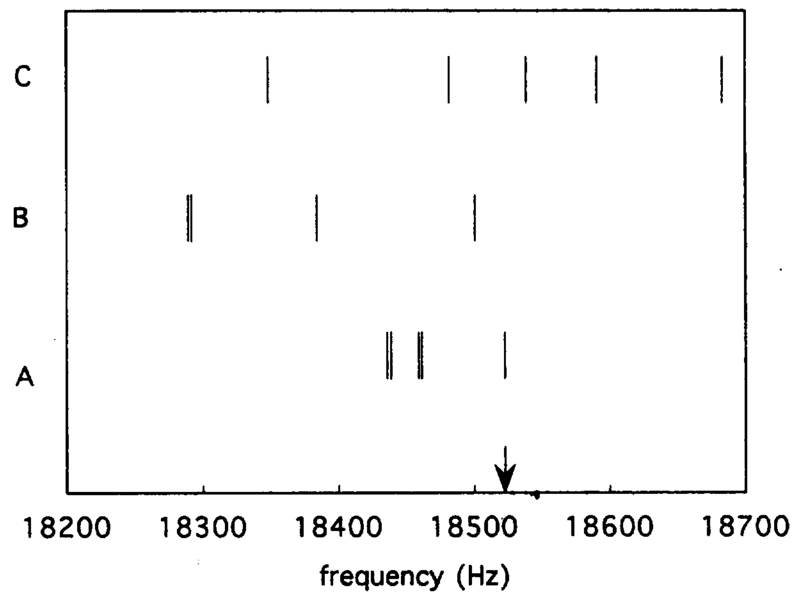

Figure 2 – Eigenfrequencies of the lowest quadrupole mode of the three spherical resonators. The frequency doublets of samples A and B are split in sample C. The arrow shows the spheroidal frequency calculated for a sphere of 153 mm diameter, with ρ = 2700 kg/m^3 and μ = 3.05 10^{10} N/m^2.

splitting is smaller for sample A which has only one hole (8 mm diameter) extending to about half the diameter and is larger for sample B, which has also another hole (4 mm diameter) through the other half. In the explosion-welded sample the degeneracy is fully lifted, indicating that the different welded interfaces (nearly parallel to the suspension) do affect the symmetry of the sphere. This effect is also seen in the t modes (Fig. 3). The splitting of the modes will probably be a useful tool to evaluate the (density) inhomogeneities of the spherical samples.

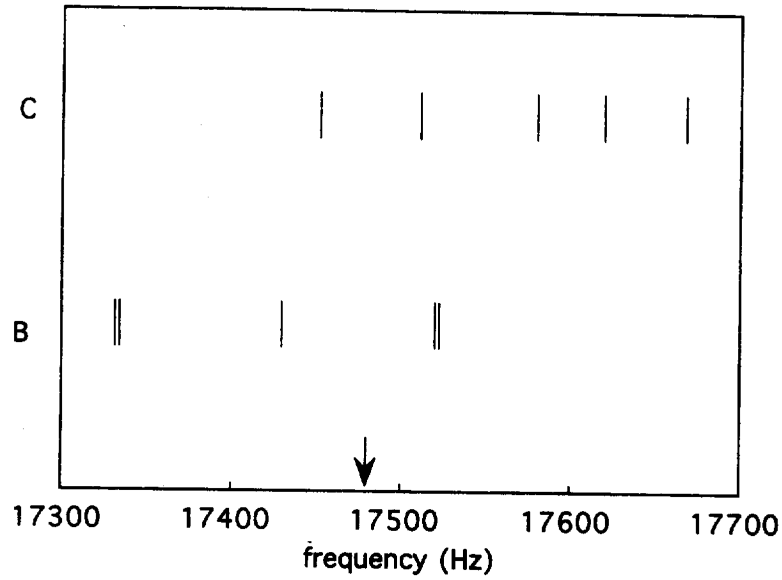

Figure 3 – Doublet splitting as observed for the toroidal mode.

The independence of the modes in the multiplet was checked within one part in 10^5, by exciting with a voltage not exceeding 50-100 mVpp a transducer for a long enough time at one of the resonance frequencies and observing that none of the neighboring frequencies was excited.

The Q values of the lower quadrupole modes are shown in Figs. 4 to Fig. 8. In Fig. 4 we

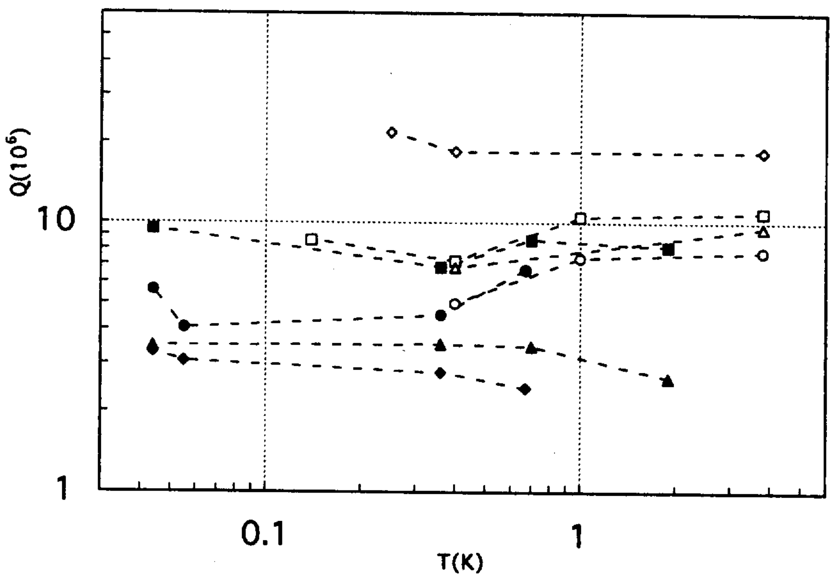

Figure 4 – Q-values of the spheroidal quadrupole mode of samples A (■ = 18436.53 Hz, ▲ = 18437.89 Hz, ● = 18459.76 Hz, ◆ = 18460.28 Hz) and sample B (○ =18289.61 Hz, □ = 18290.62 Hz, ◊ = 18383.99 Hz, Δ = 18500.36 Hz).

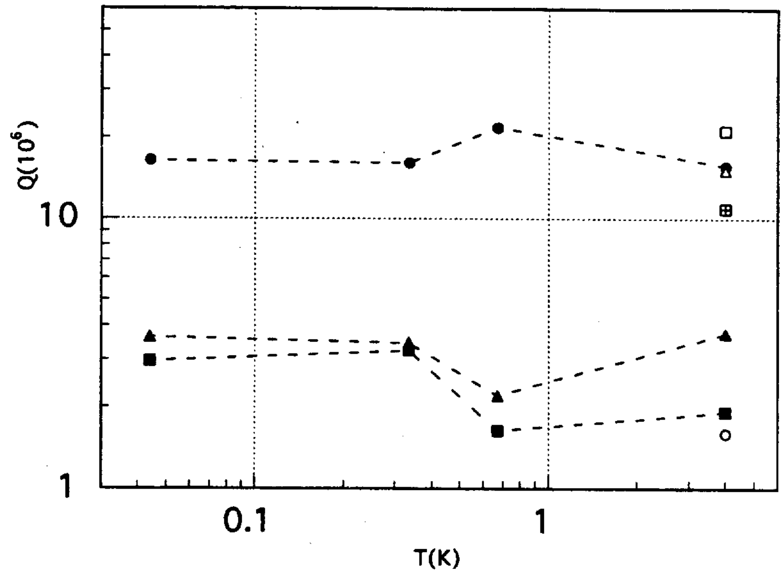

Figure 5 – Q-values of the toroidal mode of samples A (● = 17403.32 Hz, ■ = 17404.20 Hz, ▲ = 17506.04 Hz) and sample B (O = 17331.53 Hz, Δ = 17332.54 Hz, ❑ = 17429.39 Hz, ⊞ = 17520.20 Hz).

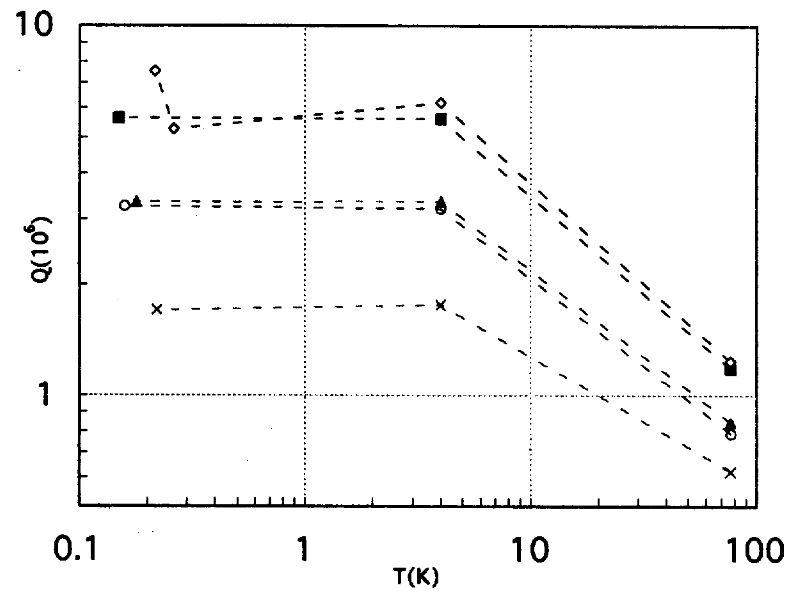

Figure 6 – Q-values of the spheroidal quadrupole mode of sample C, obtained by explosive welding (O = 18348.20 Hz, ■ = 18481.81 Hz, ▲ = 18538.57 Hz, ◊ =18590.45 Hz, X = 18683.10 Hz).

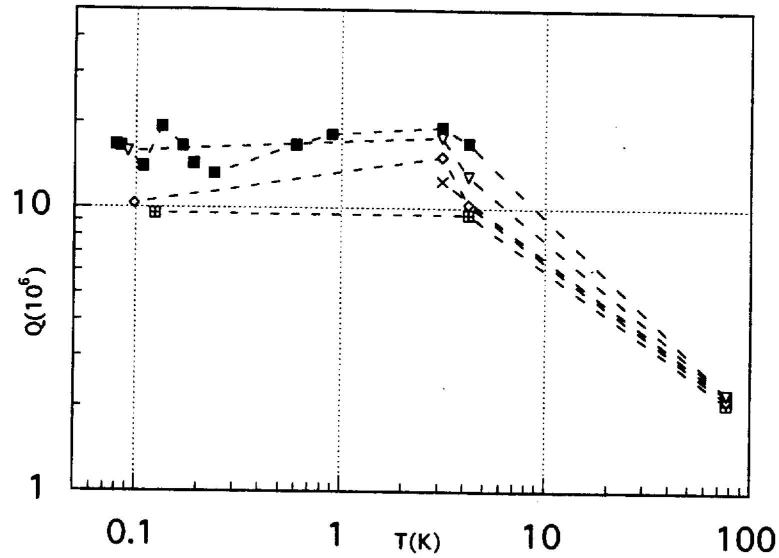

Figure 7 – Q-values of the toroidal mode of sample C (⊞ = 17451.90 Hz, ■ = 17511.11 Hz, X = 17580.20 Hz, ◊ = 17619.60 Hz, ∇ = 17667.96 Hz).

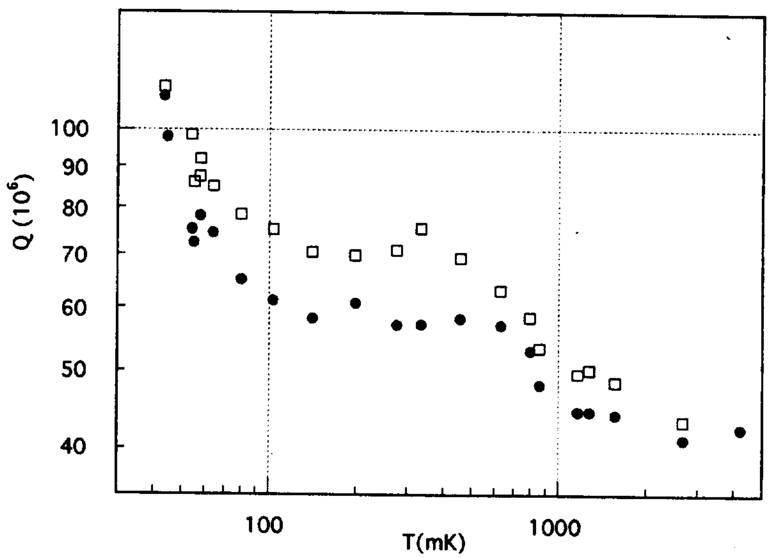

Figure 8 – Q-values of two higher order modes (● = 35354 Hz, ❑ = 35359 Hz) of sample A down to very low temperature.

see that the Q values of sample A are somewhat lower than those of sample B. This may be due to the different amount and position of the PZT's coupled to the resonator and/or to the different suspension. Fig. 5 shows the Q values of some toroidal modes of sample A and sample B (this last measured only at 4 K). The Q values of the spheroidal quadrupole modes of sample C are shown in Fig. 6. This sample gave slightly lower values than sample B although it had the same suspension. The Q values of the t modes of sample C are shown in Fig. 7. All have rather high values, between 10 and 20 million, with no temperature variation between 4 K and 80 mK.

We found a doublet resonance in sample A, at about 35 kHz, which has a very high Q (Fig. 8). It belongs quite probably to the 9 resonances of the l=4 n=1 toroidal mode (the identification is not trivial since these frequencies are very close to those of the l=2 n=2 spheroidal mode). At 44 mK both frequencies displayed Q values as high as 120 million which are the highest published values for this alloy [27]. The steep increase with temperature certainly indicates that even higher values are possible at lower temperatures. We have measured the same doublet in sample B and found similar values with a maximum of 80 million at 100 mK, the lowest temperature achieved in that sample. No indication of such high values was seen in sample C for frequencies up to 50 kHz; the highest Q value around 35 kHz was 13.5 million at 265 mK.

5. Conclusion

The result of our measurements is that Al 5056 spherical resonators have independent modes of vibration with high Q, as necessary for gw research.

We performed the low temperature Q measurements both in a bulk sample and in a explosion welded sample. Although both samples have shown reasonably high Q values, the bulk sphere reached higher values than the explosion-welded one. The l=2 n=1 spheroidal mode Q's in particular, which are the ones we are aiming for in gw detection, never reached values above 8 million as compared to 20 million for the bulk sphere with similar suspension. We observed that the lowest l=2 modes are split in 5 frequencies, following the structure predicted by the theory. They are easily identifiable and are independent of each other within 1 part in 10^5. We measured a very high Q of 1.2×10^8 for the bulk sphere. We think that the possible effect of the suspension on the measured Q values, particularly for the spheroidal modes, requires accurate finite element calculations in order to find the best suspension for a spherical gw detector.

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