A $\mu^+ \mu^-$ Quantum Collider Using Novel Crystal-Based Accelerator Components*

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Abstract

We outline a concept of a 250 × 250 GeV $\mu^+ \mu^-$ collider that uses bent crystals for beam confinement and steering instead of conventional magnets. The collider ring is based on a novel bending–focusing crystal cell. Beam optics of the proposed model cell has all the features of the alteranting gradient FODO cell. Furthermore, alternating (horizontal-vertical) focusing provides unique betatron phase stability in both planes, while bending of particle trajectories due to crystal curvature is fully achromatic. We also explore the ionization energy loss of channeling muons interacting with the electron gas in a crystal channel as a possible cooling mechanism. Finally, a use of low Z binary crystals (such as LiF) for final focus at the interaction point is proposed. Bringing the $\mu^+ \mu^-$ into collision inside a crystal channel results in quantum confinement at the collision point. For such a collider the number of required $\mu^\pm$ may be very low ($\sim 10^8$ μ per pulse), so that this collider has little problems from $\mu \rightarrow e$ backgrounds or heating, which is a virtue of paramount importance. A low intensity hadronic $\mu^\pm$ source ($p + A \rightarrow \pi \rightarrow \mu$) can be utilized providing that an effective method of fast muon cooling is used. For example, the use of frictional cooling for low energy $\mu^\pm$ beams could initially reduce the longitudinal phase-space, before the final transverse cooling is applied. Here, we outline such a 'crystal cooler' that explores ionization energy loss in the ultra-strong focusing environment of a crystal channel. Employing all above mentioned novel crystal-based accelerator components, a possible luminosity of about 10^{32} cm^{-2} sec^{-1}, is estimated for the proposed quantum collider. We also discuss a list of problems one needs to solve in order to make such a collider a real possibility.

1. Introduction

The future and vitality of elementary particle physics requires ever innovative instruments. Recent experiments in the U.S., Europe and Russia have shown impressive progress in high-efficiency steering of charged particle beams by means of the bent-crystal channeling. The scope of these experimental studies has been focused on variety of possible applications of bent crystal components for high energy accelerators – to mention just a few: parasitic beam extraction from a multi-TeV machine[1,2], spin precession of short-lived particles[3], large angle beam bending required in a search for long-base neutrino oscillations[4], and others.

The development of a novel collider that uses $\mu^+ \mu^-$ collisions promises to give a new window for the study of elementary particles[4,5,6]. This collider requires very cold $\mu^+ \mu^-$ beams and the study of muon cooling is just starting[7]. The physics opportunities are abundant[8]. To list just a few: (1) $\mu^+ \mu^- \rightarrow t\bar{t}$. (2) Direct s-channel $\mu^+ \mu^- \rightarrow h^0$. Coupling to the Higgs is 4×10^4 times greater for muons than electrons (it scales as mass square). (3) Polarized beams may uncover non-left-handed supersymmetric effects. (4) Probing the 10 TeV/c mass region.

Vigorously discussed over the last few years muon collider option, although promising great new physics opportunities is full of serious challenges, which may not be solved by conventional accelerator components stretched to the limit of projected state-of-the-art technology. Going beyond conventional solutions and accelerator techniques a futuristic muon collider scheme that uses bent crystals, rather than magnets for the circular confinement is proposed here. The conceptual basis for this collider is that muon beams are not destroyed by either radiation like electrons and positrons or hadronic interactions like protons and antiprotons, thus allowing for a full crystal-based collider. The number of revolutions goes like $300 \times B$, where B is the bending field. As we discuss later in the paper, for a bent Si crystal B is of the order of 2000 Tesla, giving 600,000 turns in a storage ring before the next muon beam is injected. The muon life-time is doubled with the use of a Ge crystal, and it will be increased by one order of magnitude if one uses a Tungsten crystal instead.

We outline here a conceptual scheme for a 250 × 250 GeV muon collider complex. It involves fast muon cooling via ionization energy loss in a novel bent crystal cooling ring as well as quantum confinement[9] of colliding $\mu^{\pm}$ beams inside a low Z binary crystal. A binary crystal provides a unique configuration of 'potential wells', which allows for accommodation of both $\mu^{\pm}$ species in a crystal. Thermal non-equilibrium or metastable equilibrium states induced by the lattice vibrations (finite temperature) could be employed as a 'mixing' mechanism[13], which may bring both muon species to a collision in a crystal channel.

Starting with an initially 'cool' muon beam (coming from a conventional hadronic source $p + A \rightarrow \pi \rightarrow \mu$) one can further reduce the longitudinal phase-space for low energy $\mu^{\pm}$ beams, e.g. by means of the frictional cooling[11,12], before the final transverse cooling is applied. Our model calculation, presented here, shows that one can decrease emittance to less than $\varepsilon_N = 10^{-8}$ m rad by passing the muon beam through a cascade of many cooling modules. Experimental demonstration of fast ionization crystal cooling is under way; e.g. E763 (muon cooling and acceleration experiment) at TRIUMF[7]. Dominant heating process (due to multiple scattering on electron gas inside a crystal channel) has also been taken into account. Derived transverse emittance 'cooling equation' shows[6] that the minimum achievable emittance (equilibrium cooling limit) is of the order of $\varepsilon_N^{min} = 10^{-9}$ m rad, while the characteristic transverse emittance damping length is about 62.5 m.

Practical demonstration of the transverse cooling in a focusing crystal channel and verification of predicted here quantun confinement luminosity enhancement in the 10^{32} cm^{-2} sec^{-1} range, are the first crucial steps towards making this novel $\mu^{+} \mu^{-}$ collider a real possibility[4].

2. Bent Crystal Channeling

We explore unique properties of relativistic channeling of charged particles in a bent crystal[1] as a technique for particle beam steering[2]. Particularly we are interested in the circular confinement of muon beams in a crystal with a strain imposed curvature[3] as a possible functional element of a high energy storage ring.

The beam dynamics of charged particles channeling through a straight (no bending) crystal corresponds to a transverse harmonic oscillator moving relativistically in the longitudinal direction – crystal channel plays the role a strongly focusing transfer line characterized by the beta function, β, (or alternatively by the betatron wavelength, λ) both expressed by the following formula

$$\beta = \frac{\lambda}{2\pi} = \sqrt{\frac{E_\mu}{\phi}} \quad . \tag{2.1}$$

The focusing strength of the crystal channel for [110] planar channeling in Silicon has been experimentally measured[3] and has a value of $\phi = 6 \times 10^{12}$ GeV m^{-2}. Assuming 25 GeV muons channeling in Silicon the corresponding beta function has a very small value: $\beta = 2 \times 10^{-6}$ m.

We consider motion of planar channeled particles in a crystal, which is bent elastically in a direction perpendicular to the particle velocity and to the channeling planes. The effect of bending introduces a centripetal force to the equation of transverse motion[3] (by adding a linear piece to the crystal potential), which is equivalent to lowering one side of the continuum potential well and raising the other. The equilibrium planar trajectory moves away from the midpoint of the planar channel toward the plane on the convex side of the curved planar channel. Although, such shift would cause some fraction of the channeled particles to leave the potential well (dechannel)[2]. The curvature at which no particle can remain channeled is reached when the equilibrium point of planar channeled motion is shifted to the position of the planar wall on the outside of the curve. This critical radius of curvature, known as the Tsyganov radius[14], ρ_T

$$\rho_T = \frac{2E_\mu}{\phi a} . \tag{2.2}$$

Using simple formula linking equivalent magnetic bending field, B, with the trajectory's curvature, ρ, namely

$$B[\mathrm{Tesla}] \times \rho_T[\mathrm{m}] = 3.34 \times E_\mu[\mathrm{GeV}] , \tag{2.3}$$

one can calculate the maximum available equivalent bending field corresponding to the Tsyganov curvature. From Eqs.(2.1)–(2.3) this field is given by

$$B_T[\mathrm{Tesla}] = 3.34 \times \frac{1}{2} \phi \ a \ . \tag{2.4}$$

Its numerical value for Silicon is evaluated as follows: $B_T = 2 \times 10^3$ Tesla. We notice in passing, that the maximum bending field is energy independent.

Assuming 250 GeV muons channeling through a 8 cm-long section of a Silicon crystal the maximum bending angle, θ_T, derived from Eq.(2.2), is equal to about 2×10^{-1} rad. This value of the critical bending angle will be used later in the paper.

3. Bent Crystal Collider Ring

Here we employ previously discussed properties of the planar channeling of high energy muons in Silicon to design components of a storage ring. Particularly, we are interested in a section of bent crystal followed by two straight pieces providing alternating horizontal – vertical focusing. A basic guiding cell is depicted schematically in Figure 1. One can notice, that the induced configuration of guiding fields in this element is equivalent to a powerful alternating gradient achromat.

Relativistic muons channeling through a Si crystal are confined between two neighboring atomic planes – they experience strong focusing electrostatic crystal-potential in the direction perpendicular to these planes, while there is virtually no confinement in the direction parallel to the planes (no focusing or defocusing). The focusing gradient, $k = 1/\beta$, equivalent to the magnetic quadrupole strength, k_1, (magnetic gradient)

$$k_1 = \frac{1}{B\rho}\frac{\partial B_y}{\partial x} \quad , \tag{3.1}$$

which is given by the following formula

$$k = \frac{\phi}{E_\mu} \quad , \tag{3.2}$$

where E_μ is the total muon energy and $\phi = 6 \times 10^{12}$ GeV m^{-2} is a material constant, related to the curvature of the potential well[17]. Assuming 250 GeV muons, crystal focusing gradient, k, yields an enormous value of 18 m^{-2} exceeding conventional quadrupole strength by three or four orders of magnitude.

As discussed in Section 2, one can bend the crystal slightly, so that channeling muons follow the curvature of the guiding field, which results in bending of muon trajectories similar to the effect of a bending magnetic field. Projecting experimental results for proton channeling in a bent Silicon crystal, one can assume that 250 GeV muons channeling through a 8 cm-long crystal should follow (without significant dechanneling effects) a bend of $\theta = 2\pi \times 10^{-2}$ rad (compare with the critical bending angle $\theta_T = 2 \times 10^{-1}$ rad). The lattice design presented here is based on these two numbers, k and θ.

Figure 1 illustrates a functional bending – focusing cell, where alternating sections of the horizontal and vertical continuous focusing channels are combined with sections of horizontally bent Si crystals. The following sequence of crystal elements: a horizontally focusing bent crystal (8 cm-long) – a short drift space (1 cm-long) – a vertically focusing straight crystal (10 cm-long) – a short (5 cm-long) horizontally focusing straight crystal – another vertically focusing straight crystal (10 cm-long) – a short drift space (1 cm-long) – another horizontally focusing bent crystal (8 cm-long) – finally, a short drift space (1 cm-long) completes the proposed elementary cell. At 250 GeV, one could close the entire collider ring using 50 of the above $F_hF_vF_hOOF_hF_vF_hO$ cells. This ultra compact collider ring would have in a circumference of 22 meters!

For a sequence of the above described cells one can find a periodic betatron trajectories in both the horizontal and vertical planes – the betatron phase stability is provided by the proposed lattice configuration (alternating horizontal/vertical focusing). By virtue of [110] planar channeling, discussed in detail in the previous section, a crystal channel provides ultra strong electrostatic focusing gradient in [110] direction with practically no confinement or defocusing in the plane perpendicular to [110] direction. This fact guarantees both local and global decoupling[2] of the horizontal and vertical betatron motions for the proposed collider lattice.

Calculating the TWISS functions for the above cell architecture, one can see from Figure 2, that the horizontal phase advances roughly as fast as the vertical one, with the vertical dispersion identically equal to zero. All lattice characteristics, collected in Table 1, are well behaved, which guaranties stable linear motion. The question of the closed orbit 'jitter' due to multiple entries (re-injection) to a focusing crystal channel should be addressed, when a realistic model of the boundary effects is constructed. Further corrections of the closed orbit imperfections could be made using conventional sextupole elements, e.g., natural chromaticity compensation, dynamic aperture etc.

4. Ionization Cooling in a Crystal Channel

Here we propose a fast muon cooling scheme based on the ionization energy loss[15] experienced by high energy muons (25 GeV) channeling through a Silicon crystal. Applying classical theory of ionization energy loss[16], a relativistic (γ) charged particle passing through a Silicon crystal of length, ΔL, looses total energy of , $\Delta E[\mathrm{MeV}] = 4 \times 10^2 \times \Delta L[\mathrm{m}]$. One can introduce a characteristic damping length, Λ

$$\frac{1}{\Lambda} = \frac{1}{E_\mu} \frac{\Delta E}{\Delta L}, \tag{4.1}$$

over which particle looses all its energy. Relativistic muons passing though the crystal loose energy uniformly in both the transverse and longitudinal directions according to Eq.(4.1). After passing through a short section of a crystal ($\Delta L << \Lambda$) muons are re-accelerated longitudinally to compensate for the lost longitudinal energy. This leads to the transverse emittance shrinkage.

Introducing normalized transverse emittance, $\varepsilon_N = \gamma\, \sigma_x\, \sigma_{x'}$, one can write down the normalized emittance budget in form of the following cooling/heating equation

$$\frac{d\varepsilon_N}{dL} = -\frac{\varepsilon_N}{\Lambda} + \left(\frac{\Delta\varepsilon_N}{\Delta L}\right)_{\mathrm{scatt}} . \tag{4.2}$$

The last term in the above equation accounts for transverse heating processes contributing to the beam divergence increase according to the following relationship[13]

$$\left(\frac{\Delta\varepsilon_N}{\Delta L}\right)_{\mathrm{scatt}} = \frac{1}{2}\gamma\,\beta\, \frac{\Delta\langle\theta\rangle^2_{\mathrm{scatt}}}{\Delta L} , \tag{4.3}$$

Here β is the beta function of a focusing crystal channel, which has enormously small value ($\beta = 2 \times 10^{-6}$ m, for 25 GeV muons channeling through a Silicon crystal).

For muon channeling in a dielectric crystal the dominant scattering process comes from elastic (Rutherford) muon scattering off the conduction electrons present in the channel. One can integrate the Rutherford cross section over the solid angle, which yields the following formula

$$\alpha = \left(\frac{\Delta\varepsilon_N}{\Delta L}\right)_{\text{scatt}} = 40\,\pi\, n\, \frac{r_\mu^{\,2}}{\gamma}\, \beta \ , \tag{4.4}$$

Here, $r_\mu = 1.4 \times 10^{-17}$ m, is the classical muon radius and $n = 6 \times 10^{30}$ m^{-3} is the concentration of the conduction electron gas in Silicon crystal.

Integrating the cooling equation, Eq.(4.2), one obtains the following compact solution in terms of the normalized transverse emittance evolution

$$\varepsilon_N = \varepsilon_N^{\,0}\; e^{-\frac{L}{\Lambda}} + \Lambda\alpha\,(1 - e^{-\frac{L}{\Lambda}}) \ , \tag{4.5}$$

The last term in Eq.(4.5) sets the equilibrium cooling limit of

$$\varepsilon_N^{\,\min} = \Lambda\alpha \quad , \quad L \rightarrow \infty \tag{4.6}$$

Assuming 25 GeV muons one gets: $\Lambda = 62.5$ m and the equilibrium limit of the normalized emittance of

$$\varepsilon_N^{\,\min} = 2.5 \times 10^{-9} \text{ m rad} \ . \tag{4.7}$$

This value of the normalized emittance will be used in our achievable luminosity estimate.

Practical realization of muon cooling at 25 GeV could be done in a bent crystal based cooling ring, similar to the collider ring described in the previous section. One could scale down the proposed bending – alternating focusing cell architecture (see Section 3) to ten times lower energy, by increasing the bend angle by the factor of ten ($\theta = 2\pi \times 10^{-1}$ rad). This way only five of the functional cells would be needed to complete the cooling ring. Its effective circumference would be equivalent to 2.2 meters of Silicon crystal. Assuming characteristic damping lenghth, Λ, of 62.5 meters, the energy loss per turn (suffered by the muon beam) is equal to 880 MeV. In principle, a conventional high gradient (30 MeV/m) acceleration inserts (6 meter long rf in every straight section of the ring) could be used to replenish the suffered energy loss (5×6 m $\times$ 30 MeV/m = 880 MeV). The proposed cooling ring of five fold symmetry is illustrated schematically in Figure 3. It has a nominal circumference of 32.2 meters!

Our goal is to start with the initial muon phase-space of the normalized emittance of 2.5×10^{-7} m rad and cool it down to the final emittance of 2.5×10^{-9} m rad. One can see from Eq.(4.7) that to achieve this goal muons have to pass through the total Silicon crystal length of

$$L = 2 \log 10 \times \Lambda = 280 \text{ m} \quad . \tag{4.8}$$

In the proposed cooling cell architecture the total cooling medium (Silicon) length of L = 280 m is equivalent to about 130 turns of the beam circulation in the ring. The lost energy is replenished every $\Delta L = 2.2$ m, which satisfies the adiabatic re-acceleration condition ($\Delta L \ll \Lambda = 62.5$ m).

To go beyond the above simple analytic calculation, we are planning to carry out a realistic computer simulations of planar channeling in bent crystals. One should tracks a charged particle through the distorted crystal lattice with the use of a realistic continuous-potential approximation and taking into account the processes of both single and multiple scattering on electrons, nuclei as well as on various defects and imperfections of the crystal lattice.

5. Quantum Collider – Achievable Luminosity

For our model calculation, we consider two counter propagating 250 × 250 GeV $\mu^\pm$ beams brought to collision inside the ultra strong focusing environment of a crystal channel. Both muon species are 'confined' to the corresponding crystal channels inside a low Z binary crystal; e.g. LiF.

One can estimate the luminosity of the collider in the following way[18]; the luminosity per single channel is given by

$$L_c = \frac{\gamma\, n_+\, n_-\, f}{4\pi\beta^* \varepsilon_N^{min}}, \tag{5.1}$$

where n_+ and n_- are numbers of muons (both species) present in the channel (here we assume $n_+ \sim n_- \sim 10^5$), $\beta^* = 6 \times 10^{-6}$ m is the equivalent beta star inside a binary crystal, $\varepsilon_N^{min} = 2.5 \times 10^{-9}$ m rad is the normalized transverse emittance (achieved by ionization crystal cooling) and f denotes the revolution frequency (we assume $f \sim 3 \times 10^6\ \mathrm{sec}^{-1}$).

In order to take advantage of the quantum confinement[9] inside a crystal channel the beam size, σ, has to be no greater than the crystal channel width (about 10 Angstroms). As one can see from the following formula

$$\sigma = \sqrt{\frac{\varepsilon_N^{min}\beta^*}{\gamma}}, \tag{5.2}$$

the $\mu^\pm$ beams with emittances of $\varepsilon_N^{min} = 2.5 \times 10^{-9}$ m rad can easily fit inside a crystal channel, $\sigma \sim 10^{-9}$ m. Applying the above numerical values into Eq.(5.1) yields the single channel luminosity of $L_c \sim 10^{29}\ \mathrm{cm}^{-2}\ \mathrm{sec}^{-1}$.

To estimate the net luminosity of a whole crystal, we need to know how many crystal channels are 'occupied' – we conservatively assume $N_c \sim 10^3$) The total luminosity is then given by:

$$L = N_c L_c \sim 10^{32}\ \mathrm{cm}^{-2}\ \mathrm{sec}^{-1}. \tag{5.3}$$

We notice in passing that this luminosity is obtained with only $N_c n_\pm \sim 10^8$ μ per pulse. We are currently studying whether this type of process is possible in a real binary crystal.

6. Conclusions

We pointed out that initially cool muons obtained from a hadronic source could be used as a starting point for a high energy, 250 × 250 GeV, $\mu^+ \mu^-$ collider complex, providing that ultra-fast muon cooling scheme is available. We suggest employing ionization energy loss in a focusing crystal channel as a cooling mechanism. Initially small muon phase space allows for efficient channeling through long sections of Silicon crystal. The ultra-strong focusing in a crystal channel combined with alternating bending makes it a powerful focusing cell characterized by a very small beta function, $\beta \sim 6 \times 10^{-6}$ m.

High energy muon colliders in the 250 × 250 GeV range, offer extraordinary physics, if challenges like capturing enough $\pi^\pm$ from a target and cooling enough $\mu^\pm$ from $\pi^\pm$ decay can be met. Efficient/fast muon cooling must be made to work if a 10^{32} cm^{-2} sec^{-1} luminosity muon collider is to be built. Using a storage ring made of 50 crystal bending – focusing cells, a 250 GeV muon will orbit $\sim 3 \times 10^6$ times while one lifetime, or 2.2×10^{-6} sec., elapses in its rest frame. To meet these ambitious luminosity challenges one has to employ radically new accelerator technology; rapid cooling combined with quantum confinement at the interaction point. Here, we suggest exploring ultra-strong guiding/focusing fields found in crystals (averaged electrostatic fields of a periodic arrangement of ion cores spaced by a few Angstroms)[17]. We outline a conceptual design of an accelarator-collider complex based on bending/focusing combined function cells made out of specially tailored crystal structures. We showed that a design of a 250 × 250 GeV $\mu^+ \mu^-$ collider exists and it provides all standard phase-stability feature of conventional high-energy storage rings. Its compactness (22 m in circumference) sets the scale (new standard) for future high energy colliders.

Our simple estimate shows that the proposed quantum crystal collider promises luminosity $\sim 10^{32}$ cm^{-2} sec^{-1}, with relatively low initial fluxes of $\mu^\pm$ (only $N_c n_\pm \sim 10^8$ μ per pulse), benefit of extremely tight localization of colliding beams inside a low Z binary crystal.

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Figure Captions

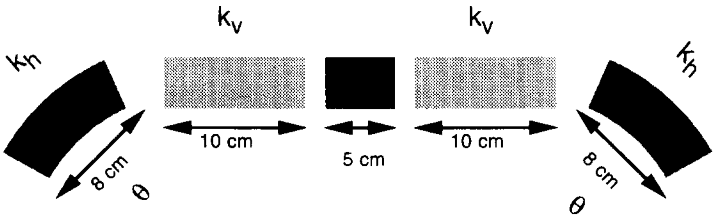
Figure 1. Layout of a 'bending – focusing' functional cell.

Figure 2. a) Horizontal betatron motion across two cells. b) Vertical beta function advancing 'out of step' with its horizontal companion (approximately the same phase advance in both planes). The vertical dispersion is identically equal to zero

Figure 3. Layout of a 'cooler ring' consisting of five bending – focusing cells and five rf inserts at every straight

Table 1. 250 × 250 GeV $\mu^+ \mu^-$ Crystal Collider Ring – Lattice Characteristics.

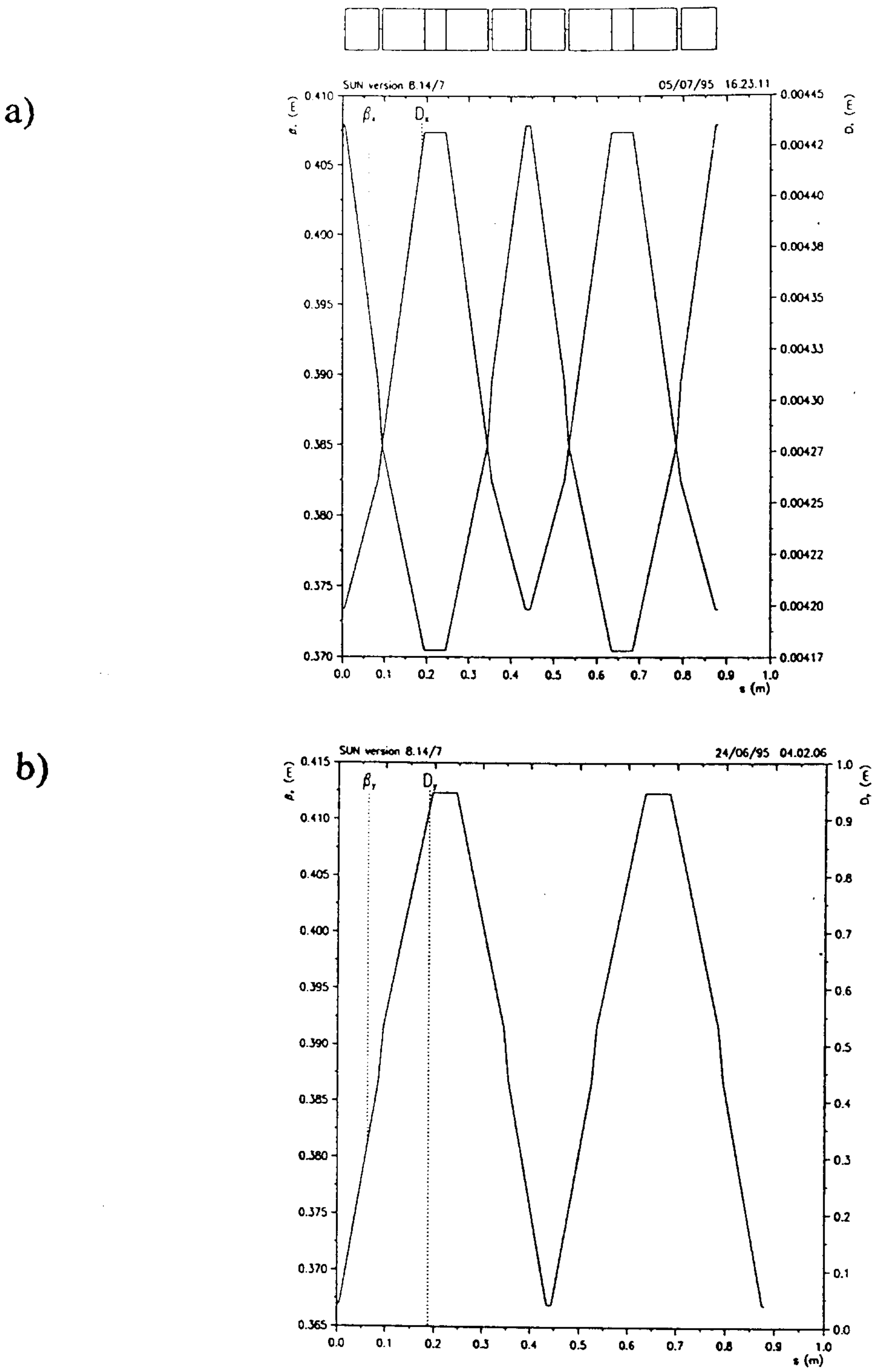
Bending – Focusing Cell



$k_h = k_v = 18\ \mathrm{m}^{-2}$

$\theta = 2\pi \times 10^{-2}$ rad

Figure 1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SUN version 8.14/7
05/07/95 16.23.11
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D_x (m)
β_x
D_x
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0.400
0.395
0.390
0.385
0.380
0.375
0.370
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0.00438
0.00435
0.00433
0.00430
0.00427
0.00425
0.00422
0.00420
0.00417
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
s (m)
b)
SUN version 8.14/7
24/06/95 04.02.06
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D_y (m)
β_y
D_y
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1.0
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0.8
0.7
0.6
0.5
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0.3
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s (m)

Figure 2

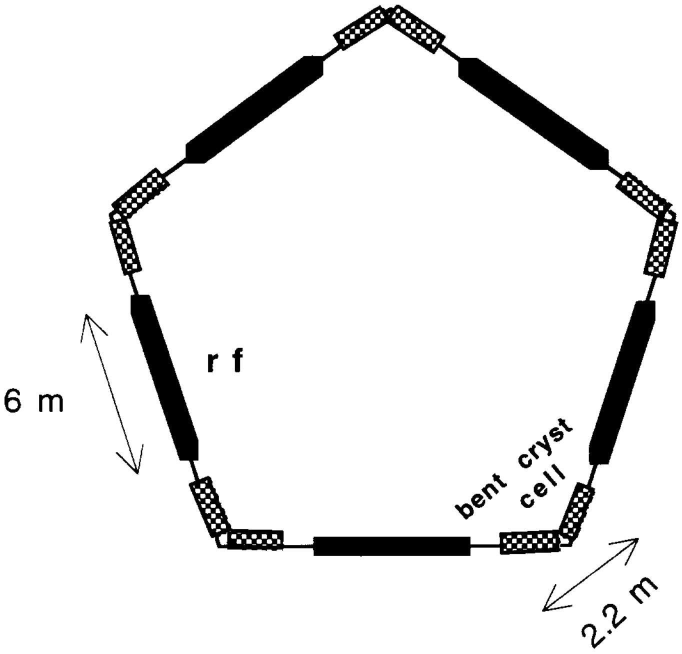

Figure 3

Table 1. 250 × 250 GeV $\mu^+ \mu^-$ Crystal Collider

Conceptual Lattice Design

Tunes	
horizontal	91.097
vertical	89.081
Beta max	
horizontal	0.4079 m
vertical	0.4123 m
Max Dispersion	
horizontal	4.43×10^{-3} m
vertical	0.0 m
Natural Chromaticity	
horizontal	−41.164
vertical	−45.891
Total Length	22 m
Number of Cells	50
Bending Angle per Cell	$4\pi \times 10^{-2}$ rad
Transition γ	91.091

Table 1