

Searching for the Nuclear Liquid-Gas Phase Transition in Au + Au Collisions at 35 MeV/nucleon

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Abstract

Within the framework of Classical Molecular Dynamics, we study the collision Au + Au at an incident energy of 35 MeV/nucleon. It is found that the system shows a critical behaviour at peripheral impact parameters, revealed through the analysis of conditional moments of charge distributions, Campi Scatter Plot, and the occurrence of large fluctuations in the region of the Campi plot where this critical behaviour is expected. When applying the experimental filters of the MULTICS-MINIBALL apparatus, it is found that criticality signals can be hidden due to the inefficiency of the experimental apparatus. The signals are then recovered by identifying semi-peripheral and peripheral collisions looking to the velocity distribution of the largest fragment, then by selecting the most complete events.

PACS : 25.70.-z, 25.70.Mn, 05.70.Jk, 64.70.Fx

I. INTRODUCTION

From several years, the idea that nuclear systems may show up some evidence for the occurrence of a critical behaviour related to a liquid-gas phase transition has stimulated lots of investigations both from theoretical and experimental sides [1–8]. This idea has initiated more than ten years ago with the observation by the Purdue-Fermilab group of asymptotic fragment charge distributions exhibiting a power law [9]. Such a power law, as described by the Fisher’s droplet model, is expected for cluster formation near the critical point of a liquid-gas phase transition [10]. This interest increased recently with the attempt by the EOS Collaboration to extract critical exponents of fragmenting nuclear systems produced in the collision of 1 GeV/nucleon Au nuclei with a carbon target [11], and with the extraction by the ALADIN Collaboration of a caloric curve resulting from the fragmentation of the quasiprojectile formed in the collision Au + Au at 600 MeV/nucleon exhibiting a behaviour expected for a liquid-gas phase transition [12].

In the present report, we study within the framework of Classical Molecular Dynamics (CMD) model, the reaction $^{197}\text{Au} + ^{197}\text{Au}$ at an incident energy of 35 MeV/nucleon, then analyse the results in terms of critical behaviour by studying fragment charge distributions, their moments, and the occurrence of large fluctuations in terms of intermittency analysis, and as shown by the fluctuations of the size of the largest fragment. Our aim for this analysis is three fold. First, a critical behaviour has been observed in the fragmentation of Au nuclei in the previously mentioned experiments at high beam energies (600 and 1000 MeV/nucleon), and we are interested to see if such a behaviour can still be observed at an energy notably lower. Second, we want to identify the critical events (if any) and study the effects of the efficiency of experimental apparatus (by applying for example the experimental filters of the MULTICS-MINIBALL apparatus [13,14]). Doing this we can see whether experimental inefficiencies can completely wash out the signals of criticality or it is still possible to recover these signals from the filtered results. Finally, we aim to apply the same procedure to the experimental data obtained by the MULTICS-MINIBALL collaboration

for the same reaction [15].

This paper is organized as follows. We give in section II a brief description of the CMD model used for this study. A more complete description can be found in Refs. [16,17]. Section III contains the analysis of the moments of charge distributions, the Campi scatter plot and the analysis of the scaled factorial moments in terms of the intermittency signal. Section IV is devoted to the study of the effects of experimental inefficiencies, applying to the CMD results the filters of the MULTICS-MINIBALL apparatus, and how it is possible to recover the signals of criticality selecting well detected events. Finally, we give in section V our summary and conclusions.

II. BRIEF DESCRIPTION OF THE CMD MODEL

In the CMD model, we assume that each nucleus is made up of 197 nucleons (79 protons + 118 neutrons) that move under the influence of a two-body potential V consisting of two different interactions [16]:

$$\begin{aligned}
 V_{nn}(r) &= V_{pp}(r) = V_0 [\exp(-\mu_0 r)/r - \exp(-\mu_0 r_c)/r_c] \\
 V_{np}(r) &= V_r [\exp(-\mu_r r)/r - \exp(-\mu_r r_c)/r_c] \\
 &\quad - V_a [\exp(-\mu_a r)/r - \exp(-\mu_a r_a)/r_a]
 \end{aligned} \tag{1}$$

$r_c = 5.4 \text{ fm}$ is a cutoff radius. The first interaction, for identical nucleons, is purely repulsive so no bound state of identical nucleons can exist (to simulate in some sense the Pauli principle), and the second, for proton-neutron interaction, is attractive at large distances and repulsive at small ones [16]. The various parameters entering Eq.(1) are defined with their respective values in Ref. [16]. This potential gives an Equation of State (EOS) of classical matter having about 250 MeV of compressibility (set M in Ref. [16]), and which strikingly resembles that of nuclear matter (i.e. equilibrium density $\rho_0 = 0.16 \text{ fm}^{-3}$ and energy $E(\rho_0) = -16 \text{ MeV/nucleon}$). Furthermore, in Refs. [16,18], it is shown that many experimental data on heavy-ion collisions are reasonably explained by this classical model.

Of course this is not accidental but it is due to the accurate choice of the parameters of the two-body potentials [16]. The classical Hamilton's equations of motion are solved using the Taylor method at the order $O[(\delta t)^3]$ where δt is the integration time step [19]. Energy and momentum are well conserved. Both nuclei are initialized in their ground state by using the frictional cooling method [20], then they are boosted towards each other with the CM kinetic energy. In the present calculations, the Coulomb interaction is explicitly taken into account. Note that even though this model is completely classical, it solves exactly (within the numerical error bars) the classical many-body problem, taking into account all order correlations.

III. RESULTS

Calculations for the reaction Au + Au at 35 MeV/nucleon are carried out for several impact parameters, from 1 to 13 fm by steps of 1 fm . One intuitively imagines the following scenario for this reaction. For central collisions, since the incident energy is rather high as compared to the Coulomb barrier, the two heavy nuclei will come in contact for a short time. The total charge of the intermediate system is very high and it will quickly explode due mainly to the high Coulomb repulsion [15]. For increasing impact parameter, two or may be three excited primary fragments might be formed. By tuning the impact parameter, we might hope to obtain some primary sources which have a combination of excitation energy, Coulomb charge, and angular momentum sufficient to bring the system into the instability region (if any). The possibility that such a scenario might apply to heavy-ion collisions has been shown in microscopic calculations [17,21]. In particular, it has been shown that the "critical" excitation energy decreases when the system is either charged and/or rotating [17,21–23]. Thus a combination of all these ingredients might give the desired result. Following this scenario, one would expect to see a critical behaviour (if any) for peripheral collisions.

In Fig. 1, we have plotted the dynamical evolution in the $x - z$ plane for this reaction

for four different times (after the two nuclei came in contact), and four different impact parameters; $\hat{b} = 0.15$ (first line panels), $\hat{b} = 0.38$ (second line panels), $\hat{b} = 0.62$ (third line panels) and $\hat{b} = 0.85$ (fourth line panels). For central and semi-central collisions (first and second rows) the two nuclei come in contact with each other and form a unique deformed source (the source is less deformed for more central collisions) which decays through light particles and fragments emission [15]. For semi-peripheral and peripheral collisions (third and fourth lines panels), one sees clearly the formation of two big sources (the quasitarget and quasiprojectile) with the formation between them of a third smaller source in the neck region. The size of this "neck" is smaller for more peripheral collisions and it completely disappears for the most peripheral ones.

One of the most powerful methods used to characterize the critical behaviour of a system undergoing a multifragmentation is the method of conditional moments introduced by Campi [24]. The moments of asymptotic cluster charge distributions are defined as:

$$m_k^{(j)} = \sum_Z Z^k n^{(j)}(Z) / Z_{tot} \quad (2)$$

where $n^{(j)}(Z)$ is the multiplicity of clusters of charge Z in the event j , $Z_{tot} = 158$, and the summation is over all the fragments in the event *except the heaviest one*, which corresponds to the bulk liquid in an infinite system. If the system keeps some trace of the phase transition for some particular events, the moments m_k should show up some strong correlations between them [24]. In particular, the second moment m_2 , which in macroscopic thermal systems is proportional to the isothermal compressibility, diverges at the critical temperature [11,25,26]. Of course in finite systems, the moments m_k remain finite due to finite size effects. In the upper part of Fig. 2, we have plotted versus the reduced impact parameter \hat{b} the second moment m_2 , calculated taking out the two largest fragments instead of only the largest one because, if one expects the critical behaviour at peripheral impact parameters and as the system is symmetric, one should subtract both bulk fragments coming from the quasitarget and the quasiprojectile. As expected, the second moment m_2 shows a peak for an impact parameter $\hat{b} \approx 0.8$. If one does not take off the second largest fragment (lower part of Fig.

2), we observe a continuous rise of m_2 and the peak disappears because we are summing with small fragments, a very big one (bulk) to the square (or power k for the highest moments m_k). In Fig. 3, we have plotted the same quantity m_2 versus the multiplicity of charged particles N_c (with $Z \geq 1$), calculated without the two largest fragments (upper part) and only without the largest one (lower part). The second moment m_2 shows also a peak versus N_c for a multiplicity around 20-25, and this peak disappears when taking into account the second largest fragment. In the following, the analysis of the non-filtered results is done taking off the two largest fragments.

Another quantity proposed by Campi to give more insight into the critical behaviour is the relative variance γ_2 defined as [24]:

$$\gamma_2 = \frac{m_2 m_0}{m_1^2} \quad (3)$$

It was shown by Campi that this quantity presents a peak around the critical point which means that the fluctuations in the fragment size distributions are the largest near the critical point [24]. In Fig. 4, we have plotted the relative variance γ_2 versus the reduced impact parameter \hat{b} (upper part), and versus the charged particle multiplicity N_c (lower part). One clearly notes that the relative variance γ_2 shows a peak in both plots, for a reduced impact parameter \hat{b} around 0.8, and for $N_c \approx 20 - 25$.

Moreover, we have considered another variable which is the normalized variance of the charge of the maximum fragment σ_{NV} . As charge distributions are expected to show the maximum fluctuations around the critical point [27], this quantity is expected to present some maximum at the critical point [24,28] This normalized variance is defined as

$$\sigma_{NV} = \frac{\sigma_{Z_{max}}^2}{\langle Z_{max} \rangle} \quad (4)$$

where

$$\sigma_{Z_{max}}^2 = \langle Z_{max}^2 \rangle - \langle Z_{max} \rangle^2 \quad (5)$$

The brackets $\langle \cdot \rangle$ indicate an ensemble averaging. We have plotted in Fig. 5 the normalized variance σ_{NV} versus \hat{b} (upper part), and versus N_c (lower part). In this case

also, we observe a peak for this quantity in both plots at almost the same values of \hat{b} and N_c . This means that the fluctuations in the charge of the maximum fragment (thus in charge distributions) are the largest around these values of the impact parameter and charged particle multiplicity.

The upper part of Fig. 6 shows a scatter plot of $\ln(Z_{max}^j)$ versus $\ln(m_2^j)$ for each event j , commonly known as Campi scatter plot. It was shown that if the system keeps some trace of the phase transition, the correlation between these two quantities exhibits two characteristic branches, an upper branch with an average negative slope corresponding to undercritical events, and a lower branch with a positive slope that corresponds to overcritical events, and the two branches meet close to the critical point of the phase transition [17,24,29]. The results of Fig. 6 show two branches corresponding to undercritical and over critical events, similar to the predicted ones. Note that the upper branch is made mainly by events having an impact parameter $\hat{b} > 0.85$, while the lower branch is made by events having $\hat{b} < 0.77$. The region where the two branches meet is made by events having $0.77 \leq \hat{b} \leq 0.85$. In the following, we will show that the central region where the two branches meet and where the critical behaviour is expected, is characterized by the occurrence of large fluctuations, revealed through an intermittency analysis [30]. In the lower part of Fig. 6, we have plotted the logarithm of the scaled factorial moments (SFM) defined as [31]

$$F_i(\delta s) = \frac{\sum_{k=1}^{Z_{tot}/\delta s} \langle n_k \cdot (n_k - 1) \cdot \dots \cdot (n_k - i + 1) \rangle}{\sum_{k=1}^{Z_{tot}/\delta s} \langle n_k \rangle^i} \quad (6)$$

$i = 2, \dots, 5$ versus the logarithm of the bin size δs . In the above definition of the SFM, i is the order of the moment. The total interval $1 - Z_{tot}$ ($Z_{tot} = 158$) is divided in $M = Z_{tot}/\delta s$ bins of size δs , n_k is the number of particles in the k -th bin for an event, and the brackets $\langle \cdot \rangle$ denote the average over many events. An intermittent pattern of fluctuations is characterized by a linear rise of the logarithm of the SFM's versus $-\ln(\delta s)$ (i.e. $F_i \propto \delta s^{-\lambda_i}$) which corresponds to the existence of large fluctuations which have self-similarity over the whole range of scales considered [29–31]. Even though this quantity is ill defined for fragment distributions [32,33], it has been shown in several theoretical studies

that critical events give a power law for the SFM versus the bin size [17,18,31,34,35]. In the figure, the logarithm of the SFM's exhibits a linear rise versus the logarithm of the bin size indicating a strong intermittency signal in the region of the Campi plot where the critical behaviour is expected. To understand whether these large fluctuations are due to a simple event mixing by considering different impact parameters inside Cut 2, we fixed the impact parameter to say $\hat{b} = 0.85$. The resulting SFM are shown in Fig. 7. One notes that the signal is still there even much weaker than previously (the absolute values of the SFM are smaller). This allows us to conclude that the intermittency signal is not due to the mixing of events and this mixing only increases the absolute values of the SFM.

At the end of this section, we would like to say few words about the mixing of different sources in the calculations of the previous quantities. First of all, we note that it is not evident to separate the different sources which might be formed after the first stages of the collision when they are still overlapping (we mean by overlap distances smaller than the range of the two-body interaction used, i.e. $r_c = 5.4$ fm), as one can see from Fig. 1. Thus it is not obvious to distinguish which fragments come from the different sources, even for a simple dynamical model like CMD. For the calculations of the second moment m_2 for instance, one should consider only one source (that entering the critical region). For central collisions, only one source is formed and m_2 is calculated according to Eq.(2) with $Z_{tot} \approx 158$. For peripheral impact parameters, one should calculate the second moment only from one source (the PLF or TLF assuming two sources), and in this case Z_{tot} should be around 79 ($158/2$) in Eq.(2). As we are dealing with a symmetric reaction, we can say that both the PLF and the TLF enter separately the critical region. So calculating m_2 using Eq.(2) with $Z_{tot} \approx 158$ is equivalent calculating it by summing on the fragments coming from only one source and dividing by $Z_{tot} \approx 79$, which gives the same results as those of Fig. 2. This discussion holds for all the moments m_k , thus for the reduced variance γ_2 . For the normalized variance of the charge of the largest fragment, one should be careful to consider the largest fragment coming from only one source (this was not done for the previous calculations of σ_{NV}). For central collisions, we have only one source, and the results do not change. For peripheral

collisions, by considering only the largest fragment with a positive velocity in the centre of mass, the obtained peak in σ_{NV}) is higher than that obtained previously (3.8 instead of 2.4 of Fig. 4). This result is in some sense obvious because we were previously smoothing the fluctuations of the largest fragment on both sources (PLF and TLF). For the Campi plot, we have plotted the logarithm of the size of the largest fragment versus the logarithm of m_2 both calculated for the fragments emitted in the forward direction (with $v_{CM} \geq 0$ to select roughly the PLF source). The obtained results are very similar to those reported on Fig. 6 and making a gate on the central region of the plot, we obtained almost the same SFM with the same absolute values as those reported on the lower part of Fig. 6.

In this section, we have seen that the analysis of the reaction Au + Au at 35 MeV/nucleon shows a signal of critical behaviour in peripheral collisions. This behaviour is revealed through the analysis of the second moment of charge distributions, the reduced variance, the large fluctuations of the size of the largest fragment, the characteristic shape of the Campi scatter plot and the occurrence of large fluctuations in the region of the Campi plot where the critical behaviour is expected.

IV. EFFECTS OF EXPERIMENTAL INEFFICIENCY

As indicated in the introduction, one of the aims of this study is to apply the same procedure of critical behaviour identification to the experimental data obtained by the MULTICS-MINIBALL Collaboration for the same reaction, Au + Au at 35 MeV/nucleon. To do so, we have filtered our results using the angular acceptance and energy thresholds of the MULTICS-MINIBALL apparatus.

First of all, we have checked that at least for semi-peripheral and peripheral collisions, the efficiency of the apparatus automatically eliminates the largest fragment coming from the target-like, so we calculate the moments of charge distributions m_k (Eq.(2)) by subtracting only the largest fragment (and not the two largest ones as for the unfiltered results). The upper part of Fig. 8 shows the second moment m_2 versus \hat{b} . The second moment m_2 does

no more show the peak observed for unfiltered results around $\hat{b} = 0.8$, even though one notes some remaining of that peak. We note also the appearance of a bump for more central collisions, around $\hat{b} = 0.38$. The situation is worst for the plot of m_2 versus N_c in the lower panel of the figure where one observes only a quasi-linear rise. The reduced variance γ_2 drawn in Fig. 9, shows a bit different behaviour. One still observes a smooth bump at $\hat{b} = 0.8$ but γ_2 is almost constant for $\hat{b} < 0.8$ and not rising as it is the case for unfiltered results. The same is for the plot of γ_2 versus N_c . The normalized variance of the size of the largest fragment represented in Fig. 10, still shows a peak but a little shifted versus higher impact parameters (upper part of the figure, see Fig. 4) and lower charged particle multiplicity (lower part). More drastic is the change in the shape of the Campi scatter plot shown in Fig. 11. This plot does no more show any particular shape characteristic to the occurrence of a critical behaviour (observed in the unfiltered results) and one is no more able to identify the upper and lower branches neither the meeting zone.

The effects of apparatus inefficiencies can thus be more or less drastic depending on the variable we are looking at. To recover the signals of criticality, we adopted the following procedure:

i) As the critical behaviour was observed at peripheral impact parameters, we identify semi-peripheral and peripheral collisions, eliminating more central ones, by selecting those events in which the velocity of the largest fragment along the beam axis is larger or equal to 75% of the beam velocity, which means that we are selecting those events in which there is a remnant of the projectile flying with the velocity of the quasiprojectile. Doing this, we hope to select only those reactions where two or three primary sources are formed (semi-peripheral and peripheral reactions) and eliminate the reactions where only one source is formed at mid-rapidity (central collisions);

ii) we select the most complete events imposing that the total detected charge is larger than 70 ($Z_{tot} \geq 70$).

Moreover, we have checked that condition *i)* does not automatically eliminate all central

collisions and to do so one needs to impose a maximum limit to the total detected charge, say $Z_{tot} \leq 90 - 95$. We note also that changing condition *i)* from 75% to 85% of the beam velocity for example does not change significantly the results, and only decreases the statistics.

In Figs. 12, 13 and 14, we have plotted the second moment m_2 , the reduced variance γ_2 and the normalized variance σ_{NV} versus the reduced impact parameter \hat{b} (upper part of the figures) and versus charged particle multiplicity N_c (lower part). One sees that the signals observed for non-filtered results are recovered at the same impact parameter. One notes also that this selection eliminates central collisions with $\hat{b} \leq 0.38$.

Figure 15 displays the Campi scatter plot for the filtered events with the selection on the velocity of the largest fragment and the total detected charge. We see that one recovers the characteristic shape of the Campi plot, in that it shows the upper branch with a negative slope and the lower branch with a positive slope, already observed in the unfiltered results. To better clarify the characteristics of these two branches and of the meeting zone, we have made three cuts in this plot selecting the upper branch (Cut 1), the lower branch (Cut 3) and the central region (Cut 2), and analysed the events falling in each of the three cuts. The upper part of Fig. 16 shows the impact parameter distributions of the events falling in the three cuts of the Campi plot. One sees that the three cuts select different regions of the impact parameter distribution; Cut 1 (left panel) selects the most peripheral collisions with a distribution peaked at $\hat{b} = 0.92$, Cut 2 (central panel) selects peripheral impact parameters with a distribution going from $\hat{b} = 0.65$ to 0.95 while Cut 3 (right panel) selects more central collisions. In the lower part of the same figure, we have plotted the charged particles multiplicity distributions for the three cuts. Cut 1 shows a multiplicity distribution from 2 to 10 while Cut 3 shows a distribution at higher multiplicities from 30 to 45. The situation is different for Cut 2. The multiplicity distribution covers a wider range of N_c values from 2 to 30. Note that this large multiplicity distribution is not due, as one can think, to a large impact parameter mixing (see upper part of the figure), but can be due to the occurrence of large fluctuations (as we will see) as expected near the critical point.

Figure 17 displays in the upper part the fragment charge distributions obtained in the three cuts [36] with, in the lower part, the corresponding scaled factorial moments calculated according to Eq.(6) [36]. Cut 1 (left part of the figure) corresponds to undercritical events and hence one obtains a charge distribution with a "U" shape characteristic to evaporation events, while for Cut 3 (right part) one observes a rapidly decreasing charge distribution with an exponential shape characteristic to highly excited systems going to vaporization. For Cut 2 (central part), we obtain a fragment charge distribution exhibiting a power law $Z^{-\tau}$, with $\tau \approx 2.2$, which is expected, according to the droplet model of Fisher, for fragment formation near the critical point indicating a liquid-gas phase transition, and consistent with the scaling laws of critical exponents [10]. In the lower part of the figure, for region 1 corresponding to evaporation events, the logarithms of the scaled factorial moments $\ln(F_i)$ are always flat and independent on $-\ln(\delta s)$ and there is no intermittency signal. For Cut 2 the situation is different. The logarithms of the SFM's are positive and almost linearly increasing versus $-\ln(\delta s)$ and a strong intermittency signal is observed (note the absolute values of $\ln(F_i)$). Cut 3 gives negative logarithms of the SFM's and we have also in this case no intermittency signal. Note that this behaviour of the scaled factorial moments is exactly the same as that observed in percolation and molecular dynamics models for undercritical, critical and overcritical events, respectively [17,37].

V. CONCLUSIONS AND OUTLOOKS

In conclusion, we have studied the reaction Au + Au at an incident energy of 35 MeV/nucleon within the framework of Classical Molecular Dynamics. The results show evidence for the occurrence of a critical behaviour revealed through the shape of the second moment of charge distributions, the reduced variance, the normalized variance of the size of the largest fragment, the particular shape of the Campi scatter plot and through the presence of large fluctuations as indicated by the intermittency analysis in the region of the Campi plot where the critical behaviour is expected. We have also seen that when our results are fil-

tered using the geometrical acceptance and energy thresholds of the MULTICS-MINIBALL apparatus, experimental inefficiencies can hide more or less the signals of criticality. Moreover, we have shown that these criticality signals can be recovered by identifying the most complete semi-peripheral and peripheral events selecting those events in which the largest fragment has a velocity along the beam axis larger or equal to 75% of the beam velocity and for which the total detected charge is $70 \leq Z_{tot} \leq 90$.

We would like to note at the end that the same procedure for characterizing the critical behaviour has been successfully applied to the experimental data obtained by the MULTICS-MINIBALL Collaboration for the same reaction Au + Au at 35 MeV/nucleon, and that a critical behaviour has been identified [38]. As an example, we show in Fig. 18 the experimental Campi scatter plot [38] obtained making more or less the same event selection as for the CMD results. Note the strong similarity with the theoretical Campi plot shown in Fig. 15. Moreover, we show in Fig. 19 the experimental scaled factorial moments [38] obtained in the three cuts made on the Campi plot of Fig. 18. Once again note the similarity of these results with those of the CMD results. The authors of the previous reference have also extracted the other quantities discussed in this paper (variance of the charge of the largest fragment, etc..) from the experimental data [39]. These quantities behave very similarly to what is discussed here for the CMD case thus strengthening our findings. A very similar behaviour to the one discussed here has also recently been observed in Xe + Sn collisions at 55 MeV/nucleon measured with the detector INDRA again for peripheral collisions [40]. Work now is in progress to characterize the fragmenting sources leading to the critical behaviour and maybe to extract the critical exponents.

ACKNOWLEDGMENTS

One of us (M. Belkacem) thanks the Physics Department of the University of Trieste for financial support and the Physics Department of the University of Bologna, where part of this work has been done, for warm hospitality and financial support.

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FIGURES

FIG. 1. Dynamical evolution. The r-space distribution is projected on the $x - z$ plane.

FIG. 2. The second moment of charge distributions m_2 versus the reduced impact parameter \hat{b} . The upper panel: without the two largest fragments, the lower panel: without only the largest fragment.

FIG. 3. The second moment of charge distributions m_2 versus charged particle multiplicity N_c . The upper panel: without the two largest fragments, the lower panel: without only the largest fragment.

FIG. 4. The reduced variance γ_2 versus the reduced impact parameter \hat{b} (upper panel) and versus charged particle multiplicity N_c (lower panel). The calculations are done without the two largest fragments.

FIG. 5. The normalized variance of the size of the largest fragment σ_{NV} versus the reduced impact parameter \hat{b} (upper panel) and versus charged particle multiplicity N_c (lower panel).

FIG. 6. Upper panel: Campi scatter plot. The logarithm of the size of the largest fragment $\ln(Z_{max})$ is plotted versus the logarithm of the second moment $\ln(m_2)$. Lower panel: The logarithm of the scaled factorial moments $\ln(F_i)$ is plotted versus the logarithm of the bin size $-\ln(\delta s)$ for the events falling within the cut drawn in the Campi scatter plot, upper panel. Solid circles represent the SFM of order $i = 2$, open circles $i = 3$, open squares $i = 4$ and open triangles $i = 5$.

FIG. 7. The logarithm of the scaled factorial moments $\ln(F_i)$ is plotted versus the logarithm of the bin size $-\ln(\delta s)$ for the events with $\hat{b} = 0.84$. Solid circles represent the SFM of order $i = 2$, open circles $i = 3$, open squares $i = 4$ and open triangles $i = 5$.

FIG. 8. Filtered CMD results. The second moment of charge distributions m_2 versus the reduced impact parameter \hat{b} (upper panel) and versus charged particle multiplicity N_c (lower panel).

FIG. 9. Filtered CMD results. The reduced variance γ_2 versus the reduced impact parameter \hat{b} (upper panel) and versus charged particle multiplicity N_c (lower panel).

FIG. 10. Filtered CMD results. The normalized variance of the size of the largest fragment σ_{NV} versus the reduced impact parameter \hat{b} (upper panel) and versus charged particle multiplicity N_c (lower panel).

FIG. 11. Filtered CMD results. Campi scatter plot. The logarithm of the size of the largest fragment $\ln(Z_{max})$ is plotted versus the logarithm of the second moment $\ln(m_2)$.

FIG. 12. Filtered CMD results with selection of events. The second moment of charge distributions m_2 versus the reduced impact parameter \hat{b} (upper panel) and versus charged particle multiplicity N_c (lower panel).

FIG. 13. Filtered CMD results with selection of events. The reduced variance γ_2 versus the reduced impact parameter \hat{b} (upper panel) and versus charged particle multiplicity N_c (lower panel).

FIG. 14. Filtered CMD results with selection of events. The normalized variance of the size of the largest fragment σ_{NV} versus the reduced impact parameter \hat{b} (upper panel) and versus charged particle multiplicity N_c (lower panel).

FIG. 15. Filtered CMD results with selection of events. Campi scatter plot. The logarithm of the size of the largest fragment $\ln(Z_{max})$ is plotted versus the logarithm of the second moment $\ln(m_2)$. Three cuts are employed to select the upper branch (1), the lower branch (3) and the central region (2).

FIG. 16. Filtered CMD results with selection of events. Impact parameter distributions (upper panels) and multiplicity distributions (lower panels) for the three cuts made on Fig. 15: left part Cut 1, central part Cut 2 and right part Cut 3.

FIG. 17. Filtered CMD results with selection of events. Fragment charge distributions (upper panels) and the corresponding scaled factorial moments $\ln(F_i)$ versus $-\ln(\delta s)$ for the three cuts made on Fig. 15: left part Cut 1, central part Cut 2 and right part Cut 3. The solid line on the upper-central panel indicates a power law distribution $N(Z) \propto Z^{-\tau}$ with $\tau = 2.2$. In the lower panels, solid circles represent the SFM of order $i = 2$, open circles $i = 3$, open squares $i = 4$ and open triangles $i = 5$.

FIG. 18. Experimental results from Ref. [38]. Campi scatter plot. The logarithm of the size of the largest fragment $\ln(Z_{max})$ is plotted versus the logarithm of the second moment $\ln(m_2)$. Three cuts are employed to select the upper branch (1), the lower branch (3) and the central region (2).

FIG. 19. Experimental results from Ref. [38]. Scaled factorial moments $\ln(F_i)$ versus $-\ln(\delta s)$ for the three cuts made on Fig. 18: left part Cut 1, central part Cut 2 and right part Cut 3. Solid circles represent the SFM of order $i = 2$, open circles $i = 3$, open squares $i = 4$ and open triangles $i = 5$.