ANGULAR DISTRIBUTION IN NUCLEON-NUCLEON "QUASI-ELASTIC DIFFRACTION" SCATTERING

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In the recent observations of Cocconi, Diddens, Lillethun and Wetherell [1) on the spectra of inelastically scattered protons from proton-nucleus collisions in the 10-25 GeV/c range of incident momenta, three features are especially noteworthy :

1) There appear, in the laboratory system spectra of protons scattered at small angles to the incident beam, two well-defined peaks. One has all the characteristics of elastic nucleon-nucleon scattering, while the other, exhibiting a larger energy loss, corresponds to an inelastic scattering process.

2) The inelastic peak is separated from the elastic peak by a constant momentum difference of 0.8-1 BeV/c, irrespective of the projectile momentum and of the scattering angle (in the range 20-60 milliradians, at least).

3) Both the elastic and inelastic peaks decrease in intensity with increasing scattering angle, roughly as would be anticipated for the diffraction (shadow) elastic scattering characteristic of absorption within a sphere of radius $\hbar/\mu c$, where μ is the π-meson mass.

As was pointed out by Wetherell [2)], the first two of the above-mentioned features can be understood in terms of a process in which the target nucleon is excited into a definite "isobar" state. The difficulties with this explanation lie, first, in the fact that the observed inelastic peak appears to correspond, at best, to the excitation of only one among the possible "isobars" (and not too well, at that, as far as its position is concerned) and, second, in understanding the diffraction angular distribution of the inelastic scattering process. Other explanations, which concentrate more on understanding the third of the above-mentioned features, account for the inelastic peak as resulting from the diffraction scattering of the projectile nucleon by a pion in the "cloud" of the target [3)], or invoke the process of "diffraction dissociation", previously discussed by a number of authors in other connections [4)].

However, on the basis of a number of physical arguments as well as the published data from CERN [1] and, at lower projectile energies, from Brookhaven [5], Feld and Iso [6] have concluded that the isobar excitation hypothesis provides the most likely explanation of the observations Adopting a relatively simple model for the excitation mechanism, they have concluded that both peaks observed by Cocconi et al. should, with improved experimental resolution, split into two; the first (elastic) peak was expected to contain both the true elastic peak and one corresponding to excitation of the lowest $(t=3/2,\ j=3/2^+)$ or $(3,3^+)$ isobar; the inelastic peak was conjectured to be a superposition of peaks corresponding to the next two, the $(1,3^-)$ and the $(1,5^+)$, isobars [*].

More recent observations [7] of Cocconi et al., with improved resolution (and especially as a result of their utilisation of a CH_2-C difference technique to isolate the protons scattered from free nucleons) have borne out these predictions, thereby definitely establishing the applicability of the isobar excitation model. It is the purpose of this note to derive, from a phenomenological point of view, the angular distributions of the inelastic peaks, in a manner completely analogous to the usual "optical" derivations of the angular distribution of diffraction elastic scattering. Thus, on the one hand, we shall be able to account for the similarity in behaviour of the two peaks at the (relatively large) angles at which they have been observed. On the other hand, our model leads to some significant differences at small scattering angles.

For demonstration purposes, we consider the scattering of spinless particles. (The complications introduced by the nucleon spins will be considered later). To begin with, we review briefly the derivation of the angular distribution of diffraction elastic scattering in the usual "optical" approximation. Thus, the differential elastic scattering cross-section is given by [8]

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*) Arguments, presented in the afore-mentioned paper [6], indicated that only the $(3,3)$ isobar should have been strongly excited in the Brookhaven experiments [5] while the higher energy CERN experiments should excite the first three, but not appreciably the fourth, a $(3,5?)$ isobar.

$$\frac{d\sigma}{d\Omega} = \frac{1}{k^2}\left|\sum_{\ell=0}^{\infty}(2\ell+1)(1-\eta_\ell)\,P_\ell(\cos\theta)\right|^2 \tag{1}$$

in which the scattering amplitudes

$$\eta_\ell = a_\ell\, e^{2i\alpha_\ell} \tag{2}$$

are determined by the details of the scattering process. In particular, on the assumption of strong absorption within a sphere of radius R, all $a_\ell \ll 1$ for $\ell \lesssim kR$ and all $a_\ell \cong 1$, $\alpha_\ell \cong 0$ for $\ell > kR$. With these approximations, Eq. (1) becomes

$$\frac{d\sigma}{d\Omega} \cong \frac{a^2}{k^2}\left|\sum_{\ell=0}^{kR}(2\ell+1)\,P_\ell(\cos\theta)\right|^2 , \tag{3}$$

where a is the average value of $|1-\eta_\ell|$, assumed to vary slowly with ℓ in the region $\ell \lesssim kR$. Assuming $kR \gg 1$, the scattering is confined to small angles, $\theta \lesssim kR$, for which we may use the approximation[9]

$$P_\ell(\cos\theta) = J_0(\ell\theta) \quad . \tag{4}$$

Then

$$\sum_{0}^{kR}(2\ell+1)\,P_\ell \cong 2\int_0^{kR}\ell\, J_0(\ell\theta)\, d\ell = \frac{2}{\theta^2}\,(kR\theta)\, J_1(kR\theta) \tag{5}$$

and

$$\frac{d\sigma}{d\Omega} \cong a^2 k^2 R^4\left[\frac{4J_1^2(X)}{X^2}\right] , \quad (X \equiv kR\theta) . \tag{6}$$

Eq. (6) is the well-known expression for "shadow" elastic scattering. It takes on the large value $a^2k^2R^4$ for $\theta=0$ and falls off rapidly to zero at $\theta=3.8/kR$.

The derivation of the angular distribution for the "quasi-elastic" scattering proceeds in a similar fashion. Consider a process in which one of the spin zero particles is excited to a state of internal angular momentum 1 without change of parity (this corresponds, in the case of spin-independent scattering, to the excitation of the $(3,3^+)$ isobar, especially in the limit $kR>>1$). In this case, the differential cross-section for the inelastically scattered projectile particles has the form *)

$$\frac{d\sigma}{d\Omega} = \frac{\pi}{k^2}\left|\sum_{\ell=0}^{\infty}(2\ell+1)^{1/2}\,\eta'_\ell\; Y_\ell^1(\theta,\varphi)\right|^2 . \tag{7}$$

In the optical approximation, the inelastic scattering amplitudes

$$\eta'_\ell = b_\ell\, e^{i\beta_\ell} \tag{8}$$

are assumed to be slowly varying with ℓ, in which case they may be extracted from the summation and replaced by an average value, $\overline{|\eta'_\ell|} = b$. Using the relations 9)

$$Y_\ell^1 = \left[\frac{2\ell+1}{4\pi\ell(\ell+1)}\right]^{1/2} P_\ell^1 \tag{9}$$

$$P_\ell^1(\cos\theta) = \sin\theta\,\frac{d}{d(\cos\theta)}\,P_\ell(\cos\theta) \cong -\ell\,J_0'(\ell\theta) \; , \tag{10}$$

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*) Y_ℓ^m is the associated Legendre polynomial.

and again, in accordance with the optical approximation, cutting off the summation at $\ell = kR$, Eq. (7) becomes

$$\frac{d\sigma}{d\Omega} = \frac{b^2}{k^2\theta^4}\left|\int_0^{X} x\, J_0'(x)\, dx\right|^2$$

$$= \frac{b^2}{k^2\theta^4}\left[X\, J_0(X) - \int_0^{X} J_0(x)\, dx\right]^2 \tag{11}$$

$$= b^2 k^2 R^4\, X^{-4}\left[X\, J_0(X) - 2\sum_{\nu=0}^{\infty} J_{2\nu+1}(X)\right]^2 .$$

The angular distribution corresponding to Eq. (11) is compared to that for diffraction elastic scattering, Eq. (6), in Figure 1. We note that the cross-section, for the case corresponding to the spin change, starts off at zero for $\theta=0$,

$$\frac{d\sigma}{d\Omega}(\theta\to 0) = b^2 k^2 R^4 \left(X/6\right)^2 , \tag{12}$$

but becomes comparable to the elastic cross-section (provided b^2 is not too small) at angles $\theta \gtrsim 1/kR$ showing, thereafter, a characteristic diffraction pattern which is "out of phase" with that corresponding to the elastic scattering.

The effect of nucleon spin is to introduce, into the angular distributions characteristic of the excitation process, terms whose angular distribution is similar to that of the elastic scattering but reduced in magnitude by the factor $(kR)^{-2}$, an effect which could be anticipated for large kR on the basis of correspondence principle arguments. To illustrate the effect of nucleon spin, but in a somewhat simpler situation that applies in the experiment of Cocconi et al. [1], we consider the case of diffraction excitation of nucleons by pions; in this case, the problem is simplified by having a spin-zero projectile. As a first example, we consider the excitation of a $j=\frac{1}{2}^-$ isobar. (This does not correspond to any of the known low-lying isobars, but it provides a simple illustration of the above-mentioned effect). Here, owing to the parity change (the nucleon ground state is $j=\frac{1}{2}^+$), the projectile must change its orbital angular momentum by one unit to effect the excitation. Assuming, as in the preceding, that the reaction amplitudes are slowly varying, and may be replaced by an average value, and that the summations may be terminated at $\ell = kR$, the expression for the angular distribution of the scattered pions is

$$\frac{k^2}{b^2}\frac{d\sigma}{d\Omega} = \left|\sum_{\ell=0}^{kR} P_\ell^1(\cos\theta)\right|^2 + \frac{1}{4}\left|\sum_{\ell=0}^{kR} P_\ell(\cos\theta)\right|^2 . \qquad (13)$$

The first term leads to the previously derived Eq. (11). The second is approximated by use of Eq. (4)

$$\sum_{\ell=0}^{kR} P_\ell \cong \int_0^{kR} J_0(\ell\theta)\, d\ell = \frac{1}{\theta}\int_0^{X} J_0(x)\, dx = \frac{2}{\theta}\sum_{\nu=0}^{\infty} J_{2\nu+1}(X) \qquad (14)$$

and gives, as its contribution to $d\sigma/d\Omega$ at $\theta=0$, $\frac{1}{4}b^2R^2$. While this is the dominant term at $\theta=0$, it is smaller than the elastic scattering term by the afore-mentioned factor $(kR)^{-2}$, as anticipated.

A more interesting example is the case of excitation of the $(3,3^+)$ isobar. Here, the differential cross-section contains three terms

$$\frac{k^2}{b^2}\frac{d\sigma}{d\Omega} \cong 2\left|\sum_{\ell=0}^{kR} P_\ell^1\right|^2 + \left|\sum_{\ell=0}^{kR} P_\ell\right|^2 + 0.35\left|\sum_{\ell=0}^{kR} \ell^{-2} P_\ell^2\right|^2 \qquad (15)$$

of which the first two are the same as have been discussed previously. The third term, which arises from the fact that, in this case, the projectile can give up two units of angular momentum in exciting the target, may be evaluated by use of the recursion relation [9)]

$$P_\ell^2 = 2\,\mathrm{ctgn}\,\theta\; P_\ell^1 - \ell(\ell+1)\,P_\ell \quad . \tag{16}$$

Adopting the small angle approximation

$$\sum_{\ell=0}^{kR} \ell^{-2} P_\ell^2(\cos\theta) \cong -\frac{1}{\theta}\left\{\int_0^X \left[2\frac{J_0'(x)}{x} + J_0(x)\right] dx\right\}$$

$$= \frac{1}{\theta}\int_0^X \left[\frac{2J_1(x)}{x} - J_0(x)\right] dx \quad . \tag{17}$$

The two terms in Eq. (17) cancel up to order x^2, for small x, and give rise, in the limit of $x \to 0$, to a contribution to the cross-section of

$$\left.\frac{d\sigma}{d\Omega}\right)_{\Delta\ell=2} \to \sim \frac{3}{8}\, b^2 R^2 \left(X^2/24\right)^2 \tag{18}$$

which, in addition to the reduction by the factor $(kR)^{-2}$ from that of the elastic scattering, exhibits the slow rise characteristic of a change in angular momentum by two units.

Having thus demonstrated the applicability of the spin-independent approximation in the limit $kR >> 1$, we use it to obtain the dominant terms in the angular distributions of the peaks corresponding to the excitation of the $(1,3^-)$ and $(1,5^+)$ isobars. For the first, we compute the angular distribution corresponding to the excitation of the target from the state 0^+ to 1^-; the second corresponds to the transition $0^+ \to 2^+$. In both cases, we obtain the somewhat surprising result *) that the leading term in the angular distribution

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*) While this result was somewhat surprising to me, it appears that it is well-known in observations of "diffraction inelastic scattering" of α-particles by nuclei in the low energy range [cf., J.S. Blair, Proceedings of the International Conference on Nuclear Structure, Kingston, Canada, 1960, p.824].

has the form of the diffraction elastic scattering, Eq. (6). In the second case, that of the excitation of the $(1,5^+)$ isobar, there is a small additional term which, though it is proportional to k^2R^4, goes like X^4 in the limit $\theta \to 0$, and has an angular distribution characteristic of $\Delta \ell = 2$. The details of the computations and the results are given in the Appendix.

Before we can apply these considerations to the experimental observations, a number of qualifications must be noted. First, although we have assumed the average amplitudes for isobar excitation, b, to be slowly varying with ℓ, there is no compelling reason to assume that they are independent of the variable $X \equiv kR\theta$. Thus, for example, Selove [10] has interpreted the Brookhaven results [5] as being primarily due to a one-pion exchange diagram, in which case the amplitude is a strongly varying function of the invariant momentum transfer which, in turn, depends strongly on $kR\theta$. The inherent assumption of our approach is that the available energies for excitation in the CERN experiments [1),7] are so large as to lead to isobar excitation with probabilities comparable to those corresponding to elastic scattering in essentially all the diagrams which give rise to the diffraction scattering. Under these circumstances we may be justified in applying the same "optical" approximation in both cases. However, at the energies of the experiments this extreme situation may not yet prevail, and it is still possible that the amplitudes b may depend on the momentum transfer, thereby introducing an additional dependence on the variable $kR\theta$.

Secondly, it is important to note that the observations have so far been confined to values of the variable $kR\theta$ in the range 2.5 - 9, which lie mostly outside of the first maxima of the curves plotted in Figure 1. As is well known, diffraction scattering does not normally exhibit the sharp secondary maxima predicted on the basis of the "black sphere" model, presumably owing to the "fuzzy" boundary of the absorbing sphere, as well as to the varying opacity of the meson cloud of the nucleon. These effects tend to eliminate the secondary diffraction maxima, and to yield cross-section curves which, at large X, follow more closely the envelopes of the curves in Figure 1 than the detailed variations. Consequently, those predictions of such a model which are confined within the range of the first maximum are most reliable, and a useful comparison of our predictions with the observations awaits further measurements at smaller angles.

I am pleased to acknowledge my indebtedness to many colleagues at CERN, especially to Y. Yamaguchi who first suggested the similarity of the isobar excitation process to phenomena well known in low energy nuclear physics, for example the deuteron stripping reactions. I am also pleased to acknowledge the financial support of the John Simon Guggenheim Foundation.

APPENDIX

Computation of the Angular Distributions

We consider the scattering processes

$$a + A \rightarrow a + A^* \tag{A1}$$

in which the projectile a is assumed to have zero spin, the target A to have spin S and the excited target (isobar) A^* to have spin S^*. The angular distribution of the scattered projectile a is given by [8)]

$$k^2 \frac{d\sigma}{d\Omega}(A \rightarrow A^*) = \frac{\pi}{(2S+1)} \sum_{i,f} \left| \sum_j q^j_{f,i} \right|^2 \tag{A2}$$

with

$$q^j_{f,i} = (2\ell+1)^{1/2}\, i^{(\ell-\ell')}\, C_{i,j}\, C_{j,f}\, a^j_{f,i}\, Y^{m'}_{\ell'}(\theta,\varphi) \tag{A3}$$

in which ℓ (m=0) is the orbital angular momentum of the incident particle, ℓ'(m') that of the scattered particle, the C's are Clebsch-Gordan coefficients connecting the initial and final states of the target through the possible intermediate states, with the restrictions

$$\underset{\sim}{J} = \underset{\sim}{S} + \underset{\sim}{\ell} = \underset{\sim}{S}^* + \underset{\sim}{\ell}' \tag{A4a}$$

$$m = m^* + m' \tag{A4b}$$

$$\Pi_A (-1)^{\ell} = \Pi_{A^*} (-1)^{\ell'} \quad , \tag{A4c}$$

the $Y^{m'}_{\ell'}$ are the normalised associated Legendre polynomials, and the $a^{j}_{f,i}$ are the scattering amplitudes which we shall assume to be slowly varying with ℓ, ℓ'.

We consider, first, the case of $S=0^+$.

1) $S^*=0^-$

$$\frac{d\sigma}{d\Omega}(0^+ \to 0^-) = 0 \tag{A5}$$

2) $S^*=1^+$

$$k^2 \frac{d\sigma}{d\Omega}(0^+ \to 1^+) = b^2 \left| \sum_{\ell=0}^{kR} P^1_\ell \right|^2 \tag{A6}$$

in which $b \equiv \overline{a^{\ell}_{\ell,\ell}}$.

3) $S^*=1^-$

$$k^2 \frac{d\sigma}{d\Omega}(0^+ \to 1^-) = 2\left| b \sum \ell P_\ell + \tfrac{1}{2}\left(b - \tfrac{3}{2}c\right) \sum P_\ell \right|^2 + \left| c \sum P^1_\ell - b \sum \ell^{-1} P^1_\ell \right|^2 \tag{A7}$$

with

$$b \equiv \overline{\tfrac{1}{2}\left(a^{\ell}_{\ell+1,\ell} + a^{\ell}_{\ell-1,\ell}\right)} \tag{A7a}$$

$$c \equiv \overline{\tfrac{1}{2}\left(a^{\ell}_{\ell+1,\ell} - a^{\ell}_{\ell-1,\ell}\right)} \tag{A7b}$$

4) $S^*=2^+$

$$k^2\frac{d\sigma}{d\Omega}(0^+\to 2^+) = \frac{1}{4}\left|\left(b+2\sqrt{\tfrac{3}{2}}\,\bar{b}\right)\sum \ell P_\ell - 2\left[b+\tfrac{1}{2}\sqrt{\tfrac{3}{2}}\,(\bar{b}-3c)\right]\sum P_\ell\right|^2$$
$$+\left|c\sum P_\ell^1 + \tfrac{1}{2}\left(\sqrt{\tfrac{3}{2}}\,b-2\bar{b}\right)\sum \ell^{-1}P_\ell^1\right|^2 \qquad \text{(A8)}$$
$$+\frac{1}{4}\left|\left(\sqrt{\tfrac{3}{2}}\,b-\bar{b}\right)\sum \ell^{-1}P_\ell^2 - \tfrac{1}{2}\left(\sqrt{\tfrac{3}{2}}\,b-\bar{b}+c\right)\sum \ell^{-2}P_\ell^2\right|^2$$

with

$$b \equiv \overline{a^{\ell}_{\ell,\ell}} \qquad \text{(A8a)}$$

$$\bar{b} \equiv \overline{\tfrac{1}{2}\left(a^{\ell}_{\ell+2,\ell} + a^{\ell}_{\ell-2,\ell}\right)} \qquad \text{(A8b)}$$

$$c \equiv \overline{\tfrac{1}{2}\left(a^{\ell}_{\ell+2,\ell} - a^{\ell}_{\ell-2,\ell}\right)} \qquad \text{(A8c)}$$

5) $S^*=2^-$

$$k^2\frac{d\sigma}{d\Omega}(0^+\to 2^-) = \left|b\sum P_\ell^1 - \tfrac{5}{2}c\sum \ell^{-1}P_\ell^1\right|^2$$
$$+\left|c\sum \ell^{-1}P_\ell^2 + \tfrac{1}{4}(b-2c)\sum \ell^{-2}P_\ell^2\right|^2 \qquad \text{(A9)}$$

where b and c are defined by equations (A7a) and (A7b), respectively.

We have also worked out some angular distributions corresponding to the processes $\pi + N \to N^* + \pi$ $(S=\frac{1}{2}^+)$.

6) $S^* = \frac{1}{2}^-$

$$k^2 \frac{d\sigma}{d\Omega}\left(\tfrac{1}{2}^+ \to \tfrac{1}{2}^-\right) = \left| c \sum \ell P_\ell + \tfrac{1}{2}(b+c) \sum P_\ell \right|^2 + b^2 \left| \sum P_\ell^1 \right|^2 \tag{A10}$$

with

$$b \equiv \tfrac{1}{2}\overline{\left(a_{\ell+1,\ell}^{\ell+\frac{1}{2}} + a_{\ell-1,\ell}^{\ell-\frac{1}{2}} \right)} \tag{A10a}$$

$$c \equiv \tfrac{1}{2}\overline{\left(a_{\ell+1,\ell}^{\ell+\frac{1}{2}} - a_{\ell-1,\ell}^{\ell-\frac{1}{2}} \right)} \quad . \tag{A10b}$$

7) $S^* = \frac{3}{2}^+$

$$k^2 \frac{d\sigma}{d\Omega}\left(\tfrac{1}{2}^+ \to \tfrac{3}{2}^+\right) = (a^2 + a'^2) \left| \sum P_\ell^1 \right|^2 + \frac{1}{16} \left| \left(2\sqrt{3}\, a' + \tfrac{1}{2} a\right) \sum P_\ell \right|^2 + \frac{3}{64} \left| \left(a + \sqrt{3}\, a'\right) \sum \ell^{-2} P_\ell^2 \right|^2 \tag{A11}$$

with

$$a \equiv \overline{a_{\ell,\ell}^{\ell \pm \frac{1}{2}}} \tag{A11a}$$

$$a' \equiv \overline{a_{\ell \pm 2,\ell}^{\ell \pm \frac{1}{2}}} \quad . \tag{A11b}$$

The method of evaluating the summations, as already indicated in the text, assumes [9] the small angle approximations

$$P_{\ell}\ (\cos\Theta) \cong J_0(\ell\,\Theta) \tag{A12a}$$

$$P_{\ell}^{1}\ (\cos\Theta) \cong -\ell J_0'(\ell\,\Theta) \tag{A12b}$$

$$P_{\ell}^{2}\ (\cos\Theta) \cong \frac{2}{\Theta} P_{\ell}^{1}\ (\cos\Theta) - \ell^2\ P_{\ell}(\cos\Theta)\ . \tag{A12c}$$

The summations over ℓ may be replaced by integrals, e.g.

$$\sum_{0}^{kR} \ell^{n}\ P_{\ell} \rightarrow \frac{1}{\Theta^{n+1}} \int_{0}^{X} x^{n}\ J_0(x)\ dx\ . \tag{A13}$$

with $X \equiv kR\Theta$. Thus, we obtain

$$\sum \ell\, P_{\ell} \cong \tfrac{1}{2}\, k^2R^2 \left[2J_1(X)/X\right] \equiv \tfrac{1}{2}\, k^2R^2\ F_0(X) \tag{A14a}$$

$$\sum P_{\ell} \cong kR\left[X^{-1} \int_0^X J_0(x)dx\right] = kR\ \left[2X^{-1} \sum_{v=0}^{\infty} J_{2v+1}(X)\right] \equiv kRG_0(X) \tag{A14b}$$

$$\sum P_{\ell}^{1} \cong k^2R^2 \left[X^{-2} \left\{\int_0^X J_0(x)dx - XJ_0(X)\right\}\right] \equiv k^2R^2\ F_1(X) \tag{A14c}$$

$$\sum \frac{1}{\ell} P_{\ell}^{1} \cong kR \left[X^{-1} \left\{1-J_0(X)\right\}\right]. \equiv kR\ G_1(X) \tag{A14d}$$

$$\sum \frac{1}{\ell} P_{\ell}^{2} \cong k^2R^2\left[2X^{-2}\left\{1-J_0(X) - \frac{X}{2}\, J_1(X)\right\}\right] \equiv k^2R^2\ F_2(X) \tag{A14e}$$

$$\sum \frac{1}{\ell^2} P_{\ell}^{2} = kR\left[X^{-1} \int_0^X \left\{\frac{2J_1(x)}{x} - J_0(x)\right\} dx\right] \equiv kR\ G_2(X) \tag{A14f}$$

Each of these exhibits its own "diffraction" behaviour, such as shown in Fig. 1 for F_0^2 and F_1^2. At small X, the F_i and G_i behave like X^i. Thus as $X \to 0$

$$F_0 \to 1 - \frac{1}{8} X^2 \qquad \text{(A14a')}$$

$$G_0 \to 1 - \frac{1}{12} X^2 \qquad \text{(A14b')}$$

$$F_1 \to X/6 \qquad \text{(A14c')}$$

$$G_1 \to X/4 \qquad \text{(A14d')}$$

$$F_2 \to X^2/32 \qquad \text{(A14e')}$$

$$G_2 \to X^2/24 \quad . \qquad \text{(A14f')}$$

Provided that $kR >> 1$, we may in general neglect all terms leading to the functions G_i. Making the further approximations $c=0$ and $\bar{b}=b$, the angular distributions, Eqs. (A6) - (A9) become

$$(kR^2)^{-2} \frac{d\sigma}{d\Omega}(0^+ \to 1^+) \simeq b^2 F_1^2(X) \qquad \text{(A6')}$$

$$(kR^2)^{-2} \frac{d\sigma}{d\Omega}(0^+ \to 1^-) \simeq \frac{1}{2} b^2 F_0^2(X) \qquad \text{(A7')}$$

$$(kR^2)^{-2} \frac{d\sigma}{d\Omega}(0^+ \to 2^+) \simeq \frac{3}{4} b^2 F_0^2(X) + 0.013\, b^2 F_2^2(X) \qquad \text{(A8')}$$

$$(kR^2)^{-2} \frac{d\sigma}{d\Omega}(0^+ \to 2^-) \simeq b^2 F_1^2(X) \quad , \qquad \text{(A9')}$$

while (A10) and (A11) reduce to (A6'). Thus, in the spin independent approximation, the angular distributions for elastic scattering and for excitation of the $(1,3^-)$ and $(1,5^+)$ isobars should be similar, Eqs. (A7') and (A8'), while that for the $(3,3^+)$ isobar is different from these, Eq. (A6').

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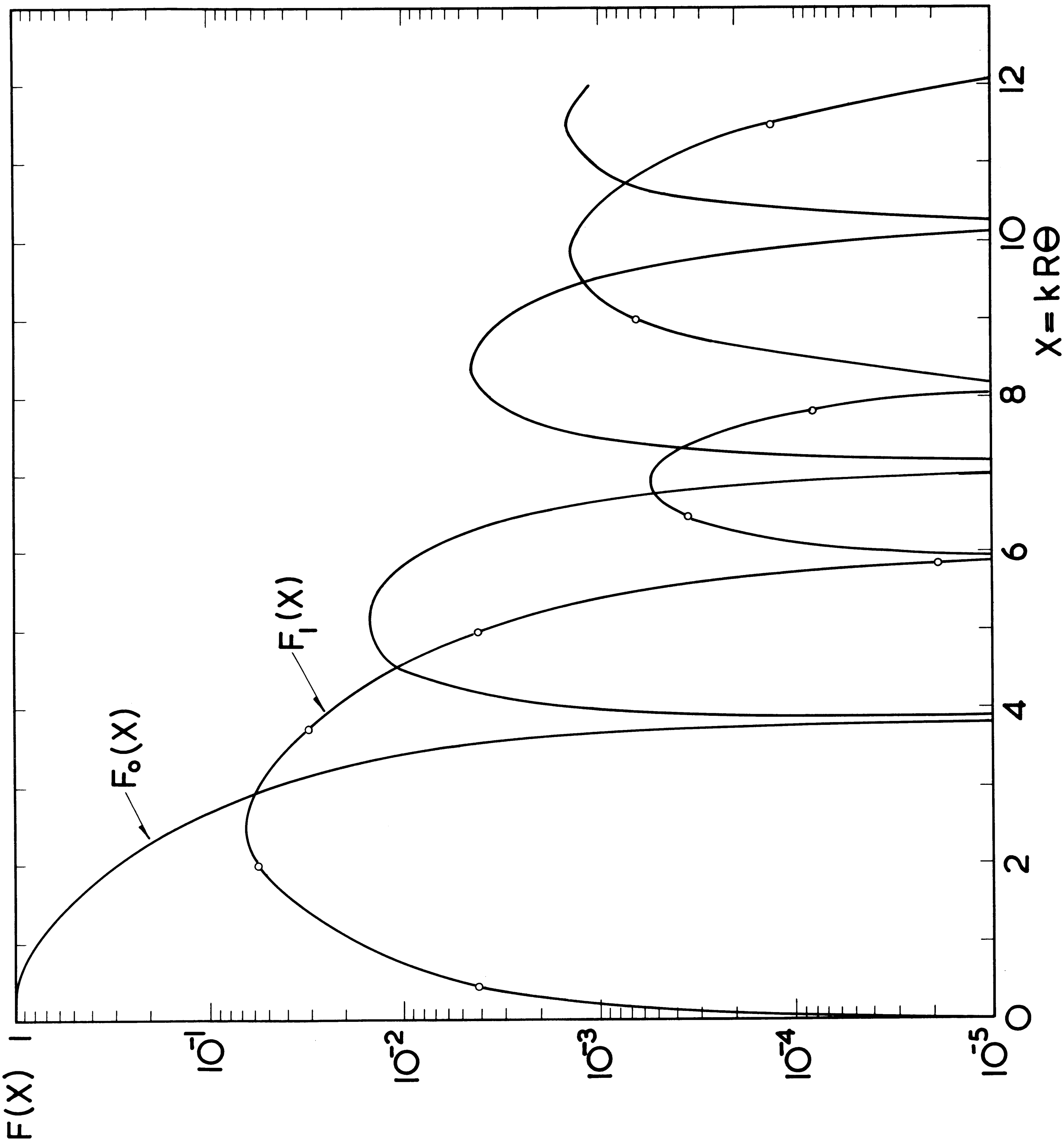
FIGURE CAPTION

Fig. 1 Predictions of the optical model for the angular distributions of the elastic and the quasi-elastic peaks, on the assumption of a "black" sphere of radius R. The curve labelled $(0^+ \rightarrow 0^+)$ is a plot of $\left[2J_1(X)/X\right]^2$, while that labelled $(0^+ \rightarrow 1^+)$ is a plot of

$$\left[X\,J_0(X) - \int_0^X J_0(x)dx\right]^2/X^4$$

The former is expected to apply also to the excitation of the $(1,3^-)$ and $(1,5^+)$ isobars (as well as to the elastic peak), while the latter is expected to be characteristic of the $(3,3^+)$ isobar (see text).



F(X)
$F_0(X)$
$F_1(X)$
$X = kR\Theta$