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BARYON SPECTROSCOPY AND CHIRAL SYMMETRY*

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Abstract

The status of light-quark baryon spectroscopy is reviewed. There are 57 well-established (3- and 4-star) resonances and 46 weak candidates (1- and 2-star). All are (qqq) states. Based on SU(3) relations, the established N* and Δ* states imply the existence of twice the known number of Λ*, Σ*, Ξ* and Ω* states. Quark models predict the existence of many more states. At least half the established resonances belong to clusters with the same pole position but different spin or parity. Over half the resonances come in parity doublets. There are several peculiar S-wave resonances with a strong coupling to the η-meson. The up-down quark mass difference obtained from the average of 16 isospin-multiplet mass-splittings is $m_d - m_u = +(3.1 \pm 0.3)$ MeV. The strange-down quark mass difference determined from 7 baryon SU(3) multiplet mass-splittings yields $m_s - m_d = 157 \pm 30$ MeV and 9 meson multiplets yield 108 ± 20 MeV.

1. Introduction

Hadron Spectroscopy has impressive credentials. It has led directly to three major advances in particle physics that have become cornerstones of QCD. Firstly was the application of SU(3) symmetry by Gell-Mann and Ne'eman. This was followed by the introduction of three light quarks, the u, d, and s, as the elementary constituents of the known hadrons by Gell-Mann and Zweig. Thirdly, baryon spectroscopy, in particular the properties of the Δ^{++}, required the existence of a new degree of freedom, that of color.

The discovery of the J/Ψ meson in 1974 stunted the growth of light-quark hadron spectroscopy. Lured by prospects for novel hadronic matter containing heavy quarks, many a hadron spectroscopsist hopped on the charm bandwagon. New particle-physics facilities such as Tristan and Doris were devoted to the pursuit of heavy quarks. Potent new proton accelerators for intense secondary beams, LAMPF II, The European Hadron facility and KAON, failed to be funded and light-quark spectroscopy famished. The Japanese Hadron Project is a major breakthrough for light-quark spectroscopy. Together with the upgraded AGS, the necessary facilities will finally be available to attack the major problems in nuclear and particle physics such as non perturbative QCD, quark confinement at low temperatures and the role of chiral symmetry.

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We hope to explore the intriguing regularities displayed by the baryonic resonances such as the clustering of states and the occurrance of parity doublets. We can search for low energy manifestations of the gluon and make accurate determinations of the basic quark masses of QCD, which are uncertain to 50% at present.

2. Baryon Resonances

The Review of Particle Properties [1] lists 103 candidates for the light-quark baryon resonances. All 103 are considered to be (qqq) states of flavor SU(3). There is no solid evidence for the existence of exotica such as: hybrids, the H - particle, strangelets, Z^* states and ($qqq\bar{q}q$) pentaquarks. The different resonances can be grouped into six SU(3) families, each one is identified by the light-flavor quantum numbers of isospin and strangeness, see Table 1. To indicate the quality and status of a resonance candidate the Particle Properties Review group assigns "stars" to each state with the following meaning: 3 or 4 stars implies an established state, while 1 or 2 stars indicates a questionable one.

Table 1. Summary of the Light-Quark Baryon Resonance Candidates.

I	S	Family Symbol	Total # of Candidates	****	***	**	*
$\frac{1}{2}$	0	N^*	22	11	3	5	3
$\frac{3}{2}$	0	Δ^*	22	7	4	4	7
0	-1	Λ^*	18	9	5	1	3
1	-1	Σ^*	26	6	4	8	8
$\frac{1}{2}$	-2	Ξ^*	11	2	4	2	3
0	-3	Ω^*	4	1	1	2	-
Totals			103	36	21	22	24

* = hint ** = questionable *** = reasonably established **** = well established

The numbers of resonances in each of the SU(3) families are related to one another. For example, $n_6(\Omega^-)^* = n_2\,(\Delta^*)$ where n_6 is the number of $(\Omega^-)^*$ resonances and n_2 the number of Δ^*'s. Table 2 gives the relation between the different n_i's. For the ground state multiplet the SU(3) flavor singlet Λ is forbidden by Fermi statistics.

Table 2. SU(3) Requirements

n_i	Name	n (observed) (4 + 3 stars)	n minimal
n_1	N^*	14	14
n_2	Δ^*	11	11
n_3	Λ^*	14	27
n_4	Σ^*	10	25
n_5	Ξ^*	6	25
n_6	Ω^{-*}	2	11
		Totals: 57	113

When one considers only 3- and 4-star states to be bonafide resonances there are 57 established light-flavor baryon resonances. The SU(3) relations of Table 2 imply that there should be double that number, while quark models predict even more states [2]. There is a great need for a vigorous program in light baryon spectroscopy. Particularly dismal is the status of the Ξ^* and $(\Omega^-)^*$ families. This situation may be remedied by the JHP. It requires a separated 10 GeV K^- beam and a modern, large acceptance detector such as an expanded version of the Crystal Barrel spectrometer at LEAR.

Recently, the AGS has approved a major new program in baryon spectroscopy of neutral states [3]. The chief detector is the famous SLAC Crystal Ball, CB, which has recently been moved to BNL. The CB consists of 672 N_a I crystals, individually viewed. It covers 94% of the 4π solid angle and it has good angular and energy resolution for multi-photon final states, π^o, η, ω, η'. The program covers various N^*, Δ^*, Λ^* and Σ^* resonances and candidate - resonances, reactions being studied are listed in Table 3.

Table 3. Program of Experiments in Baryon Spectroscopy*

Resonance Family	Initial States	Final States
N^*	$\pi^- p$	$n\gamma$, $n\pi^0$, $n\eta$, $n\eta'$, $n\omega$, $n\pi^0\pi^0$, $n\pi^0\eta$
Δ^*	$\pi^- p$	$n\gamma$, $n\pi^0$, $n\pi^0\eta$, $n\eta'$, $n\pi^0\pi^0$
Λ^*	$K^- p$	$\Lambda\gamma$, $\Lambda\eta$, $\Lambda\eta'$, $\Lambda\omega$, $\Lambda\pi^0\pi^0$, $\Sigma^0\gamma$, $\Sigma^0\pi^0$, $\Sigma^0\pi^0\eta$
Σ^*	$K^- p$	$\Lambda\gamma$, $\Lambda\pi^0$, $\Sigma^0\gamma$, $\Sigma^0\pi^0$, $\Sigma^0\eta$, $\Sigma^0\eta'$, $\Sigma^0\omega$, $\Sigma^0\pi^0\pi^0$

*To be done in the first round with the Crystal Ball at the AGS

3. Spectroscopic Regularities of Baryon Resonances

QCD does not predict the masses of the hadron resonances. Naively one might expect some regularity such as a simple spacing of the masses. It came as a surprise therefore when Hoehler pointed out that nearly half the resonances are grouped in clusters having the same pole position but different spins and parities. The same pole position implies similar but not identical masses of the resonances. For example, the first Δ– cluster includes the S_{31} (1900), P_{31}, (1910), P_{33} (1920), D_{35} (1930), F_{35} (1905) and F_{37} (1950). The origin of the clustering is unknown. It is not predicted by chiral symmetry quark or bag models or, the Skyrmion model. No clustering is found in meson spectroscopy, it is a feature of baryons only.

Another interesting property which is apparent in the compilation of baryons is the occurrence of parity doublets. It holds for over half the states. A typical example is the N* (1700), $J^P = \frac{3}{2}^+$ and its companion, the N* (1720), $J^P = \frac{3}{2}^-$. Various explanations have been offered for the existence of parity doublets such as a diquark substructure of the baryons [4]. It could also be a manifestation of chiral symmetry [5].

Another example of an interesting feature found in the Particle Data Tables [1] is the existence of a family of special S-wave resonance's, the N* (1535), Λ* (1670) and Σ* (1750). Suggestions have been made for the possible existence of an S-state Ξ* (1850); maybe, there might be even a $(\Omega^-)^*$[2220] state. The mass of each of the special S-wave resonances is close to the η- production threshold. Each resonance has a strong η- decay branch despite the phase-space reduction as compared to other decay channels. There are several interesting theoretical speculations about this [5].

The η-N and η–Λ interactions at low energy are attractive and strong. This has prompted the suggestion of the existence of new types of nuclear matter, eta mesic nuclei [6] and eta-mesic hypernuclei [7]. They await a thorough experimental exploration as do the aforementioned spectroscopic regularities. To help understand the physics of quark confinement it is clear that much more hadron spectroscopy is needed.

4. The Mass of the Light Quarks

QCD is a remarkable theory: one Lagrangian governs all strong interactions, at least in principle. QCD requires only 8 input parameters, they are: the strong coupling strength, (α_s), the parameter (Λ) that describes the energy (μ) dependence of α_s,

$$\alpha_s(\mu) \simeq 1/\ln(\mu^2/\Lambda^2), \tag{1}$$

and finally six quark-mass parameters (m_u, d_m, m_s, …). The experimental values (Ref. 1) of the 8 QCD input parameters are listed in Table 4. The uncertainty in the value of the light quark masses optimistically is 50%. Ideally all the QCD imput parameters should be known to a similar precision as α_s, which is 5%.

Table 4. The 8 Fundamental Input Parameters of QCD

$\alpha_s(m_Z) = 0.117 \pm 0.005$

$\Lambda^4_{\overline{ms}} = 234 \pm 26 \pm 50$ MeV

$m_u = 2\text{-}8$ MeV/c^2	$m_c = 1\text{-}1.6$ GeV/c^2
$m_d = 5\text{-}15$ MeV/c^2	$m_b = 4.1\text{-}4.5$ GeV/c^2
$m_s = 100\text{-}300$ MeV/c^2	$m_t \sim 170$ GeV/c^2

It will be helpful to split the QCD Lagrangian into two parts,

$$\mathcal{L}_{QCD} = \mathcal{L}_0 + \mathcal{L}_m \tag{2}$$

The $\mathcal{L}_0$ term is chiral symmetric, but the $\mathcal{L}_m$ term is not because it depends on the masses of the quarks,

$$\mathcal{L}_m = m_u \bar{u} u + m_d \bar{d} d + m_s \bar{s} s + ... \tag{3}$$

There are no free quarks and the quark masses must be extracted from properties of the multi-quark systems ($\bar{q}q$) and (qqq) which requires a model. There is no compelling theory for this. We shall use the modified minimal subtraction ($\overline{ms}$) scheme to obtain the "running" masses. A serious complication arises from the fact that the value of α_s increases with decreasing energy reaching the value $\alpha_s(m_\pi) = 1$. This excludes the use of perturbative QCD which has a successful record when as $\alpha_s << 1$ at multi GeV energies. Another problem is understanding quark-confinement at low energy into ($\bar{q}q$) and (qqq) systems, because it does not emerge from the QCD Lagrangian. This forces us to consider a phenomenological approach to confinement. A widely used method is based on the construction of an effective Lagrangian which respects chiral symmetry. The numerical values quoted for the three light-quark masses is the outcome of a systematic study of nuclear charge symmetry and SU(3) breaking of systems and reactions with small electromagnetic corrections.

Assuming m_u, m_d and m_s to be small, which follows from the approximate validity of isospin and SU(3) symmetry, we can expand M^2_h, (the mass squared of a light hadron) in a power series in the current quark masses.

$$M^2_h = A + B\,(m_u + m_d + m_s) + \text{e.m. corrections} \tag{4}$$

For mesons A = 0 leading to the well known ratio of SU(3) to SU(2) breaking [8,9],

$$R = (m_s - m_d)/(m_d - m_u) = (M_K^2 - M_\pi^2)/(M^2_{K^0} - M_{K^+}^2) \tag{5}$$

To proceed further we extract $m_d - m_u$ from charge symmetry breaking data of isospin multiplets, see Tables 5 and 6. The electromagnetic correction to the restmass of various multiplets averages to about zero. Mesons and baryons yield similar quark mass differences, the average is $(m_d - m_u) = +(3.1 \pm 0.3)$ MeV/c^2.

The mass splittings of seven baryon SU(3) multiplets are shown in Table 7. The average is $m_s - m_d = 157 \pm 30$ MeV/c^2. The nine available meson SU(3) multiplets listed in Table 8 yield $m_s - m_d = 108 \pm 20$ MeV/c^2. Further progress in understanding the possible difference of quark masses extracted from ($\bar{q}q$) and (qqq) states awaits new spectroscopic imput on the masses of various resonances.

Table 5. Mass Splittings of Baryonic Isopin-Multiplets

Particle Pairs	Quark Content	e.m. Corr. (MeV)	Mass Diff. (MeV)
n - p	(d - u) ud	-0.6	+1.3
$\Sigma^- - \Sigma^o$	(d - u) ds	+2.2	+4.9
$\Sigma^o - \Sigma^+$	(d - u) us	+0.2	+3.1
$\Sigma^{*-} - \Sigma^{*o}$	(d - u) ds	+1.3	$+3.5 \pm 1.2$
$\Sigma^{*o} - \Sigma^{*+}$	(d - u) us	-0.4	$+0.9 \pm 1.1$
$\Xi^- - \Xi^o$	(d - u) ss	+2.8	$+6.4 \pm 0.6$
$\Xi^{*-} - \Xi^{*o}$	(d - u) ss	+1.5	$+3.2 \pm 0.7$
$\Delta^o - \Delta^+$	(d - u) ud	-0.6	$+2.7 \pm 0.3$
$\Sigma_c^{++} - \Sigma_c^o$	(d - u) uc	-3.4	$+0.7 \pm 1.2$
$\Sigma_c^o - \Sigma_c^+$	(d - u) dc	-1.6	$-1.4 \pm 0.5 \pm 0.3$
$\Xi_c^o - \Xi_c^+$	(d - u) sc	-1.5	$+5.2 \pm 2.2$
	Average	-0.1	2.8 ± 0.3

Table 6. Mass-Splittings of Mesonic Isospin-Multiplets

Particle Pairs	Quark Content	e.m. Corr. (MeV)	Mass Diff. (MeV)
$K^0 - K^+$	$(d - u)\bar{s}$	-2.3	+4.0
$K^{*0} - K^{*+}$	$(d - u)\bar{s}$	-1.4	$+4.5 \pm 0.5$
$D^- - \bar{D}^0$	$(d - u)\bar{c}$	+3.2	$+4.8 \pm 0.3$
$D^{*-} - \bar{D}^{*0}$	$(d - u)\bar{c}$	+2.5	$+3.3 \pm 0.7$
$B^0 - B^+$	$(d - u)\bar{b}$	-1.2	$+0.3 \pm 0.3$
		Average: +0.2	3.2 ± 0.4

Table 7. The Mass-Splitting of Baryonic SU(3)-Multiplets

SU(3) Multiplet	Baryons	Quark Content	Mass. Diff. (MeV)
$\frac{3}{2}^+$ Decuplet	$\frac{1}{3}$ [Ω^-(1672) - Δ(1232)]	$3 \times$ (s-d)	147
$\frac{1}{2}^+$ Octet	$\frac{1}{2}$ [Ξ (1315) - n(939)]	$2 \times$ (s-d)u	188
$\frac{3}{2}^+$ Octet	$\frac{1}{2}$ [Ξ (1823) - N*(1520)]	$2 \times$ (s-d)u	152
$\frac{5}{2}^+$ Octet	$\frac{1}{2}$ [Ξ (2025) - N*(1680)]	$2 \times$ (s-d)u	173
$\frac{1}{2}^+$ Octet	$\frac{1}{2}$ [Λ(1600) + Σ(1660)] - N*(1440)]	ud(s-d)	190 ± 30
$\frac{1}{2}^-$ Octet	$\frac{1}{2}$ [Λ (1800 + Σ(1750)] - N*(1650)]	ud(s-d)	125 ± 25
$\frac{5}{2}^-$ Octet	$\frac{1}{2}$ [Λ 1830) + Σ(1775)] - N*(1675)]	ud(s-d)	127 ± 27
		Average:	157 ± 30

Table 8. The Mass Splitting of Mesonic SU(3)-Multiplets

Mesons	Mass Diff. (MeV)
$K^*(892)^0 - \frac{1}{2}[p^0(768) + \omega(752)]$	116
$\frac{1}{2}[\phi(1019) - K^{*0}(892)]$	127
$K_2^*(1432) - \frac{1}{2}[a_2(1318) - f_2(1275)]$	135 ± 22
$f_2'(1525) - K_2^*(1432)$	93
$K_3^*(1770)^0 - \frac{1}{2}[\rho_3(1691) + \omega_3(1668)]$	90
$D_s(1969)^+ - D^+(1869)$	100 ± 2
$D_s^*(2110)^+ - D^*(2010)^+$	100 ± 3
$D_{s1}(2535)^+ - D_1(2433)$	112 ± 6
$B_s(5375)^0 - B(5279)^0$	96 ± 7
	Average: 108 ± 20

In order to obtain the values of the quark masses from the above mass differences, one must assess the validity of the ratio of SU(3) to SU(2) breaking, Eq. 5, this depends on the importance of the higher terms in the expansion of $M^2{}_h$ which we have ignored in Eq. 4. Corrections in Eq. 4 are expected to be of order $(m_k / \Lambda)^2$ or 25% leading to a 50% uncertainty in the quark masses listed in Table 4. Extensive hadron spectroscopy is needed to reduce the errors in Tables 5-8 and to expand the listing to include more multiplets. The theoretical evaluation depends very much on the applicability of chiral perturbation theory, χPTh, which is discussed elsewhere [10].

The not-so-rare decay modes of the η meson and various η-decay spectra are particularly suitable to test the use of χPTh. The reasons for focusing on the η are several. The η is the heaviest Goldstone boson and is therefore extra sensitive to the higher order terms in the energy expansion. The electromagnetic decays such as $\eta \to \pi^+\pi^-\gamma$ are driven by the chiral anomaly. The η is subject to SU(3) octet-singlet mixing. Finally the η decays mainly via a G-parity forbidden strong interaction and by second order electromagnetic interactions.

A comparison of η decays and the prediction of χPTh are given in Table 9. There are no glaring discrepancies.

Table 9: Tests of Chiral Perturbation Theory in η Decay

Parameter	χPT	Ref.	Experimental Value	Ref.	Note
$\Gamma(\eta \to 3\pi^0)/\Gamma(\eta \to \pi^+\pi^-\pi^0)$	1.43 ± 0.03	22	1.37 ± 0.04	1, 21	1
$\eta \to \pi^+\pi^-\pi^0$ Dalitz plot "a"	-1.3	22	-1.0 ± 0.1	1, 21	2
$\eta \to \pi^+\pi^-\pi^0$ Dalitz plot "b"	0.38	22	0.1 ± 0.1	1, 21	2
$A(\eta \to \pi^+\pi^-\gamma)$ (GeV^{-3})	6.81	4	6.47 ± 0.25	23	3
$\Gamma(\eta \to 2\gamma)$(keV)	0.59	4	0.46 ± 0.04	1	4
$\Gamma(\eta \to \pi^0 \gamma\gamma)$(eV)	0.42 ± 0.20	24	0.85 ± 0.28	1	5
$\Gamma(\eta \to \pi^+\pi^-\pi^0)$keV)	0.16 - 0.24	22	0.27 ± 0.02	1, 21	6

Summary

Hadron spectroscopy is our only available means to determine the masses of the quarks which are fundamental parametered of QCD. The average of 16 isospin mass splitting yields the value $m_d - m_u = +(3.1 \pm 0.3)$ MeV/c^2. The SU(3) mass splitting averaged over eleven sets of baryon resonances is $m_s - m_d = +(157 \pm 30)$ MeV/c^2; while five sets of mesons give $+(108 \pm 30)$ MeV/c^2.

The inventory of the known light-quark baryon resonances coupled with the SU(3) relations indicate that less than half of the expected resonances have been found. There is no solid evidence for the existence of exotica.

Many baryons may be grouped in clusters with the same pole position but different spins and parities and there is abundant evidence that baryons come in parity doublets. There exists at least one family of peculiar S-wave resonances which have an anomalously strong coupling to the η meson.

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Table I. Tests of cPT in Strong and Electromagnetic Interactions

Parameter	Symbol	Unit	cPT	Ref.	Exp.	Ref.	Note
Scatt. Length	a^- (πN)	$10^{-3}\ m_\pi^{-1}$	88 - 96	8	86 ± 2	5, 6, 7	1
Scatt. Length	a ($\pi\pi$)	$10^{-2}\ m_\pi^{-1}$	20 ± 1	12	26 ± 5	9	2
S-wave Photoprod.	E_{0+} (π^0p)	$10^{-2}\ m_\pi^{-1}$	-1.2	14	-1.31 ± 0.08	13	3
S-wave Photoprod.	E_{0+} (π^- p)	$10^{-3}\ m_\pi^{-1}$	-31.3	16	-34.7 ± 1	15	4
S-wave Photoprod.	E_{0+} (π^+n)	$10^{-3}\ m_\pi^{-1}$	26.3	16	28.4 ± 0.5	16	5
P-wave Photoprod.	P_1(π^0p)/ q	GeV^{-2}	-0.48	14	-0.47 ± 0.01	13	6
Proton Elect. Pol.	$\bar{\alpha}$p	10^{-4} fm^3	10.5	15	12.1 ± 0.9	17	7
Proton Mag. Pol.	$\bar{\beta}$p	10^{-4} fm^3	3.5	15	2.1 ± 0.9	17	7
Neutron Elect. Pol.	$\bar{\alpha}$n	10^{-4} fm^3	13.4	15	12.3 ± 2.5	18	7
Neutron Mag. Pol.	$\bar{\beta}$n	10^{-4} fm^3	7.8	15	3.1 ± 2.8	18	7
Pion Pol.	$a_\pi + b_\pi$	10^{-4} fm^3	0	4	1.4 ± 4	4	8
πN Sigma-term	s	MeV	45 ± 10	20	60 ± 10	20	9

Table III. Mass Splittings of Baryonic Isopin-Multiplets

Particle Pairs	Quark Content	e.m. Corr. (MeV)	Mass Diff. (MeV)
n - p	(d - u) ud	-0.6	+1.3
$S^- - S^0$	(d - u) ds	+2.2	+4.9
$S^0 - S^+$	(d - u) us	+0.2	+3.1
$S^{*-} - S^{*0}$	(d - u) ds	+1.3	+3.5 ± 1.2
$S^{*0} - S^{*+}$	(d - u) us	-0.4	+0.9 ± 1.1
$X^- - X^0$	(d - u) ss	+2.8	+6.4 ± 0.6
$X^{*-} - X^{*0}$	(d - u) ss	+1.5	+3.2 ± 0.7
$\Delta^0 - \Delta^+$	(d - u) ud	-0.6	+2.7 ± 0.3
	(d - u) uc	-3.4	+0.7 ± 1.2
	(d - u) dc	-1.6	-1.4 ± 0.5 ± 0.3
	(d - u) sc	-1.5	+5.2 ± 2.2
	Average	-0.1	2.8 ± 0.3

Table IV. Mass-Splittings of Mesonic Isospin-Multiplets

Particle Pairs	Quark Content	e.m. Corr. (MeV)	Mass Diff. (MeV)
$K^0 - K^+$	$(d - u)\bar{s}$	-2.3	+4.0
$K^{*0} - K^{*+}$	$(d - u)\bar{s}$	-1.4	+4.5 ± 0.5
$D^- - \bar{D}^0$	$(d - u)\bar{c}$	+3.2	+4.8 ± 0.3
$D^{*-} - \bar{D}^{*0}$	$(d - u)\bar{c}$	+2.5	+3.3 ± 0.7
$B^0 - B^+$	$(d - u)\bar{b}$	-1.2	+0.3 ± 0.3
	Average:	+0.2	+3.4 ± 0.4

Table V. The Mass-Splitting of Baryonic SU(3)-Multiplets

SU(3) Multiplet	Baryons	Quark Content	Mass. Diff. (MeV)
$\frac{3}{2}^+$ Decuplet	$\frac{1}{3}$ [Ω^-(1672) - Δ(1232)]	3 ¥ (s-d)	147

$\frac{1}{2}^+$ Octet	$\frac{1}{2}$ [X (1315) - n(939)]	2 ¥ (s-d)u	188
$\frac{3}{2}^+$ Octet	$\frac{1}{2}$ [X (1823) - N*(1520)]	2 ¥ (s-d)u	152
$\frac{5}{2}^+$ Octet	$\frac{1}{2}$ [X (2025) - N*(1680)]	2 ¥ (s-d)u	173
$\frac{1}{2}^+$ Octet	$\frac{1}{2}$ [L(1600) + S(1660)] - N*(1440)]	ud(s-d)	190 ± 30
$\frac{1}{2}^-$ Octet	$\frac{1}{2}$ [L (1800 + S(1750)] - N*(1650)]	ud(s-d)	125 ± 25
$\frac{5}{2}^-$ Octet	$\frac{1}{2}$ [L 1830) + S(1775)] - N*(1675)]	ud(s-d)	127 ± 27
		Average:	157 ± 30

Table V. The Mass-Splitting or Baryonic SU(3)-Multiplets

SU(3) Multiplet	Baryons	Quark Content	Mass. Diff. (MeV)
$\frac{3}{2}^+$ Decupulet	$\frac{1}{3}$ [Ω-(1672) - Δ(1232)]	3 x (s-d)	147
$\frac{1}{2}^+$ Octet	$\frac{1}{2}$ [Ξ^o(1315) - n(939)]	2 x u(s-d)	188
$\frac{3}{2}^+$ Octet	$\frac{1}{2}$ [Ξ (1823) - N*(1520)]	2 x u(s-d)	152
$\frac{5}{2}^+$ Octet	$\frac{1}{2}$ [Ξ (2025) - N*(1680)]	2 x u(s-d)	173
$\frac{1}{2}^+$ Octet	$\frac{1}{2}$ [Λ(1600) + Σ(1660)] - N*(1440)]	ud(s-d)	190 ± 30
$\frac{1}{2}^-$ Octet	$\frac{1}{2}$ [Λ (1800 + Σ(1750)] - N*(1650)]	ud(s-d)	125 ± 25
$\frac{5}{2}^-$ Octet	$\frac{1}{2}$ [Λ 1830) + Σ(1775)] - N*(1675)]	ud(s-d)	127 ± 27
		Average:	157 ± 30

Table VI. The Mass Splitting of Mesonic SU(3)-Multiplets

Mesons	Mass Diff. (MeV)
	116
$\frac{1}{2}$ [f(1019) -	127
	135 ± 22

$f_2'(1525)$ -	93
	90
	100 ± 2
	100 ± 3
$D_{s1}(2535)^+$ - $D_1(2433)$	112 ± 6
$B_s(5375)^0$ - $B^0(5279)$	96 ± 7
	Average: 108 ± 20

Table VII. Summary of the Light-Quark Baryon States.

I	S	Family Symbol	Total # of Resonances	Star Assignment ****	***	**	*
$\frac{1}{2}$	0	N^*	22	11	3	5	3
$\frac{3}{2}$	0	Δ^*	22	7	4	4	7
0	-1	L*	18	9	5	1	3
1	-1	S*	26	6	4	8	8
$\frac{1}{2}$	-2	X*	11	2	4	2	3
0	-3	Ω^*	4	1	1	2	–
Totals			103	36	21	22	24

Table 7: Tests of Chiral Perturbation Theory in η Decay

Parameter	χPT	Ref.	Experimental Value	Ref.	Note
$\Gamma(\eta \to 3\pi^0)/\Gamma(\eta \to \pi^+\pi^-\pi^0)$	1.43 ± 0.03	22	1.37 ± 0.04	1, 21	1

$\eta \rightarrow \pi^{+}\pi^{-}\pi^{o}$ Dalitz plot "a"	-1.3	22	-1.0 ± 0.1	1, 21	2
$\eta \rightarrow \pi^{+}\pi^{-}\pi^{o}$ Dalitz plot "b"	0.38	22	0.1 ± 0.1	1, 21	2
$A(\eta \rightarrow \pi^{+}\pi^{-}\gamma)$ (GeV^{-3})	6.81	4	6.47 ± 0.25	23	3
$\Gamma(\eta \rightarrow 2\gamma)$(keV)	0.59	4	0.46 ± 0.04	1	4
$\Gamma(\eta \rightarrow \neq^{o}\ \gamma\gamma)$(eV)	0.42 ± 0.20	24	0.85 ± 0.28	1	5
$\Gamma(\eta \rightarrow \pi^{+}\pi^{-}\pi^{o})$(keV)	0.16 - 0.24	22	0.27 ± 0.02	1, 21	6