EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

ISR-300/GS/68-3

A METHOD OF REDUCING THE PHASE OSCILLATION FREQUENCY IN A FAST CYCLING BOOSTER

by

W. Hardt and D. Möhl

Geneva - 31st January, 1968

1. Introduction

One of the difficulties in the design of a booster injector for a large proton synchrotron is the fact that an unusually large number of phase oscillations per revolution (Q_s) results from a choice of parameters which in other respects would seem to be appropriate.

The drawbacks of a high Q_s value are:

i) reduction of bucket area due to unequally distributed r.f. gaps[1];

ii) critical conditions for the phase lock system[2];

iii) transverse beam blow-up and an increase in closed orbit deviations, due to coupling between betatron and synchrotron oscillations[3][4].

In Part 2 of this report, the influence on Q_s of the basic machine parameters is discussed. In Part 3 it is proposed to reduce Q_s by the introduction of higher harmonic components into the magnet current waveform.

2. Dependence of maximum Q_s on machine parameters

Typical design studies for a fast cycling booster arrive at a biased sinewave type of magnet cycle and a sinusoidal accelerating voltage together with an r.f. programme for which the bucket area is very nearly a constant during the first part of the cycle. The solid line of Fig. 1 is an example of the $Q_s(t)$ curve of such a 'standard booster'.

At injection (stationary buckets) the Q_s value is given by

$$Q_{s\ inj} = \frac{A\ h\ |\eta_{inj}|}{16\ (\beta\gamma)_{inj}} \qquad (1)$$

(the symbols used are defined in Appendix C).
The maximum Q_s value for small bucket area may be approximated by

$$\hat{Q}_s \approx 0.4\ h^{3/4}\ A^{1/2}\left(\frac{R}{c}\right)^{1/4} \omega_{rep}^{1/4}\ \gamma_{inj}^{1/4}\ \eta_{inj}^{3/4}\ (\beta\gamma)_{inj}^{-7/8}(\beta\gamma)_{ej}^{1/8} \qquad (2)$$

(see Appendix B).
For a large bucket area, $\hat{Q}_s$ occurs at injection and has the value given by eq. (1).

If the booster's r.f. frequency is fixed by the need to synchronize with the r.f. of the main ring, then

$$h \propto R \qquad (3)$$

Equations (1) to (3) enable one to assess the influence of the design parameters on $\hat{Q}_s$. Obviously its dependenc on ejection energy ($(\beta\gamma_{ej})$) and repetition rate (ω_{rep}) is rather weak. Therefore, if a low Q_s is desired one would choose:

i) a low harmonic number, i.e. a small booster radius or a low r.f. frequency;
ii) a high injection energy;
iii) a small bucket area.

Unfortunately, the choice of these parameters is constrained by other requirements.

Essentially the booster radius is limited by the need for straight sections of adequate length[5]. The r.f. frequency is fixed in so far as it is desirable to have synchronization with the main ring and to work with mechanically tuned cavities. The bucket area must be large enough to cope with the energy spread of the injected beam and to preserve the required space charge capability of the booster. The possibility of raising the injection energy is constrained by cost.

A compromise between these conflicting requirements is the recently proposed[5] 0.2 - 8 GeV twin booster with 60 m radius. In this machine, for a bucket area of 6 mrad (in units of $\Delta p/m_o c$ x r.f. phase) the maximum Q_s value is 0.12. Although this would seem to be a safe value at low intensities, it is not entirely clear whether it may not adversely affect the transverse beam quality at large space charge levels.

Two other methods of reducing Q_s may be considered, which are essentially independent of the basic machine parameters discussed above:

i) The introduction of higher harmonics into the r.f. voltage.
ii) The introduction of higher harmonics into the magnet current waveform.

The first of these methods has recently been studied by Morton and Gram[6]. The modification of the magnet current waveform is discussed in the remainder of this report.

By addition of a second harmonic r.f. system as proposed by Morton and Gram, Q_s can in principle be reduced for all energies and its maximum can even be made somewhat smaller than the "injection value" given by eq. (1). The harmonic distortion of the magnet current waveform which seems to present fewer technical problems leaves the injection value unchanged but flattens the maximum of Q_s to differ less from $Q_{s\ inj}$.

3. Modifications to the magnet cycle

By the introduction of higher harmonic components into the magnet current waveform (Fig. 2), the minimum of the guiding field can be flattened (front porch!). This reduces the maxima of Q_s (Fig. 1), and of $\Delta Q_{sc} \propto (B\beta\gamma^2)^{-1}$ (Fig. 3), the Q-shift due to space-charge forces, which determines the transverse space charge limit. Ultimately the excursions of Q_s and $B\beta\gamma^2$ can thus be flattened to differ only slightly from their values at injection.

The discussion which follows is restricted to the insertion of one additional harmonic of an order between two and five. Possible combinations of two additional harmonics have also been studied. Although these alternatives seem to be attractive from the point of view of cost, results indicate an increase in complexity which in our opinion is hardly justified by the additional improvement.

We will assume in the following section, that the higher magnet harmonic is introduced in such a way as to provide optimum gain in Q_s. It was found that in this case the improvement in $B\beta\gamma^2$ (space charge limit) is at the same time very nearly a maximum. A somewhat different setting of the parameters (phase and amplitude of the higher harmonic) might be preferable to optimize r.f. parameters such as tuning speed $\dot{\beta}/\beta$ or voltage rise after injection, or to make $Q_s(t)$ and $B\beta\gamma^2$ decrease monotonically. During commissioning that setting of these parameters which gives optimum performance will be found empirically.

An estimate of these parameters for optimum gain in Q_s is derived in Appendix A. For more general cases a computer programme to calculate r.f. and space-charge parameters, assuming that the bucket area is a constant, was written. It was used together with a minimization routine (Minros) of the CERN computer library to optimize the magnetic field shape due to given criteria.

3.1 Discussion

Results for the example of the twin booster are summarized in Table 1 and in Figs. 1-5. Assuming a bucket area of 6 mrad, $\hat{Q}_s$ is reduced from 0.12 to 0.09 with a second magnet harmonic. At the same time, the maximum Q shift due to space-charge forces is reduced by about 15%. Similar improvements (see Table 2) could be achieved by raising the injection energy from 200 to at least 250 MeV. The costs for both the second magnet harmonic and a linac energy augmented by 50 MeV are comparable.

For a fourth harmonic system an equivalent increase of injection energy would be from 200 to at least 230 MeV. However, in this case the costs for the modification of the power supply are less than one third of the extra cost for 230 MeV linac energy.

The gain in space-charge capability by harmonic shaping of the B field is especially pronounced in cases, where Q_s restricts the permissible Q shift. Assuming that the space-charge limit is given by the first order sideband[4] of the nearest half integer resonance $n/2 < Q_o$ the permissible Q shift is:

$$\Delta Q_{sc} (1 - C^2/2) \leq (Q_o - n/2) - Q_s$$

(where $C \leq 1$ is the relative synchrotron oscillation amplitude). There is then a twofold gain in space-charge capability; firstly because ΔQ_{sc} is reduced and secondly because Q_s is lower. For the example of the twin booster the total improvement in intensity would be about 40% with the second, and 30% with the fourth magnet harmonic.

Further it is noted from Figs. 1 and 3 that the range where ΔQ_{sc} and Q_s increase is much reduced with higher harmonics, so there is less probability of crossing sidebands of the nearest half integer resonance in the unfavourable direction where the nonlinearity of space-charge forces does not limit the amplitude growth.

Besides Q_s and space charge, other parameters which enter directly into the design of the r.f. system are changed. The peak r.f. voltage required to accelerate a constant bucket area (Fig. 4) is somewhat increased (20 - 25%) but the sharp voltage rise immediately after injection (Fig. 4) and the maximum tuning speed (Fig. 5) are both considerably reduced. It seems that these changes would make it easier to provide the high accuracy of r.f. operation necessary in the early part of the cycle, where one would like to accelerate nearly full buckets.

The reduction in tuning speed is especially pronounced for the addition of a second magnet harmonic (Fig. 4). Also the gain in Q_s and $B\beta\gamma^2$ is highest. However, a fourth harmonic seems to be the most reasonable choice, since it is comparatively cheap (Table 2). A disadvantage of the third harmonic is the relatively large increase in peak r.f. voltage.

Odd harmonics can be used to flatten both the maximum and the minimum of the B field. With even harmonics the maximum is sharper for the cases discussed above. Even harmonics thus reduce the time available for phasing the booster bunches to the main ring buckets (by a factor of about 1.4 if the jitter of the fundamental harmonic remains unchanged). The time available seems still sufficient both for the quick phasing scheme proposed for the 300 GeV machine[2)] and for long range homing. If, however, an extended flat top would be desirable, a fifth harmonic might be the most reasonable choice. In all other respects, a fourth harmonic seems to be preferable.

Modification of a fast cycling magnet system to introduce a higher harmonic has been examined in connection with flat topping schemes for electron synchrotrons (see e.g. ref. 7). The insertion of a fourth harmonic is being studied at DESY and results of model tests should soon be available.

The basic modification consists in adding suitable LC-elements to the White circuit, so that it becomes resonant at the fundamental frequency and at the frequency of the higher harmonic component.

Since no cost figures for a second harmonic system seem to be available in literature we shall discuss cost calculations in Appendix B.

4. Summary and Conclusion

The introduction of a higher harmonic into the magnet current waveform seems to be a relatively cheap way of improving the performance of a fast cycling booster.

The gain in Q_s, space-charge capability, r.f. tuning speed and speed of voltage rise together with possible improvement in other r.f. requirements seems to justify the increase in cost and complexity of the magnet system. If one seeks an improvement in only Q_s and space-charge effects the addition of a fourth harmonic is definitely the most economic choice.

It is our opinion, that the power supply should be designed and built to include a higher harmonic current from the beginning. This would minimize costs, and provide the possibility to take full profit of the improved performance and flexibility during commissioning and later operation.

Acknowledgements

It is a pleasure to thank our colleagues W. Schnell and C. Zettler for their contributions with regard to the implications for the r.f. system.

APPENDIX A

Maximum Q_s and its reduction

a) Approximate formula for Q_s

The small amplitude synchrotron oscillation frequency is given by:

$$Q_s = \frac{h\,|\eta|\,A}{16\,(\beta\gamma)} \; \frac{(\cos\varphi_s)^{1/2}}{\alpha(\varphi_s)} \tag{A1}$$

The r.f. programme for a synchrotron is determined by the equations for bucket area and energy gain per turn:

$$A = 16\,\alpha_{(\varphi_s)} \left(\frac{\hat{U}\,\gamma}{U_p \cdot 2\pi\,h\,\eta} \right)^{1/2} \tag{A2}$$

$$\hat{U}\sin\varphi_s = 2\,\pi\,R\,(\dot{\beta\gamma})\,\frac{U_p}{c} \tag{A3}$$

For an analytical treatment the approximation:

$$\frac{(\cos\varphi_s)^{1/2}}{\alpha(\varphi_s)} \approx 2\,\frac{(\sin\varphi_s)^{1/4}}{(\alpha)^{1/2}} \tag{A4}$$

is used. The quality of this approximation can be seen from Table A1. From (A2) and (A3):

$$\frac{(\sin\varphi_s)^{1/2}}{\alpha(\varphi_s)} = \frac{16}{A} \left(\frac{\gamma\,R\,(\dot{\beta\gamma})}{|\eta|\,h\,c} \right)^{1/2}$$

Table A1

$\Gamma = \sin \varphi_s$	$\frac{(\cos \varphi_s)^{1/2}}{\alpha}$	$2 \cdot \left(\frac{(\sin \varphi_s)^{1/2}}{\alpha}\right)^{1/2}$
0	1	0
0.05	1.1	1
0.1	1.2	1.2
0.2	1.5	1.6
0.3	1.8	2
0.4	2.2	2.4
0.5	2.8	2.9
0.6	3.6	3.5
0.7	5	4.5
0.8	7.8	6
0.9	16.2	9.8
1	∞	∞

Using this relation, one obtains from (A1) and (A4):

$$Q_s \approx \frac{h^{3/4}\, A^{1/2}}{2} \left(\frac{R}{c}\right)^{1/4} \frac{\left[(\dot{\beta\gamma})\, \gamma\right]^{1/4} |\eta|^{3/4}}{(\beta\gamma)} \qquad \text{(A5)}$$

(for $0.1 \leq \sin \varphi_s \leq 0.7$).

b. <u>Maximum of Q_s for biased sinusoidal field</u>

Assuming that the maximum occurs near injection, the variation with γ and η can be neglected. For constant bucket area A one obtains from (A5):

$$Q_s \propto \frac{(\dot{\beta\gamma})^{1/4}}{\beta\gamma} \qquad \text{(A6)}$$

For $(\beta\gamma) = \overline{\beta\gamma} - \widetilde{\beta\gamma} \cos(\varphi_{(t)})$; $\varphi = \omega_{rep} t$ the magnet phase φ_M at the maximum is given approximately by:

$$\sin \varphi_M \approx \frac{1}{2} \left(\frac{(\beta\gamma)_{inj}}{\widetilde{\beta\gamma}}\right)^{1/2} \qquad \text{(A7)}$$

Thus $(\dot{\beta\gamma})$ at the maximum is given by:

$$(\dot{\beta\gamma}) = \omega_{rep}\, \widetilde{\beta\gamma} \sin \varphi_M = \frac{\omega_{rep}}{2} \left((\beta\gamma)_{inj}\, \widetilde{\beta\gamma}\right)^{1/2} \qquad \text{(A8)}$$

Substituting for β, γ, and η the values at injection and assuming $\widetilde{\beta\gamma} \approx \frac{1}{2} (\beta\gamma)_{ej}$ the expression (2) (Section 2 of this report) is derived from (A5) and (A8). The approximations made above are valid if

$$(\beta\gamma)_{ej} \gg 2(\beta\gamma)_{inj}.$$

c. Change of field shape to reduce Q_s

For an estimate the field law $(\beta\gamma)(t)$ which gives a constant $Q_s(t)$ may be calculated from (A6). By integration of (A6) for Q_s = const. the corresponding field law:

$$\beta\gamma = (c_1 - c_2 t)^{-1/3} \tag{A8}$$

$$c_1 = (\beta\gamma)^{-3}_{inj}$$

$$c_2 = (\beta\gamma)^{-3}_{inj} - (\beta\gamma)^{-3}_{ej} \frac{\omega_{rep}}{\pi}$$

is obtained.

We approximate this field by a sinusoidal field with two harmonics

$$\beta\gamma = \overline{\overline{\beta\gamma}} - \widetilde{\widetilde{\beta\gamma}} \left[\cos(\omega_{rep} t) - b_n \cos n (\omega_{rep} t - \varphi_o)\right] \tag{A9}$$

in such a way that injection and ejection energy correspond to the minimum and the maximum of the field and require that the first two derivations are the same at t = 0 for the fields (A8) and (A9). An approximate solution for the system of equations thus obtained is:

$$b_n = \frac{1}{n^2}$$

$$\varphi_o = \frac{(\beta\gamma)_{inj}}{\widetilde{\widetilde{\beta\gamma}}} \cdot \frac{1}{3\pi} \quad \text{(rad)}$$

$$\left.\begin{aligned} \widetilde{\widetilde{\beta\gamma}} &= \widetilde{\beta\gamma} = \frac{1}{2}\left((\beta\gamma)_{ej} - (\beta\gamma)_{inj}\right) \\ \overline{\overline{\beta\gamma}} &= \overline{\beta\gamma}\left(1 - \frac{1}{n^2}\right) \end{aligned}\right\} \text{ for } n = 2, 4, \ldots$$

$$\left.\begin{array}{lll}\widetilde{\widetilde{\beta\gamma}} & = & \widetilde{\beta\gamma}\,(1 + \frac{1}{n^2}) \\ \overline{\overline{\beta\gamma}} & = & \overline{\beta\gamma}\end{array}\right\} \quad \text{for } n = 3, 5, \ldots$$

(for $(\beta\gamma)_{ej} \gg (\beta\gamma)_{inj}$).

For n = 2, $(\beta\gamma)_{inj} = 0.686$, $(\beta\gamma)_{ej} = 9.48$ we obtain from these relations:

$$b = 0.25$$

$$\varphi_o \approx 1^o$$

$$\widetilde{\widetilde{\beta\gamma}} \approx \widetilde{\beta\gamma}$$

$$\overline{\overline{\beta\gamma}} = 0.75\ \overline{\beta\gamma}$$

The exact values for this case found numerically to give the lowest maximum Q_s and highest space-charge limit are:

$$b \approx 0.24$$

$$\varphi \approx 1.8^o$$

$$\widetilde{\widetilde{\beta\gamma}} = 0.99\ \widetilde{\beta\gamma}$$

$$\overline{\overline{\beta\gamma}} = 0.8\ \overline{\beta\gamma}\ .$$

APPENDIX B

Cost Figures for a Modified White Circuit

The normal White circuit is conveniently subdivided into groups one of which is shown in Fig. 6a. Ideally the impedance of each group (as seen between points A and B of Fig. 6a) should vanish for the angular frequencies $\omega = 0$ (d.c.) and $\omega = \omega_{rep}$ (repetition frequency) to limit the voltage to earth. In case of a simultaneous n^{th} harmonic the impedance should also disappear at the frequency:

$$\omega = n\,\omega_{rep}$$

In order to simplify the analytical treatment the following units are introduced:

- L for inductances (L being the inductance of a magnet group (Fig. 6a)).
- $\frac{1}{L\omega^2_{rep}}$ for capacitances.
- I_1 for currents (I_1 being the amplitude of the current through L).
- $\omega_{rep} I_1 L$ for voltages.
- ω_{rep} for frequencies.

The elements of the basic circuit are then described by the simple notation used in Fig. 6b. The higher harmonic component requires more elements. The simplest solutions are obtained by the insertion of parallel LC circuits either at points E, F or G (Fig. 6b) or by bridging D and G with a series LC circuit in such a way that the resonance conditions are met. A reasonable measure of the cost of the system is the stored energy in the capacitors and the stored energy in the chokes, the latter being less heavily weighted. A few simplifications will be made. The higher harmonic current is assumed to be:

$$I_n = \frac{I_1}{n^2}$$

which is only approximately true. The peak voltage across the capacitors is assumed to be given by $\hat{U}_1 + \hat{U}_n$. Under normal operating conditions the peak voltage will be slightly less than this but, on the other hand,

i) during commissioning this peak value can occur if the phase between the two harmonics escapes control;
ii) the difference between the real stored energy and the assumed one is small.

The insertion of a parallel circuit at points E or G (Fig. 6b) will not be discussed since the costs of these solutions compare unfavourably with those discussed below.

For the insertion of a series circuit (Fig. 6c) the resonance condition demands that:

$$x = \frac{1 + \frac{1}{\ell}}{n^2\alpha}$$

$$y = x\,(1 - \alpha)(n^2\alpha - 1) \quad \text{with} \quad x = my$$

The voltages across the capacitors are:

$$U_{1x} = 1 \qquad U_{1y} = \frac{1}{1 - \alpha}$$

$$U_{nx} = \frac{1}{n} \qquad U_{ny} = \frac{1}{n(n^2\alpha - 1)}$$

The total stored electric energy becomes:

$$2W_c = \frac{1 + \frac{1}{\ell}}{n^4\alpha} \left\{ \frac{1 - \alpha}{n^2\alpha - 1} + n^2 \frac{n^2\alpha - 1}{1 - \alpha} + 1 + 4n + n^2 \right\}$$

Optimum values for α are

$$\alpha \approx 0.5 \quad \text{for} \quad n = 2$$

$$\alpha \approx 0.2 \quad \text{for} \quad n = 4$$

Thus the optimum stored energies are given by:

$$2W_c \approx 2.69\,(1 + \tfrac{1}{\ell}) \quad \text{for} \quad n = 2$$

$$2W_c \approx 1.51\,(1 + \tfrac{1}{\ell}) \quad \text{for} \quad n = 4$$

These values have to be compared with $2W_c = (1 + 1/\ \ )$, the stored energy for the case in which there is no higher harmonic.

For the insertion of a parallel circuit (Fig. 6d) the resonance conditions require:

$$x = (1 + \tfrac{1}{\ell}) \frac{1}{n^2\alpha}$$

with $\alpha = my$

$$y = (1 + \tfrac{1}{\ell}) \frac{\alpha}{(1 - \alpha)(n^2\alpha - 1)}$$

The voltages across the capacitors are:

$$U_{1x} = n^2 \alpha \qquad U_{1y} = (n^2\alpha - 1)$$

$$U_{nx} = \frac{\alpha}{n} \qquad U_{ny} = \frac{1 - \alpha}{n}$$

The stored electric energy becomes:

$$2W_c = (1 + \frac{1}{\ell}) \frac{\alpha}{n^2} \left\{ \frac{1 - \alpha}{n^2\alpha - 1} + n^2 \frac{n\alpha^2 - 1}{1 - \alpha} + \frac{1 + 4n^3 + n^6}{n^2} \right\}$$

Optimum values for α are:

$$\alpha \approx \frac{2}{7} \quad \text{for} \quad n = 2$$

$$\alpha \approx \frac{1}{15} \quad \text{for} \quad n = 4$$

The optimized stored energies become:

$$2W_c \approx 2.15 \; (1 + \frac{1}{\ell}) \quad \text{for} \quad n = 2$$

$$2W_c \approx 1.196 \; (1 + \frac{1}{\ell}) \quad \text{for} \quad n = 4$$

The choke inductance ℓ must be optimized, taking into account the capital cost of the choke and its operation costs. ℓ is expected to be in the range $2 \leq \ell \leq 3$.

We conclude that the increase in the cost of capacitors for n = 2 is about 115% while for n = 4 the increase is only 20%. More complicated circuits exist which are slightly cheaper. Nevertheless, a second harmonic will roughly double the power supply cost, while a fourth harmonic costs at most 20% more.

This conclusion remains true, even when other components of the power supply which were neglected above, are taken into account. By similar calculations one arrives at an increase of costs for the power supply of about 60% in the case of a third harmonic and about 15% in the case of a fifth harmonic.

APPENDIX C

Some notations used

A	Total r.f. bucket area in units of $\Delta\beta\gamma$ x r.f. phase
B	Bucket area/(2π x bucket height): bunching factor for a full bucket at uniform phase space density
b_n	Amplitude of n^{th} magnet harmonic in units of amplitude of fundamental harmonic
c	Velocity of light
h	R.f. harmonic number
n	Order of higher magnet harmonic
Q	Number of betatron oscillations per revolution
Q_s	Number of synchrotron oscillations per revolution
ΔQ_{sc}	Incoherent Q-shift due to space-charge forces
R	Machine radius
T	$= 2\pi/\omega_{rep}$ time of one machine cycle
t_{ac}	Acceleration time ($\approx T/2$)
$\hat{U}$	Peak r.f. voltage per turn
eU_p	Particle rest energy (938 MeV for protons)
$\alpha(\varphi_s)$	R.f. bucket area in normalized units ($\alpha = 1$ for stationary bucket)
β	$= v/c$

$\beta\gamma$	$= p/m_o c$, also used for magnetic field strength (in units of $m_o c/e\varrho$)

The two harmonic field is denoted by

$$\beta\gamma(t) = \overline{\overline{\beta\gamma}} - \widetilde{\widetilde{\beta\gamma}} \left[\cos (\omega_{rep} t) - b_n \cos n (\omega_{rep} t - \varphi_o)\right]$$

γ	$= \frac{W}{m_o c^2}$ total energy in rest mass units
γ_{tr}	γ at transition energy
ω_{rep}	Angular magnet repetition frequency = frequency of fundamental magnet harmonic
φ_s	R.f. synchronous phase
φ_o	Constant phase angle, phase difference between fundamental and higher magnet harmonic
τ	$= 180^o \cdot t/t_{ac}$ normalised time
η	$= \frac{1}{\gamma^2} - \frac{1}{\gamma_{tr}^2}$.

TABLE I

R.f. and magnet parameters for twin booster with higher magnet harmonic

$$(B_{(t)} \propto \beta\gamma = (\overline{\overline{\beta\gamma}}) - \widetilde{\widetilde{\beta\gamma}} \left[\cos\varphi - b_n \cos n(\varphi - \varphi_o)\right], \quad \varphi = \omega_{rep} t$$

Higher harmonic	None	2nd	3rd	4th	5th
1. R.f. Parameters (Constant bucket area A = 6 mrad)					
Maximum Q_s	0.12	0.09	0.095	0.096	0.099
Minimum $(B\beta\gamma^2)$ ("space charge limit")	0.41	0.47	0.455	0.45	0.44
Maximum tuning speed $\dot{\beta}/\beta$ (sec^{-1})	105	62	81	88	96
Peak voltage (KV)	710	890	1000	840	820
2. Magnet Parameters					
DC part of field $(\overline{\overline{\beta\gamma}})$	5.08	4.05	5.08	4.81	5.08
Amplitude of basic harmonic $\widetilde{\widetilde{\beta\gamma}}$	4.39	4.35	4.83	4.37	4.58
Relative amplitude of higher harmonic b_n	-	0.24	0.1	0.06	0.04
Phase difference between basic and higher harmonic φ_o	-	1.8^o	2.2^o	3^o	3.5^o
Additional cost (in per cent of original power supply)	-	100%	60%	20%	15%

TABLE 2

Comparison of a booster with second or fourth magnet harmonic with a standard booster with higher injection energy

	Booster with 2nd magnet harmonic	Equivalent standard booster
Injection energy	200 MeV	250 MeV
Maximum Q_s	0.09	0.1
Minimum of $(B\beta\gamma^2)$	0.47	0.47
Peak r.f. voltage/turn (KV)	890	700
Maximum r.f. turning speed $\dot{\beta}/\beta$ (sec^{-1})	62	90
Frequency swing β_{max}/β_{min}	1.76	1.65
Voltage rise near injection $\dot{U}$ (MV/sec)	70	200
Additional cost (1967 prices)	100% of original power supply ≈ 9 MSF	+ 50 MeV linac energy ≈ 10 MSF
	Booster with 4th magnet harmonic	**Equivalent standard booster**
Injection energy	200 MeV	230 MeV
Maximum Q_s	0.096	0.105
Minimum of $(B\beta\gamma^2)$	0.45	0.45
Peak r.f. voltage/turn (KV)	840	705
Maximum r.f. turning speed $\dot{\beta}/\beta$ (sec^{-1})	88	95
Frequency swing β_{max}/β_{min}	1.76	1.67
Voltage rise near injection $\dot{U}$ (MV/sec)	90	200
Additional cost (1967 prices)	20% of original power supply ≈ 1.8 MSF	+ 30 MeV linac energy ≈ 6 MSF

Figure Captions

Fig. 1 $Q_s(t)$ for a standard booster (solid line) and a booster with second or fourth magnet harmonic.

Fig. 2 Shape of magnetic field B(t) with second or fourth magnet harmonic.

Fig. 3 Inverse of Q-shift due to space-charge for uniformly filled buckets.

Fig. 4 R.f. voltage per turn, required to accelerate a constant bucket area A = 6 mrad.

Fig. 5 R.f. tuning speed $\dot{\beta}/\beta$.

Fig. 6 Basic group of a White circuit (a, b) and possible modifications for the introduction of a higher harmonic.

References

1. K.R. Symon, V.D. Streben, L.J. Laslett, in Proc. Int. Conf. on High-Energy Accelerators, Frascati, p. 296 (1965).

2. CERN Study Group on New Accelerators, AR/Int. SG/64-15.

3. Ju. F. Orlov, Soviet Phys. JETP 5, p. 45-48 (1957).

   Ju. F. Orlov, E.K. Terassov, in Proc. Int. Conf. on High-Energy Accelerators, CERN, p. 263-73 (1959).

   K.W. Robinson, (H) CEA-54 (1958).

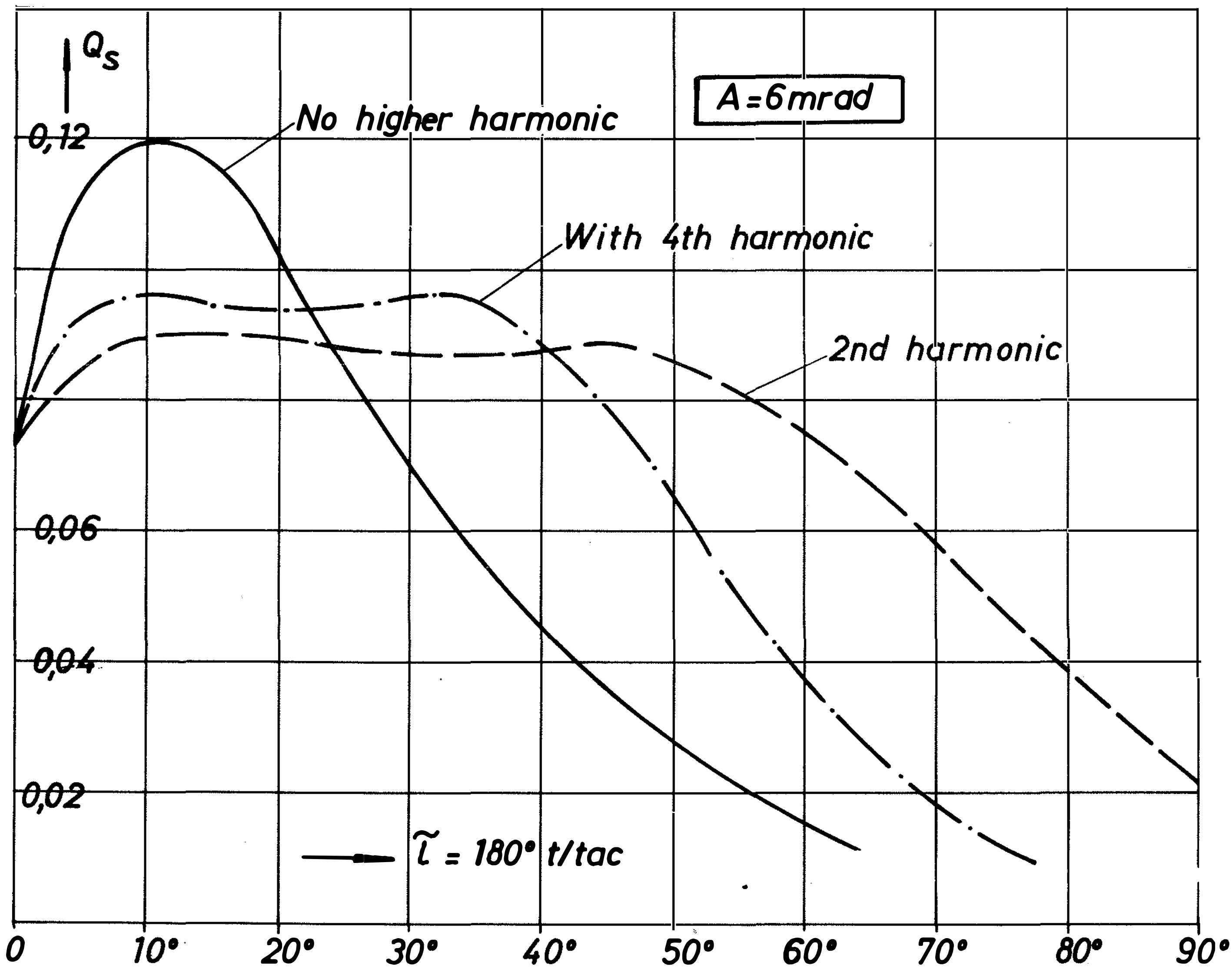
4. D. Möhl, CERN ISR-300/GS/67-32 (1967).

5. W. Schnell, CERN ISR-RF/67-21 (1967).

   R. Billinge, W. Schnell, CERN ISR-300/GS/67-46 (1967).

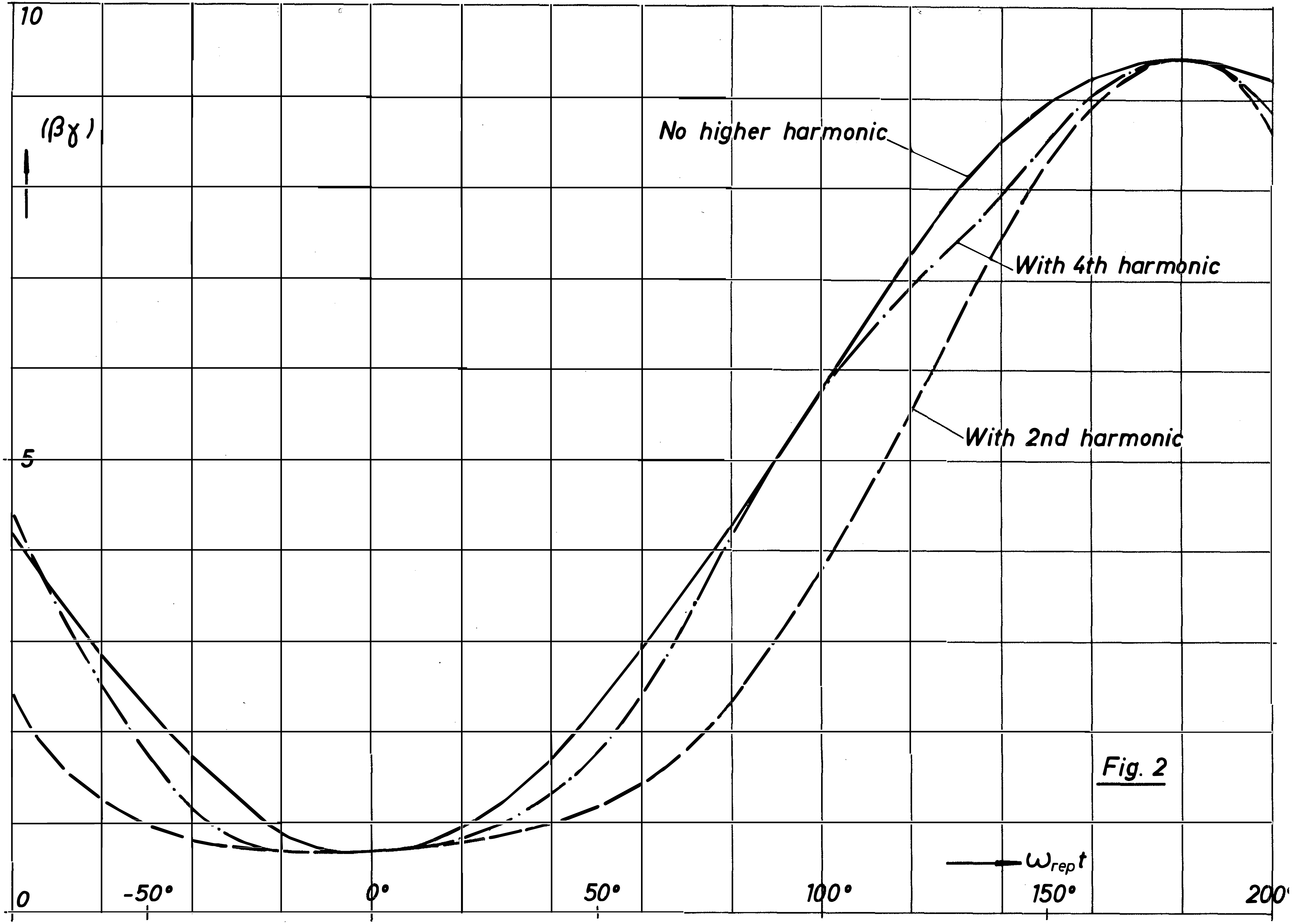
6. R. Gram, P. Morton, Advantages of a set of second harmonic r.f. cavities, Draft of a SLAC report (1967).

7. J. Riedel, PPAD 581 D (1966).

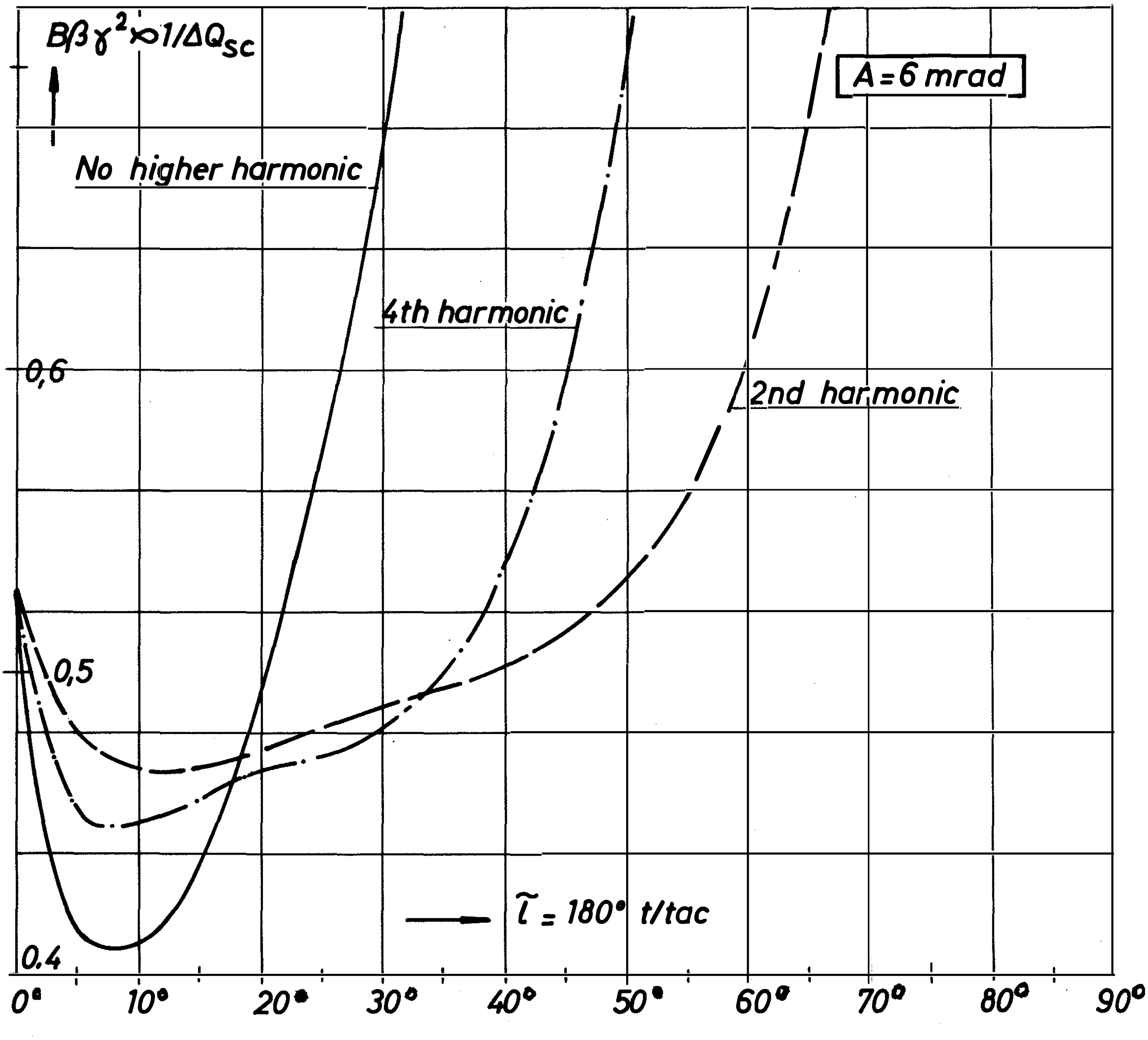


Q_s
A = 6 mrad
No higher harmonic
With 4th harmonic
2nd harmonic
0,12
0,06
0,04
0,02
τ = 180° t/tac
0
10°
20°
30°
40°
50°
60°
70°
80°
90°

Fig. 1

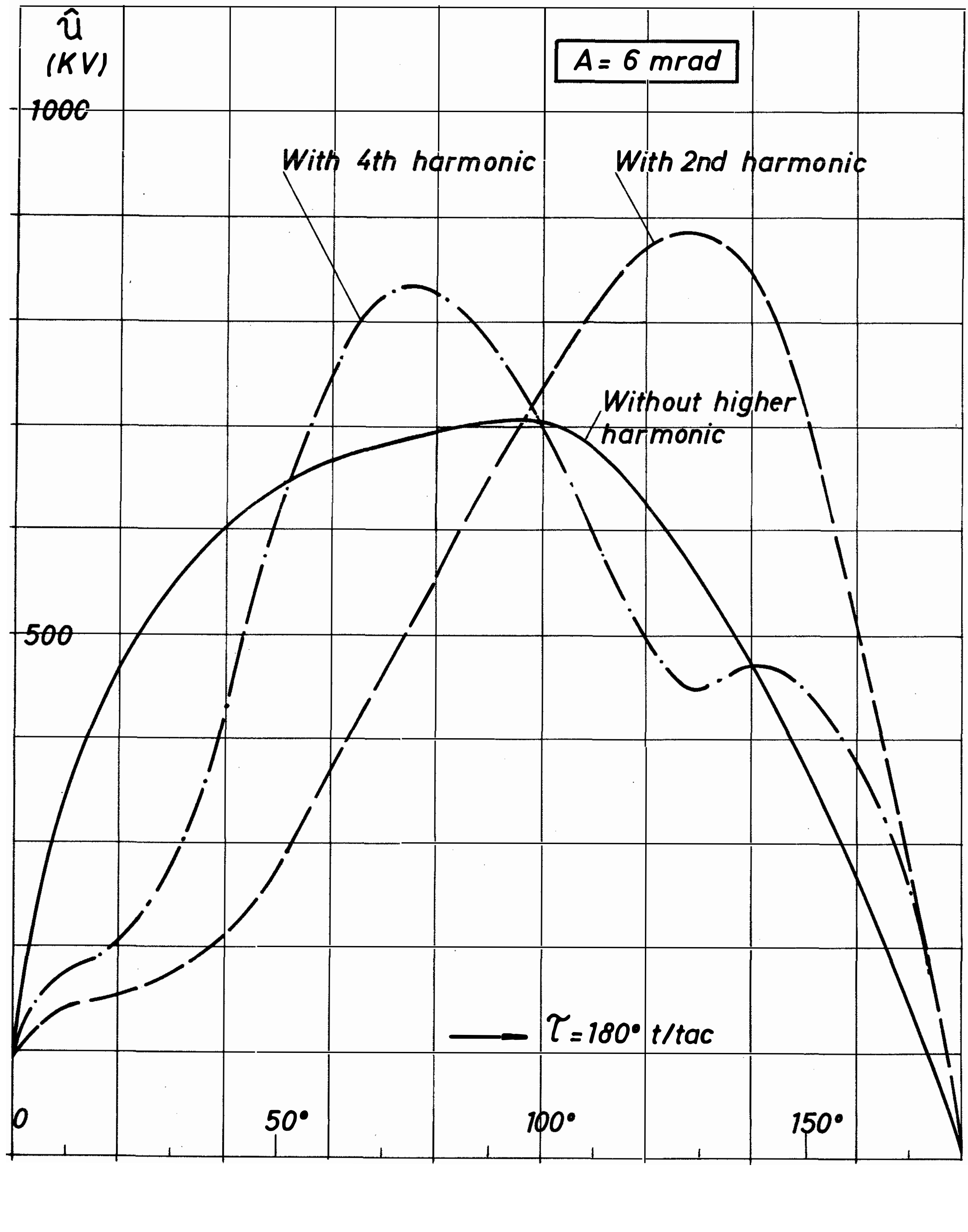


10
$(\beta\gamma)$
No higher harmonic
With 4th harmonic
With 2nd harmonic
5
Fig. 2
$\omega_{rep} t$
0
-50°
0°
50°
100°
150°
200°



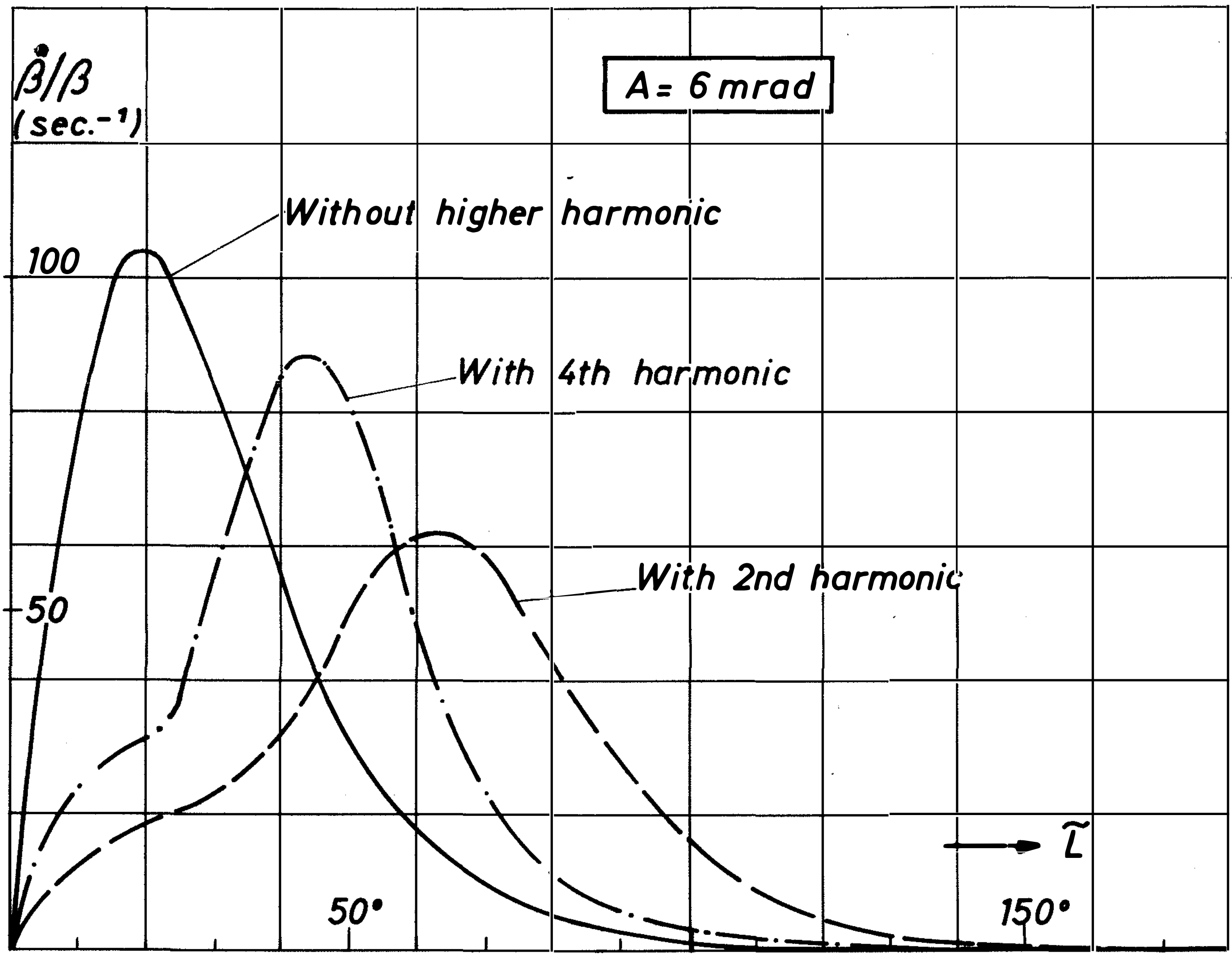
$B\beta\gamma^2 \propto 1/\Delta Q_{sc}$
A = 6 mrad
No higher harmonic
4th harmonic
2nd harmonic
0,6
0,5
0.4
$\tilde{\tau}$ = 180° t/tac
0°
10°
20°
30°
40°
50°
60°
70°
80°
90°

Fig. 3



û
(KV)
1000
500
A = 6 mrad
With 4th harmonic
With 2nd harmonic
Without higher harmonic
τ = 180° t/tac
0
50°
100°
150°

Fig. 4



$\dot{\beta}/\beta$
(sec.$^{-1}$)
A = 6 mrad
Without higher harmonic
100
With 4th harmonic
With 2nd harmonic
50
50°
150°
$\tilde{L}$

Fig. 5

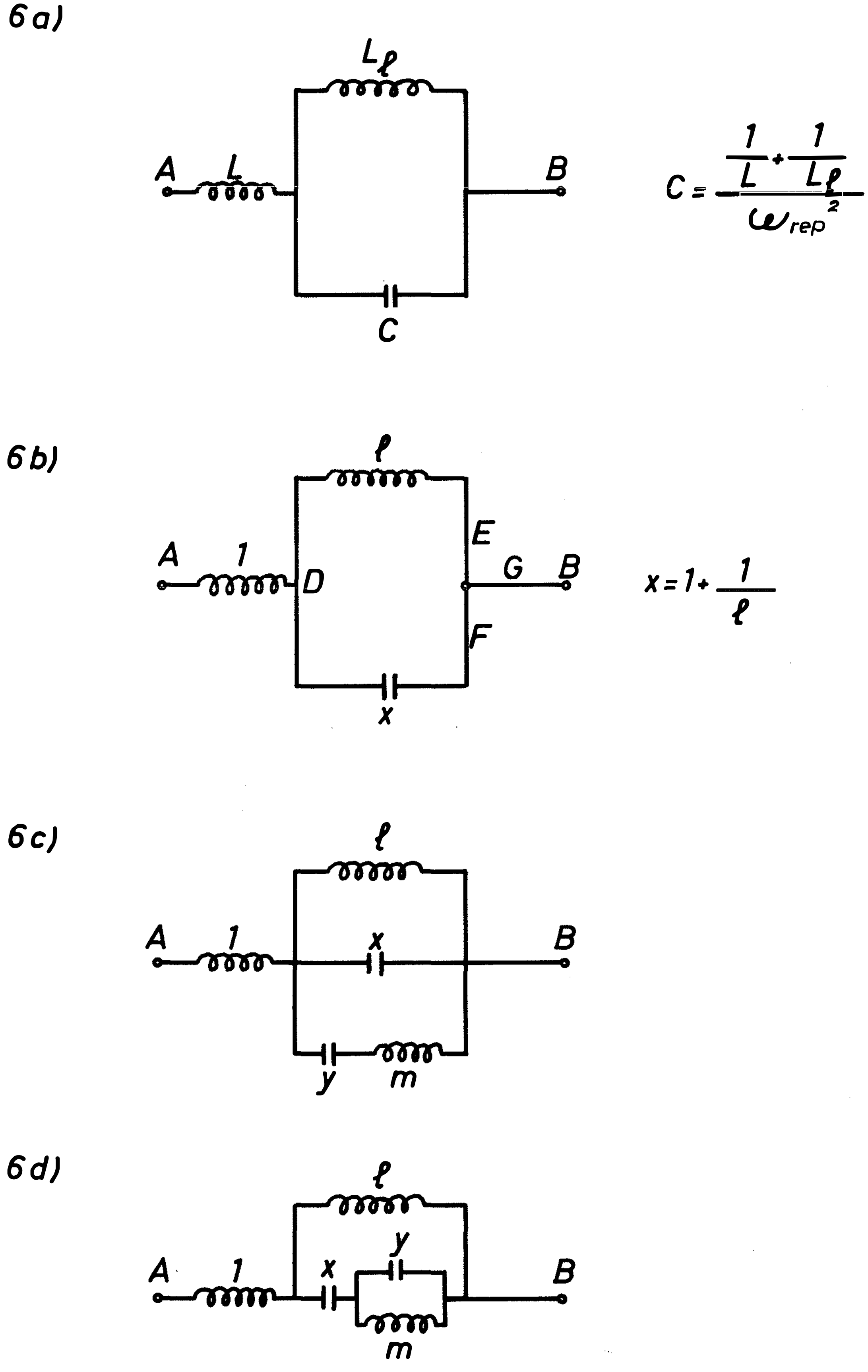

6 a)
L_ℓ
A
L
B
C
$C = \dfrac{\frac{1}{L} + \frac{1}{L_\ell}}{\omega_{rep}^2}$
6 b)
ℓ
A
1
D
E
G
B
F
x
$x = 1 + \dfrac{1}{\ell}$
6 c)
ℓ
A
1
x
B
y
m
6 d)
ℓ
y
A
1
x
B
m

Fig. 6