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Determination of the Bilocal Field in a QCD-based Model Field Theory*

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Abstract

The vacuum configuration of the bilocal field in a QCD-based model field theory(GCM) is shown to be equivalent to the dynamical mass of quarks. Combining the equivalent relation with the theory of many instantons, the bilocal field and the QCD effective action are obtained. Some meson properties are derived and calculated results agree with experimental data quite well. A quark confinement potential in hadron which is consistent with the empirical one is given.

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I. Introduction

It is known that quantum chromodynamics[1] (QCD) is a non-Abelian gauged quantum field theory of interacting quarks and gluons which is believed to be fundamental theory of hadrons. However, the main techniques for studying QCD are QCD Sum rules[2], instanton approximation[3], phenomenological models (MIT bag model, chiral bag model, etc)[4,5] and others. In QCD sum rules, the non-perturbative effects of QCD which is related to the underlying physics of hadrons come from the quark condensate and gluon condensate resulting from the extension of operator product expansion (OPE) technique. Since the coupling constant g in QCD may become very large in the case of low energy, the extension of the OPE technique of perturbative calculation is still in doubt. Meanwhile the condensates are defined through experiments. In instanton approximation, only the effect of pure Yang-Mills solution is emphasized. In the phenomenological models, some properties of hadrons can be described well, even though the consequences of the dynamically broken chiral symmetry are not accounted for properly.

To explain and unify the phenomenological models (take into account the chiral symmetry breaking and avoid exploiting the OPE technique), a QCD-based model field theory (referred to as global color symmetry model (GCM)), with QCD effective action, elementary quark fields and interacting via dressed vector boson exchange, has been put forward[6]. With the GCM, the properties of several mesons and baryon can be described[6-8] and the partial wave amplitudes in $\pi-\pi$ scattering can be reproduced well[9]. The most important ingredient of GCM is the bilocal-field representation $B(x,y)$ of the QCD generating functional, from which a bilocal field bosinization ($B(s)$) of QCD can be obtained. However, the practical determination of the vacuum configuration $B(s)$ is very difficult. To get the $B(s)$, a relation between the running coupling constant of QCD and the momentum of quarks was phenomenologically introduced[6] as $4\pi\alpha(q^2)/q^2 = 3\mu^2\delta^4(q)/16$. It is obvious that this is a quite rough approximation. Even with this assumption, the behavior of the $B(s)$ was still not good enough. In ref.[9], an improved expression of $B(s)$ was given, but three parameters were introduced.

In this paper, we propose a new way to determine the bilocal boson-type field. In Sec. II, we show the vacuum configuration of the bilocal field $B(p)$ is equivalent to the dynamical mass $m(p^2)$ of quarks. By employing the instanton dilute approximation[10,11], the vacuum configuration is the obtained. In Sec. III, some meson properties and quark confinement are discussed to check on the vacuum structure.

II. Determination of the Bilocal Field in GCM

In the global color symmetry model[6-9] (GCM), the QCD effective action[1] in Euclidean metric is given as

$$I[B^\theta] = -Tr\ln[\not\partial\delta(x-y) + \frac{1}{2}M^\theta B^\theta(x,y)] + \int \frac{B^\theta(x,y)B^\theta(y,x)}{2g^2D(x-y)}\,, \tag{1}$$

where $M^\theta = K^aC^bF^c$ is determined by Fierz transformation in color, flavor and Lorentz space[6]

It is obvious that the field in the effective action is a bilocal field. The quantization of the bilocal field $B^\theta(x,y)$ can be achieved, as usual, through the path integral $\int e^{-I[B^\theta(x,y)]}dB^\theta(x,y)$. In the following, we take $B^\theta(x,y)$, $B^\theta_q(x,y)$ to denote the classical field and the quantum field respectively. In fact, the quantum bilocal field is obtained from the bosonization of quark and anti-quark quantum fields. The vacuum structure $B^\theta(x,y)$ can then be described by the bilocal field $B^\theta(x,y)$ under the condition

$$\frac{\partial I[B^\theta(x,y)]}{\partial B^\theta(x,y)} = 0\,, \tag{2}$$

Although it is not possible to determine the vacuum state through eqs. (1) and (2), the general property of the vacuum configuration can be deduced. Since the vacuum has time-space translation invariant property, the vacuum configuration can be written as

$$B^\theta(x,y) = B^\theta(x-y)\,. \tag{3}$$

Defining the Fourier transformation of $B^\theta(x,y)$ as

$$B^\theta(p',p) = \int d^4xd^4y e^{ip'\cdot x-ip\cdot y}B^\theta(x,y)\,, \tag{4}$$

we can get

$$B^\theta(p',p)e^{ip'\cdot x-ip\cdot y} = \langle 0|B^\theta_q(x,y)|p',p\rangle^\theta\,, \tag{5}$$

where $|p,p'\rangle^\theta$ denotes the bound states of the quark and antiquark. The $B^\theta(p',p)$ is then the truncated Bethe-Salpeter amplitude as shown in figure 1. Therefore, the relation between the $B^\theta(p',p)$ and the Bethe-Salpeter amplitude can be given as

$$-S(p')M^\theta B^\theta(p',p)S(p) = \int {}^\theta\langle p',p|T(\psi(x)\bar\psi(y))|0\rangle e^{ip'\cdot x-ip\cdot y}dxdy\,, \tag{6}$$

where $S(p)$ is the propagator of the pseudoparticle. The $B^\theta(p',p)$ obtained in this way contains then all the possible excited states of various bosons. Because only the low

energy phenomena are concerned, the lowest excited states are then to be considered in the following calculation.

As all known, $B^\theta(p',p)$ can be separated into two parts: the motion of the center of mass and the relative motion. The motion of the center of mass has the same dynamical variable as that for point particle. The quantization formalism is completely similar to a local field. The relative motion dominates the internal structure of the bound states which can be naturally deduced from the effective action. Considering only the u, d quarks, the flavor symmetry is $SU(2)\otimes SU_A(2)$ and the lowest excited states are the Goldstone bosons. The quantum numbers of the Goldstone bosons are usually taken as isoscalar scalars and isovector pseudoscalars, which can be identified to be σ and π^i with a normalization

$$B^i(p',p) = \frac{\pi^i(p-p')}{f_\pi}B(\frac{p'+p}{2}) \tag{7a}$$

$$B(p',p) = \frac{\sigma(p-p')}{f_\sigma}B(\frac{p'+p}{2}) \tag{7b}$$

where $B(\frac{p'+p}{2})$ denotes the structure of the vacuum. The normalization constants f_π and f_σ can be determined by the second term of the effective action (eq.(1)). The $\pi^i(p-p')$ is the excitation against $\pi^i = 0$, and $\sigma(p-p')$ is an excited state against $\sigma = 1$. For the isovector field, eq. (6) can be rewritten as

$$-S(p)\tau^a\gamma_5\frac{B(p)}{f_\pi}S(p) = \int\langle\pi^a(0)|T(\psi(x)\bar\psi(0))|0\rangle e^{ip\cdot x}dx\,. \tag{8}$$

In order to get the vacuum configuration $B(p)$, we discuss the axial vector current in the PCAC scheme. In the massless QCD with u and d quarks, the axial vector currents are $A^a_\mu = \bar\psi\gamma_\mu\gamma_5\frac{\tau^a}{2}\psi$. The axial-vector vertex may then be written as

$$S(p')^5\Gamma^a_\mu(p',p)S(p) = -\int d^4xd^4ye^{ip'\cdot x}e^{-ip\cdot y}\langle 0|T(\psi(x)A^a_\mu(0)\bar\psi(y))|0\rangle\,. \tag{9}$$

where the vertex ${}^5\Gamma^a_\mu(p',p)$ can be represented as figure 2.

From Lorentz invariant analysis we know

$$q^\mu\,{}^5\Gamma^a_\mu(p',p) = q^2K(q^2) + q^\mu\gamma_\mu A(q^2)\,, \tag{10}$$

where $q = p'-p$. Since the second term on right hand side does not contain the pole, the limit of $q^\mu\gamma_\mu A(q^2)$ as $q\to 0$ approaches to zero. Otherwise we are interested in the vacuum excitation which corresponds the case $q\to 0$, we can consider only the term $q^2K(q^2)$.

According to the Landau and Cutkosky rule[12], the dispersion relation of $q_\mu K(q^2)$ can be written as

$$q_\mu K(q^2) = \frac{q_\mu R}{q^2} + \frac{q_\mu}{\pi}\int \frac{\mathrm{Im}(K(q'^2))}{q'^2 - q^2} dq'^2 \,, \tag{11}$$

in which the second term comes from the contribution of continuous spectrum that we are not interested in when $q \to 0$ since pion as a Goldstone boson is only the pseudoparticle. The first term is the pole term where R is the residue of the pole. We should then calculate only the R. From the Landau and Cutkosky rule, we know that the contribution to the pole term of the vertex ${}^5\Gamma^a_\mu$ (see Fig. 2) can be expressed as

$$q_\mu R = -i\sum_c \langle 0|A^a_\mu(0)|\pi^c\rangle\langle\pi^c|T(\psi\bar\psi)|0\rangle^p_{tr} \,, \tag{12}$$

where tr denotes to the truncated diagram (see Fig. 3 and the difference between Fig. 3 and Fig. 2).

According to PCAC theory,

$$\langle 0|A^a_\mu(0)|\pi^c(q)\rangle = if_\pi q_\mu \delta^{ac} \,. \tag{13}$$

Since $\langle\pi^c|T(\psi\bar\psi)|0\rangle^p_{tr}$ is the truncated Bethe-Salpeter amplitude, we have

$$\langle\pi^c|T(\psi\bar\psi)|0\rangle^p_{tr} = \int e^{ip\cdot x}\langle\pi^c|T(\psi(x)\bar\psi(0))|0\rangle_{tr} d^4x \,.$$

From eq.(8), we get

$$\langle\pi^c|T(\psi\bar\psi)|0\rangle^p_{tr} = -\tau^c\gamma_5 B(p)/f_\pi \,. \tag{14}$$

Therefore, when $p' \to p$ ($q \to 0$), we have

$$q^\mu\, {}^5\Gamma^a_\mu(p', p) = -\tau^a\gamma_5 B(p) \,. \tag{15}$$

On the other hand, considering the translation invariance, we can write the right hand side of eq.(9) as

$$\begin{aligned}
&-\int d^4x d^4y e^{ip'\cdot x} e^{-ip\cdot y}\langle 0|T(\psi(x)A^a_\mu(0)\bar\psi(y))|0\rangle \\
&= -\int d^4x d^4y e^{ip'\cdot(x-z)} e^{-ip\cdot(y-z)}\langle 0|T(\psi(x)A^a_\mu(z)\bar\psi(y))|0\rangle \,.
\end{aligned} \tag{16}$$

In massless QCD

$$\begin{aligned}
&\partial^\mu_z\langle 0|T(\psi(x)A^a_\mu(z)\bar\psi(y))|0\rangle \\
&= \langle 0|T\{[A^a_0(z), \psi(x)]\delta(x_0 - z_0)\bar\psi(y)\}|0\rangle + \langle 0|T\{\psi(x)[A^a_0(z), \bar\psi(y)]\delta(z_0 - y_0)\}|0\rangle \\
&= -\langle 0|T(\gamma_5\frac{1}{2}\tau^a\psi(z)\bar\psi(y)\delta(x - z))|0\rangle - \langle 0|T(\psi(x)\bar\psi(z)\frac{1}{2}\tau^a\gamma_5\delta(z - y))|0\rangle \,.
\end{aligned} \tag{17}$$

In momentum space, eq.(17) can be written as

$$\begin{aligned}
&-\int d^4x d^4y d^4z e^{ip'\cdot x - ip\cdot y - iq\cdot z}\partial^\mu_z\langle 0|T(\psi(x)A^a_\mu(z)\bar\psi(y))|0\rangle \\
&= \int d^4x d^4y d^4z e^{ip'\cdot x - ip\cdot y - iq\cdot z}\langle 0|T(\gamma_5\frac{1}{2}\tau^a\psi(z)\bar\psi(y)\delta(x - z))|0\rangle \\
&\quad + \int d^4x d^4y d^4z e^{ip'\cdot x - ip\cdot y - iq\cdot z}\langle 0|T(\psi(x)\bar\psi(z)\frac{1}{2}\tau^a\gamma_5\delta(z - y))|0\rangle \,. \\
&= \int d^4x d^4y e^{i(p'-q)\cdot x - ip\cdot y}\langle 0|T(\gamma_5\frac{1}{2}\tau^a\psi(x)\bar\psi(y))|0\rangle \\
&\quad + \int d^4x d^4y e^{ip'\cdot x - i(p+q)\cdot y}\langle 0|T(\psi(x)\bar\psi(y)\frac{1}{2}\tau^a\gamma_5)|0\rangle \,.
\end{aligned} \tag{18}$$

Since

$$\int d^4x d^4y e^{i(p'-q)\cdot x - ip\cdot y}\langle 0|T(\gamma_5\frac{1}{2}\tau^a\psi(x)\bar\psi(y))|0\rangle = i\int d^4x d^4y e^{i(p'-q)\cdot x - ip\cdot y}\gamma_5\frac{1}{2}\tau^a S(x - y) \,. \tag{19}$$

Writing $S(x - y)$ in momentum space and calculating the integral, we can get

$$\int d^4x d^4y e^{i(p'-q)\cdot x - ip\cdot y}\langle 0|T(\gamma_5\frac{1}{2}\tau^a\psi(x)\bar\psi(y))|0\rangle = i\gamma_5\frac{1}{2}\tau^a(2\pi)^4\delta(q - p' + p)S(p) \,. \tag{20}$$

By the same way, we can

$$\int d^4x d^4y e^{ip'\cdot x - i(p+q)\cdot y}\langle 0|T(\psi(x)\bar\psi(y)\frac{1}{2}\tau^a\gamma_5)|0\rangle = iS(p')\gamma_5\frac{1}{2}\tau^a(2\pi)^4\delta(q - p' + p) \,. \tag{21}$$

Eq.(18) can then be written as

$$\begin{aligned}
&-\int d^4x d^4y d^4z e^{ip'\cdot x - ip\cdot y - iq\cdot z}\partial^\mu_z\langle 0|T(\psi(x)A^a_\mu(z)\bar\psi(y))|0\rangle \\
&= i(2\pi)^4\delta(q - p' + p)[\gamma_5\frac{1}{2}\tau^a S(p) + S(p')\gamma_5\frac{1}{2}\tau^a] \,.
\end{aligned} \tag{22}$$

Calculating the integral in the left hand side of eq.(18) directly, we have

$$\begin{aligned}
&-\int d^4x d^4y d^4z e^{ip'\cdot x - ip\cdot y - iq\cdot z}\partial^\mu_z\langle 0|T(\psi(x)A^a_\mu(z)\bar\psi(y))|0\rangle \\
&= -\int d^4x d^4y d^4z \partial^\mu_z\{e^{ip'\cdot x - ip\cdot y - iq\cdot z}\langle 0|T(\psi(x)A^a_\mu(z)\bar\psi(y))|0\rangle\} \\
&\quad -q^\mu\int d^4x d^4y d^4z e^{ip'\cdot x - ip\cdot y - iq\cdot z}\langle 0|T(\psi(x)A^a_\mu(z)\bar\psi(y))|0\rangle \,.
\end{aligned} \tag{23}$$

Paying attention to the infinite bound and the translation invariance, we know

$$\begin{aligned}
&-\int d^4x d^4y d^4z e^{ip'\cdot x - ip\cdot y - iq\cdot z}\partial^\mu_z\langle 0|T(\psi(x)A^a_\mu(z)\bar\psi(y))|0\rangle \\
&= -iq^\mu\int d^4x d^4y d^4z e^{ip'\cdot x - ip\cdot y - iq\cdot z}\langle 0|T(\psi(x)A^a_\mu(z)\bar\psi(y))|0\rangle \\
&= -iq^\mu\int d^4x d^4y d^4z e^{ip'\cdot x - ip\cdot y - i(q-p'+p)\cdot z}\langle 0|T(\psi(x)A^a_\mu(0)\bar\psi(y))|0\rangle \\
&= -iq^\mu\int d^4z e^{-i(q-p'+p)\cdot z} d^4x d^4y e^{ip'\cdot x - ip\cdot y}\langle 0|T(\psi(x)A^a_\mu(0)\bar\psi(y))|0\rangle
\end{aligned}$$

With eq.(9), we have

$$-\int d^4x d^4y d^4z e^{ip'\cdot x - ip\cdot y - iq\cdot z}\partial_z^\mu \langle 0|T(\psi(x)A_\mu^a(z)\bar{\psi}(y))|0\rangle$$
$$= iq^\mu(2\pi)^4\delta(q-p'+p)S(p')\,{}^5\Gamma_\mu^a(p',p)S(p)\ . \tag{24}$$

Comparing eq.(22) with eq.(24), we get

$$iq^\mu(2\pi)^4\delta(q-p'+p)S(p')\,{}^5\Gamma_\mu^a(p',p)S(p)\ .$$
$$= i(2\pi)^4\delta(q-p'+p)[\gamma_5\frac{1}{2}\tau^a S(p) + S(p')\gamma_5\frac{1}{2}\tau^a]\ .$$

We know thus that the axial-vector vertices satisfies relation

$$q^\mu\,{}^5\Gamma_\mu^a(p',p) = S^{-1}(p')\frac{\tau^a}{2}\gamma_5 + \frac{\tau^a}{2}\gamma_5 S^{-1}(p)\ , \tag{25}$$

where $S^{-1}(p) = A(p^2)\not{p} - m(p^2)$ is the inverse of the complete fermion propagator in which the $m(p^2)$ is the dynamical mass of the quarks[13].

When $p' \to p$, it becomes

$$q^\mu\,{}^5\Gamma_\mu^a(p,p) = -\tau^a m(p^2)\gamma_5\ . \tag{26}$$

Comparing eq. (15) with eq. (26), we get

$$B(p) = m(p^2)\ . \tag{27}$$

It indicates that the bilocal field in the GCM can be expressed as the dynamical mass of quarks and implies that the dynamical chiral symmetry is produced by the vacuum structure of the condensates of quarks and gluons.

Recently many evidences of strong interaction phenomena are in favor of the instanton structure of vacuum in QCD[10,11,14-16]. In particular, the complex configuration of the vacuum of QCD with different topological winding numbers can be constructed in the model of instanton dilute liquid[10,11]. Lattice QCD calculations[17] are also in consistent with the above point of view. In the scheme of instantons, the infrared catastrophe occurring at the low energy phenomenon can be handled by the instanton dilute liquid approximation. Because there are repulsive forces among the instantons, the size of instantons can not become large. It have been shown that the radii of instantons are $\bar{R} \approx (600\mathrm{MeV})^{-1} \approx \frac{1}{3}$ fm and the distances between the instantons $\bar{D} \approx (200\mathrm{MeV})^{-1} \approx 1$ fm[10,11]. Because instantons are distributions of the pure gauge fields, they can have weak overlapping. The background configuration of the pseudoparticles can then be written as

$$A_\mu = \sum_I A_\mu^I + \sum_{\bar{I}} A_\mu^{\bar{I}}\ , \tag{28}$$

where I denotes the contribution of instanton and $\bar{I}$ denotes the contribution of anti-instanton.

It has been shown that, in massless QCD, the quark propagator in the medium of instantons can be written as[11]

$$S(p) = [\not{p} - m(p^2)]^{-1}\ , \tag{29}$$

with dynamical mass

$$m(p^2) = \frac{\epsilon\bar{\rho}}{6}p^2\varphi'^2(p)\ . \tag{30}$$

Combining this result with eq.(27), we get the vacuum configuration of the bilocal field $B(p)$ as

$$B(p) = \frac{\epsilon\bar{\rho}}{6}p^2\varphi'^2(p)\ , \tag{31}$$

where $\varphi'(p) = \pi\bar{\rho}^2\frac{d}{dz}[I_0(z)K_0(z) - I_1(z)K_1(z)]$, in which $z = \frac{|p|\bar{R}}{2}$. $I_n(z)$ ($K_n(z)$) ($n = 0,1$) are the first (second) kind modified Bessel functions of order n. $\bar{\rho} \approx (200\ \mathrm{MeV})^4$ is the average density of the instantons. $\bar{R} = \frac{1}{3}$ fm is the average radii of the instantons. ϵ is a constant $(85\ \mathrm{MeV})^{-1}$. The general feature of $B(p)$ is illustrated in figure 4. And it has asymptotic behavior

$$B(p) = \begin{cases} \dfrac{2\pi^2\epsilon\bar{\rho}\bar{R}^2}{3} & p \ll \frac{1}{\bar{R}} \\ \dfrac{24\pi^2\epsilon\bar{\rho}}{\bar{R}^4 p^6} & p \gg \frac{1}{\bar{R}} \end{cases} \tag{32}$$

With eq. (7) and the results of the local fields of point particles, the bilocal field $B(p',p)$ can be determined.

III. Evaluation of Some Hadron Properties

From eq. (1), we know that the effective action of the Goldstone particles can be written as

$$I[\sigma,\vec{\pi}] = -Tr\ln\{\not{\partial}\delta(x-y) + B(x-y) + [\frac{\theta(\frac{x+y}{2})}{f_\sigma} + \frac{i}{f_\pi}\gamma_5\vec{\pi}(\frac{x+y}{2})\cdot\vec{\tau}]B(x-y)\}$$
$$+\frac{1}{2}\int d^4z\{[1+\frac{\theta(z)}{f_\sigma}]^2 + \frac{1}{f_\pi{}^2}\vec{\pi}^2(z)\}\int d^4w B(w)Tr[G(w)]\ , \tag{33}$$

where $G^{-1}(x,y) = \not{\partial}\delta(x-y) + B(x-y)$ and the field $\theta(z) = \sigma(z) - 1$. This effective action has to be normalized to

$$I[\sigma(x),\vec{\pi}(x)] - I[1,0] = \frac{1}{2}\int[(\partial_\mu\theta)^2 + m_\sigma{}^2\theta^2]d^4z + \frac{1}{2}\int[(\partial_\mu\vec{\pi})^2 + m_\pi{}^2\vec{\pi}^2]d^4z + \cdots\ . \tag{34}$$

From this equation the constants f_π and f_σ can be determined. For example, with eq. (33) we know that the pion decay constant f_π holds the following relation

$$I[\sigma(x),\vec{\pi}(x)] - I[1,0] = Tr\ln[\delta(x-y) + G\, i\gamma_5 \frac{\vec{\pi}\cdot\vec{\tau}}{f_\pi} B(x-y)] . \qquad (35)$$

Combining eq. (35) with eq. (34), we get the decay constant of pion in the following

$$f_\pi^2 = \frac{3}{8\pi^2}\int_0^\infty sds \left[\frac{3B^2 + 2B^3\frac{dB}{ds} + sB^3\frac{d^2B}{ds^2} + 3sB^2\left(\frac{dB}{ds}\right)^2}{(s+B^2)^2} - \frac{sB^2\left(1+2B\frac{dB}{ds}\right)^2}{(s+B^2)^3}\right] , \qquad (36)$$

where $s = p^2$, $B(p) = m(p^2)$. By the same way, we obtain the decay constant of σ-meson as

$$\begin{aligned} f_\sigma^2 = \frac{3}{8\pi^2}\int_0^\infty sds \Bigg\{ & \frac{[1+2B\frac{dB}{ds} + sB\frac{d^2B}{ds^2} - s\left(\frac{dB}{ds}\right)^2]B^2}{(s+B^2)^2} \\ & + \frac{2[1+2B\frac{dB}{ds} + sB\frac{d^2B}{ds^2} + s\left(\frac{dB}{ds}\right)^2]B^2(s-B^2)}{(s+B^2)^3} \\ & - \frac{[1+2B\left(\frac{dB}{ds}\right)^2]sB^2(s-B^2)}{(s+B^2)^4} \Bigg\} . \end{aligned} \qquad (37)$$

With the data listed in Sec. II and eqs. (36) and (37), we obtain $f_\pi = 97$ MeV, $f_\sigma = 60$ MeV. The calculated result of f_π is quite close to the experimental data $f_\pi = 93$ MeV. Comparing this result with those in references [6], [7] and [8], we know that eq.(36) is almost the same as those obtained by considering only the effective action. However, because of the difference of the vacuum configuration $B(p)$, the numerical results are quite different. Their results are 72 MeV[6], 74 MeV[8] and 84 MeV[7] respectively. Even with three parameter being introduced, the calculated f_π is 91 MeV[9]. On the other hand, in the case of taking only the instanton dilute liquid approximation into account, the result is 138 MeV[11].

As mentioned above, the vacuum structure of the nonperturbative QCD can be described by the field with $\sigma = 1$, $\vec{\pi} = 0$. The effective action of the vacuum can be given as

$$\begin{aligned} I[1,0] &= \frac{-3}{4\pi^2}\int_0^\infty \int_0^\infty sds \left\{ \left[\ln\frac{s}{s+B^2}\right] + \frac{B^2}{s+B^2(s)} \right\} d^4z \\ &= \int_0^\infty K d^4z , \end{aligned} \qquad (38)$$

where

$$K = \frac{-3}{4\pi^2}\int_0^\infty sds \left\{ \left[\ln\frac{s}{s+B^2(s)}\right] + \frac{B^2(s)}{s+B^2(s)} \right\} . \qquad (39)$$

For short distance, the effective action in a hadron can be written as

$$I = I^{(P)} + I^{(NP)} \approx I[\bar{q},q,A]^{(P)} + I[1,0]_{\text{inside}} , \qquad (40)$$

where $I[\bar{q},q,A]^{(P)}$ corresponds to the perturbative contribution to interaction potential in a hadron which can be approximated to the one gluon exchange interaction. With eq.(38), the second term can be written as

$$I[1,0]_{\text{inside}} = \int_{\text{inside}} K d^4z = \int_0^\infty \left(\int K dxdydz\right) dt .$$

We have then

$$V = \int_{\text{inside}} K dxdydz . \qquad (41)$$

where K is determined with eq. (39) (about 23 MeV/fm^3). For the distance having been pulled in one dimension, it gives $V \propto Kr$. For three-dimensional sphere, it gives $V \propto Kr^3$. This indicates that eq. (41) provides a potential of quark confinement which is consistent with the empirical one.

IV. Conclusion

We have shown that the vacuum configuration of the bilocal field in GCM (a QCD-based model field theory by considering the effective action in QCD) can be expressed as the dynamical mass of quarks. Applying the approach of instanton dilute liquid to the above QCD-based model theory, the bilocal field and the QCD effective action are given. With this scheme, a potential of quark confinement which is consistent with the empirical confinement is obtained. The decay constants of pion and σ-meson are calculated. The calculated result agrees with experimental data quite well. Moreover, the agreement is much better than that obtained by considering only either the effective action or the instanton dilute liquid approximation. It shows that, with presently proposed scheme to determine the bilocal field and the QCD effective action in the GCM, one can discuss various low energy phenomena of strong interaction, especially the meson properties, on a firm foundation.

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Figure Captions:

Figure 1. The truncated Bethe-Salpeter amplitude

Figure 2. The vertex of the axial vector

Figure 3. The truncated vertex of the axial vector

Figure 4. The momentum dependence of the bilocal field

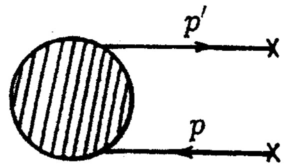

Fig. 1

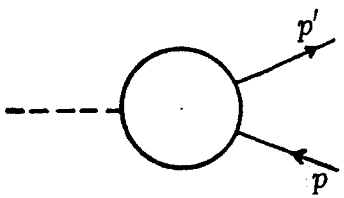

Fig. 2

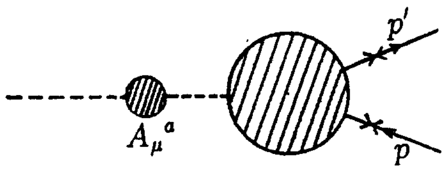

Fig. 3

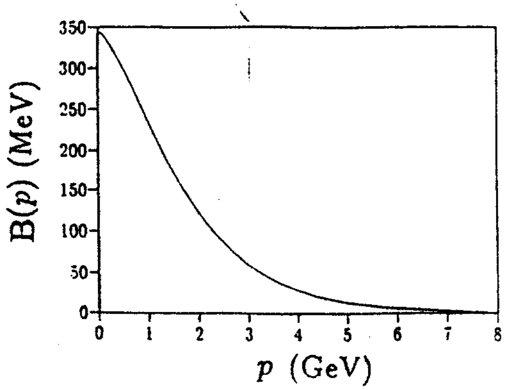

Fig. 4