

The Irrelevance of Primordial Black Hole Clustering in the LVK mass range

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Abstract. We show that in realistic models where primordial black holes are formed due to the collapse of sizeable inflationary perturbations, their initial spatial clustering beyond Poisson distribution does not play any role in the binary mergers, including sub-solar primordial black holes, responsible for the gravitational waves detectable by LIGO-Virgo-KAGRA. This is a consequence of the existing FIRAS CMB distortion constraints on the relevant scales. This conclusion might not hold for lighter masses potentially accessible by future gravitational wave observations.

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1 Introduction

The physics of Primordial Black Holes (PBHs) has garnered significant interest in recent years (see [1–5] for some reviews), largely due to the numerous detections of gravitational waves (GWs) originating from BH binary mergers [6–10] and the hypothesis that some of these may be of primordial origin [11–13].

Various mechanisms and models have been proposed as viable pathways for PBH production. Depending on the specific formation scenario and the models for PBH formation, a wide range of mass functions have been predicted (see [4] for a recent review). In this work, we adopt the standard formation scenario, where PBHs arise from the gravitational collapse of large over-densities in the primordial density contrast field upon their horizon re-entry [14–16]. Thus, to achieve a non-negligible PBH abundance, it is necessary to have an enhancement of the primordial curvature power spectrum on scales smaller than those probed by the Cosmic Microwave Background (CMB), where its spectral amplitude is approximately 10^{-9} [17]. In Ultra-Slow-Roll (USR) models of single-field inflation, for example, the peak in the power spectrum of the curvature perturbation arises from a brief phase of ultra-slow-roll which is typically followed by slow-roll or constant-roll inflation [14, 18–36]. In curvaton-like models, instead, the enhancement is due to an extra light field whose perturbations contribute to the curvature perturbation at the time of decay [37–57]. In both cases, the enhanced perturbations are accompanied by some level of non-Gaussianity (NG) [58–60].

The presence of NG induces a correlation between small and large scales. Hence PBHs are never exactly Poisson distributed at the time of formation, but are always correlated up to a given scale: small scales where the PBH is formed, in fact, are modulated by the large-scale fluctuations. Of course, the importance of clustering associated to PBH is significant only if such correlation is larger than the Poisson contribution. In addition, the initial clustering could influence the merger rate of PBH binaries, whose GWs could be detected by current and future experiments, with particular interest for the subsolar BHs which might be a smoking gun for their primordial nature [61, 62].

We show in this work that current observations prevent such a possibility for PBH masses in the LIGO-Virgo-KAGRA (LVK) mass range. The reasoning is the following. For PBH clustering to meaningfully affect the merger rate of PBH binaries within the (sub)-solar

mass range, PBHs must exhibit spatial correlations on scales of comoving kpc relevant to the current merger rate. Even by maximizing clustering with a broad curvature perturbation spectrum, current bounds on CMB distortions [63–68] significantly constrain the range of scales where clustering may play a role, see Ref. [69]. In the remaining relevant range of scales, the models commonly studied in the literature do not deliver a large enough NG to overcome the Poisson distribution. However, we will also show that, for PBH masses smaller than about $10^{-6} M_{\odot}$, whose binaries could lead to a signal potentially detectable by future experiments, clustering may indeed play a role.

The paper is organized as follows. In section 2, we briefly describe how to compute the PBH abundance in the presence of local non-Gaussianity in the curvature perturbation field, following the prescription based on threshold statistics on the compaction function. In section 3, we describe how to compute the correlation length and the bias factor in the presence of initial clustering PBHs, and then we compute these quantities for some realistic scenarios. In section 4, we show how in the relevant scales for the LVK observations and Einstein Telescope the spatial clustering does not play any role in the binary mergers. We conclude in section 5. The paper is also supplemented by one Appendix.

2 The PBH abundance

In this section, we summarize the formalism to evaluate the PBH abundance and set some useful definitions. The criterion we adapt for the formation of a PBH in a given region and for estimating the amount of clustering is based on the compaction function [70]. It is defined as twice the local mass (M) excess relative to the background value (M_b), divided by the areal radius $R(r, t) = a(t) e^{\zeta} r$ (in terms of the scale factor a and the comoving curvature perturbation ζ)

$$\mathcal{C}(r, t) = \frac{2[M(r, t) - M_b(r, t)]}{R(r, t)} = \frac{2}{R(r, t)} \int_{V_R} d^3\vec{x} \rho_b(t) \delta(\vec{x}, t). \quad (2.1)$$

On super-horizon scales, adopting the gradient expansion approximation, and assuming spherical symmetry, the density contrast is [71]

$$\delta(r, t) = -\frac{4}{9} \left(\frac{1}{aH} \right)^2 e^{-2\zeta(r)} \left[\zeta''(r) + \frac{2}{r} \zeta'(r) + \frac{1}{2} \zeta'^2(r) \right], \quad (2.2)$$

where $' \equiv d/dr$, the factor $4/9$ is for a radiation-dominated universe, and $\zeta(r)$ is assumed to be constant on super-horizon scales. We can use the super-horizon expansion since we are interested in the correlation on scales larger than the Hubble radius within which PBHs form.

In substituting the previous expression in Eq. (2.1) and performing the volume integral, the compaction function takes the form [71]

$$\mathcal{C}(r) = -\frac{4}{3} r \zeta'(r) \left[1 + \frac{r}{2} \zeta'(r) \right] = \mathcal{C}_1(r) - \frac{3}{8} \mathcal{C}_1^2(r), \quad \mathcal{C}_1(r) = -\frac{4}{3} r \zeta'(r). \quad (2.3)$$

Notice that \mathcal{C} becomes time-independent and Eq. (2.3) includes the full non-linear relation between δ and ζ . We define r_m as the scale at which the compaction function is maximized. Therefore, it verifies the condition

$$\mathcal{C}'(r_m) = 0 \quad \text{that is} \quad \zeta'(r_m) + r_m \zeta''(r_m) = 0 \quad (2.4)$$

in terms of the comoving curvature perturbation. If we define $\mathcal{C}_{\max} = \mathcal{C}(r_m)$ as the value of the compaction at the position of the maximum, PBHs form only if the maximum value of the compaction function exceeds some threshold value, $\mathcal{C}_{\max} > \mathcal{C}_c$. Notice also that, at the horizon crossing of the relevant scale $r_m = (aH)^{-1}$, the compaction function at its peak becomes equal to the fully non-linear density contrast smoothed over the horizon volume, which is the quantity we will compute the correlation of.

A crucial role in determining the PBH abundance, as well as the clustering, is the NG in the curvature perturbation [72–90]. In the literature, the local non-Gaussian behavior of the curvature perturbation ζ is usually parameterized by the expansion [91, 92]

$$\zeta = \zeta_g + \frac{3}{5}f_{\text{NL}}\zeta_g^2 + \frac{9}{25}g_{\text{NL}}\zeta_g^3 + \dots, \quad (2.5)$$

where ζ_g obeys the Gaussian statistics while the parameters $f_{\text{NL}}, g_{\text{NL}}, \dots$ (which, in full generality, depending on the scale of the perturbation) encode deviations from the Gaussian limit.

Generally, when a closed-form resummed expression for ζ is available, it has been shown that truncating the previous power series expansion at a fixed order can lead to an incorrect estimation of the PBH abundance [90, 93]. Consequently, to maintain model independence in this part of the work, we introduce primordial NGs through the following functional form

$$\zeta = F(\zeta_g). \quad (2.6)$$

In a radiation-dominated universe, the linear component of the compaction function takes the form

$$\mathcal{C}_1(r) = -\frac{4}{3}r\zeta'_g(r)\frac{dF}{d\zeta_g} = \mathcal{C}_g(r)\frac{dF}{d\zeta_g}, \quad \text{with} \quad \mathcal{C}_g(r) = -\frac{4}{3}r\zeta'_g(r). \quad (2.7)$$

Consequently, the compaction function reads

$$\mathcal{C}(r) = \mathcal{C}_g(r)F' - \frac{3}{8}\mathcal{C}_g^2(r)(F')^2, \quad (2.8)$$

where primes of the function F indicate derivatives with respect to ζ_g . The compaction function thus depends on both the Gaussian linear component \mathcal{C}_g and the Gaussian curvature perturbation ζ_g . Both are Gaussian random variables since ζ_g is Gaussian by definition, while \mathcal{C}_g is proportional to the derivative ζ'_g . We write [86]

$$\mathcal{C}_g(r) = -\frac{4}{9}r^2 \int d^3y \nabla^2 \zeta_g(\vec{y}) W(\vec{x} - \vec{y}, r) \quad (2.9)$$

and

$$\zeta_g(r) = \int d^3y \zeta_g(\vec{y}) W_s(\vec{x} - \vec{y}, r), \quad (2.10)$$

where W_s is the spherical-shell window with Fourier transform $W_s(k, r) = \sin(kr)/kr$ and W is the Heaviside-step function with Fourier transform

$$W(k, r) = 3 \left[\frac{\sin(kr) - kr \cos(kr)}{(kr)^3} \right]. \quad (2.11)$$

The two-dimensional joint PDF of ζ_g and \mathcal{C}_g can be written as

$$P_g(\mathcal{C}_g, \zeta_g) = \frac{1}{(2\pi)\sqrt{\det \Sigma_1}} \exp\left(-\frac{1}{2}\vec{Y}_1^T \Sigma_1^{-1} \vec{Y}_1\right), \quad \vec{Y}_1 = \begin{pmatrix} \mathcal{C}_g \\ \zeta_g \end{pmatrix}, \quad \Sigma_1 = \begin{pmatrix} \sigma_c^2 & \sigma_{cr}^2 \\ \sigma_{cr}^2 & \sigma_r^2 \end{pmatrix}, \quad (2.12)$$

where $\Sigma_1 = \langle \vec{Y}_1 \vec{Y}_1^T \rangle$ is the covariance matrix. The entries of Σ_1 are

$$\sigma_c^2 = \langle \mathcal{C}_g \mathcal{C}_g \rangle = \frac{16}{81} \int_0^\infty \frac{dk}{k} (kr_m)^4 W^2(k, r_m) P_\zeta^T(k), \quad (2.13)$$

$$\sigma_{cr}^2 = \langle \mathcal{C}_g \zeta_g \rangle = \frac{4}{9} \int_0^\infty \frac{dk}{k} (kr_m)^2 W(k, r_m) W_s(k, r_m) P_\zeta^T(k), \quad (2.14)$$

$$\sigma_r^2 = \langle \zeta_g \zeta_g \rangle = \int_0^\infty \frac{dk}{k} W_s^2(k, r_m) P_\zeta^T(k), \quad (2.15)$$

with $P_\zeta^T = T^2(k, r_m) P_\zeta(k)$, where $T(k, r_m)$ is the radiation transfer function and all the entries are evaluated at $r_m = 1/aH$.

After computing the inverse of Σ_1 and its determinant, and completing the square in the argument of the exponential function, Eq. (2.12) can be recast in the form

$$P_g(\mathcal{C}_g, \zeta_g) = \frac{1}{(2\pi)\sigma_c\sigma_r\sqrt{1-\gamma_{cr}^2}} \exp\left(-\frac{\zeta_g^2}{2\sigma_r^2}\right) \exp\left[-\frac{1}{2(1-\gamma_{cr}^2)} \left(\frac{\mathcal{C}_g}{\sigma_c} - \frac{\gamma_{cr}\zeta_g}{\sigma_r}\right)^2\right], \quad (2.16)$$

where

$$\gamma_{cr} = \frac{\sigma_{cr}^2}{\sigma_c\sigma_r}. \quad (2.17)$$

Using the conservation of the probability, we can therefore write

$$P(\mathcal{C} > \mathcal{C}_c) = \int_{\mathcal{D}} P_g(\mathcal{C}_g, \zeta_g) d\mathcal{C}_g d\zeta_g \quad (2.18)$$

$$\mathcal{D} = \{\mathcal{C}_g, \zeta_g \in \mathbb{R} : \mathcal{C}(\mathcal{C}_g, \zeta_g) > \mathcal{C}_c \wedge \mathcal{C}_1(\mathcal{C}_g, \zeta_g) < 4/3\}. \quad (2.19)$$

We are finally in the position to give our prescription to calculate the PBH abundance, following Ref. [90] (see also [94]) based on threshold statistics¹ on the compaction function \mathcal{C} . The abundance of PBHs is given by the integral (see e.g. [32])

$$\begin{aligned} f_{\text{PBH}}(M_{\text{PBH}}) &\equiv \frac{1}{\Omega_{\text{DM}}} \frac{d\Omega_{\text{PBH}}}{d \ln M_{\text{PBH}}} \\ &= \frac{1}{\Omega_{\text{DM}}} \int d \ln M_H \left(\frac{M_H}{M_\odot}\right)^{-1/2} \left(\frac{g_{*s}^4/g_*^3}{106.75}\right)^{-1/4} \left(\frac{\beta(M_{\text{PBH}}, M_H)}{7.9 \times 10^{-10}}\right), \end{aligned} \quad (2.20)$$

where $\Omega_{\text{DM}} = 0.264$ is the cold dark matter density of the universe, so the total abundance of PBHs is

$$f_{\text{PBH}} = \int f_{\text{PBH}}(M_{\text{PBH}}) d \ln M_{\text{PBH}}. \quad (2.21)$$

¹A discrepancy is present between peak theory and threshold statistics (see, e.g., Refs. [74, 95, 96]). A technical drawback of peaks theory is that it is not clear how to include NGs in the computation of the abundance using a generic functional form for the curvature perturbation field as in Eq. (2.6). Hence, for comparison, we limit our analysis to the threshold statistic approach.

The relation between the mass of the resulting PBH M_{PBH} and the horizon mass M_{H} is dictated by the following critical scaling law [97, 98]

$$M_{\text{PBH}}(\mathcal{C}) = \mathcal{K}M_{\text{H}}(\mathcal{C} - \mathcal{C}_c)^\gamma, \quad (2.22)$$

with $\gamma = 0.38$ [99, 100]. The horizon mass corresponds to the k with the relation

$$M_{\text{H}} \simeq 17M_{\odot} \left(\frac{g_{\star}}{10.75} \right)^{-1/6} \left(\frac{k/\kappa}{10^6 \text{Mpc}^{-1}} \right)^{-2} \quad (2.23)$$

where the factor κ depends on the shape of the power spectrum (see Ref. [101]) and in this work is fixed to $\kappa = 4.5$. The mass fraction β is obtained from the joint probability distribution function P_{g}

$$\beta(M_{\text{PBH}}, M_{\text{H}}) = \int_{\mathcal{D}} \frac{M_{\text{PBH}}}{M_{\text{H}}} \delta \left[\ln \frac{M_{\text{PBH}}}{M_{\text{PBH}}(\mathcal{C})} \right] \text{P}_{\text{g}}(\mathcal{C}_{\text{g}}, \zeta_{\text{g}}) d\mathcal{C}_{\text{g}} d\zeta_{\text{g}}, \quad (2.24)$$

where the domain of integration is given by Eq.(2.19), the multivariate Gaussian is given in Eq.(2.16) and the correlators are computed as in Eqs.(2.13)-(2.15). In this work, we have followed the prescription given in Ref. [101] to compute the values of the threshold \mathcal{C}_c and the position of the maximum of the compaction function r_m , which depend on the shape of the power spectrum. We get $\mathcal{C}_c = 0.56$. The presence of the QCD phase transitions is taken into account by considering that $\gamma(M_{\text{H}})$, $\mathcal{K}(M_{\text{H}})$, $\mathcal{C}_c(M_{\text{H}})$ and $\Phi(M_{\text{H}})$ are functions of the horizon mass around $M_{\text{PBH}} = \mathcal{O}(M_{\odot})$ [102, 103]. As we will see in the next sections, the latter has an important phenomenological consequence when a PBH mass function centered nearly around one solar mass is considered.

3 The PBH clustering

Having described how to estimate the PBH abundance, we now turn our attention to the spatial clustering of PBHs. To characterize the PBH two-point correlation function $\xi_{\text{PBH}}(x)$ (or, simply, correlation function) at any comoving separation $x = |\vec{x}|$, we can use the overdensity of discrete PBH centers at position \vec{x}_i (eventually smoothed over a sphere with a radius equal to the Hubble radius at the moment the perturbations re-enter the horizon)

$$\delta_{\text{PBH}}(\vec{x}) = \frac{1}{\bar{n}_{\text{PBH}}} \sum_i \delta_D(\vec{x} - \vec{x}_i) - 1, \quad (3.1)$$

where $\delta_D(\vec{x})$ is the three-dimensional Dirac distribution, \bar{n}_{PBH} is the average comoving number density of PBH, and i runs over the initial positions of PBH. The corresponding two-point correlation function must take the general form (see, for instance, Ref. [104] in the context of large scale structure)

$$\begin{aligned} \langle \delta_{\text{PBH}}(\vec{x}) \delta_{\text{PBH}}(0) \rangle &= \frac{1}{\bar{n}_{\text{PBH}}} \delta_D(\vec{x}) - 1 + \frac{1}{\bar{n}_{\text{PBH}}^2} \left\langle \sum_{i \neq j} \delta_D(\vec{x} - \vec{x}_i) \delta_D(\vec{x}_j) \right\rangle \\ &= \frac{1}{\bar{n}_{\text{PBH}}} \delta_D(\vec{x}) + \xi_{\text{PBH}}(x), \end{aligned} \quad (3.2)$$

where [105]

$$\bar{n}_{\text{PBH}} \simeq 30 f_{\text{PBH}} \left(\frac{M_{\text{PBH}}}{M_{\odot}} \right)^{-1} \text{kpc}^{-3}. \quad (3.3)$$

Here, $\xi_{\text{PBH}}(x)$ is the reduced PBH correlation function. We would like to evaluate the correlation function ξ_{PBH} between PBHs at a comoving distance x . Since PBHs form at peaks of the underlying radiation overdensity, their two-point correlator is biased with respect to the one of radiation [106]. For separations x much larger than the typical (comoving) size of perturbations collapsing into PBHs, we have

$$\xi_{\text{PBH}}(x) \approx b_1^2 \xi_r(x), \quad (3.4)$$

where ξ_r is the correlation function for radiation and b_1 is the linear bias factor, which will be computed later in this work. The initial PBH two-point function can also be expressed as [107]

$$\xi_{\text{PBH}}(x) = \frac{1}{2\pi^2} \int_0^\infty dk k^2 P_{\text{PBH}}(k) j_0(kx), \quad (3.5)$$

where $j_0(kx)$ is a spherical Bessel function, a_{H} is the scale factor evaluated at PBH formation time and

$$P_{\text{PBH}}(k) = \left(\frac{4}{9}\right)^2 b_1^2 P_\zeta(k). \quad (3.6)$$

To be as conservative as possible in our conclusions, we focus on the finite scale invariant curvature power spectrum that maximizes the clustering, i.e. a broad spectrum that correlates small and large scales

$$P_\zeta(k) = A_s \frac{2\pi^2}{k^3} \theta(k_{\text{max}} - k) \theta(k - k_{\text{min}}), \quad (3.7)$$

where k_{min} and $k_{\text{max}} = k_{\text{min}} \cdot \Delta$ define the minimum and maximum scales, with Δ being the dimensionless width of the spectrum. With such a power spectrum, the lightest PBHs form when the smallest scale $\sim k_{\text{max}}^{-1}$ re-enters the horizon. They dominate the mass function over the heavier PBHs formed later because their density is diluted much slower than radiation [108, 109] (an effect which has not been accounted for in Ref. [110]). As we will show, such light PBHs are clustered up to the scale $\sim k_{\text{min}}^{-1}$. The integral in Eq. (3.5) can be solved analytically, giving

$$\begin{aligned} \xi_{\text{PBH}}(x) &= \left(\frac{4}{9}\right)^2 A_s b_1^2 \left[\text{Ci}(k_{\text{max}}x) - \text{Ci}(k_{\text{min}}x) - \frac{\sin(k_{\text{max}}x)}{k_{\text{max}}x} + \frac{\sin(k_{\text{min}}x)}{k_{\text{min}}x} \right] \\ &\approx \left(\frac{4}{9}\right)^2 A_s b_1^2 \begin{cases} \ln \Delta, & x \ll k_{\text{max}}^{-1}, \\ \ln\left(\frac{1.53}{xk_{\text{min}}}\right), & k_{\text{max}}^{-1} \ll x \ll k_{\text{min}}^{-1}, \\ \mathcal{O}((xk_{\text{min}})^{-2}), & x \gg k_{\text{min}}^{-1}, \end{cases} \end{aligned} \quad (3.8)$$

where $\text{Ci}(z) \equiv -\int_z^\infty \frac{\cos(t)}{t} dt$ is the cosine integral. The second line gives the asymptotics in the regions separated by k_{max}^{-1} and k_{max}^{-2} , which hold well for $\Delta \gg 1$, that is, for the broad spectra considered in this study. Thus, $\xi_{\text{PBH}}(x)$ reaches its maximum at smallest distances $x \lesssim k_{\text{max}}^{-1}$, where it stays constant. At the largest distances, $x \gtrsim k_{\text{min}}^{-1}$ is a rapidly damped oscillating function. These oscillations are partly an artefact of the sharp cuts in the adapted power spectrum (3.7). However, they are not relevant to our analysis, which depends mostly on the intermediate range $k_{\text{max}}^{-1} \lesssim x \lesssim k_{\text{min}}^{-1}$. We also remark, that the first zero of $\xi_{\text{PBH}}(x)$ lies at $x = 2.16/k_{\text{max}}$, that is, at slightly larger distances than suggested by the approximation in the intermediate region.

The average number density of PBHs in a volume of radius R is

$$\langle N(R) \rangle = \bar{n}_{\text{PBH}} V(R) + \bar{n}_{\text{PBH}} \int_0^R d^3x \xi_{\text{PBH}}(x) = \bar{n}_{\text{PBH}} V(R) [1 + \bar{\xi}_{\text{PBH}}(R)], \quad (3.9)$$

where

$$\begin{aligned} \bar{\xi}_{\text{PBH}}(R) &= \left(\frac{4}{9}\right)^2 A_s b_1^2 \left[\text{Ci}(k_{\text{max}} R) - \text{Ci}(k_{\text{min}} R) + \frac{\cos(k_{\text{max}} R)}{k_{\text{max}}^2 R^2} - \frac{\cos(k_{\text{min}} R)}{k_{\text{min}}^2 R^2} \right. \\ &\quad \left. - \frac{\sin(k_{\text{max}} R)}{k_{\text{max}}^3 R^3} (1 + k_{\text{max}}^2 R^2) + \frac{\sin(k_{\text{min}} R)}{k_{\text{min}}^3 R^3} (1 + k_{\text{min}}^2 R^2) \right] \\ &\approx \left(\frac{4}{9}\right)^2 A_s b_1^2 \begin{cases} \ln \Delta, & x \ll k_{\text{max}}^{-1}, \\ \ln\left(\frac{2.12}{x k_{\text{min}}}\right), & k_{\text{max}}^{-1} \ll x \ll k_{\text{min}}^{-1}, \\ \mathcal{O}((x k_{\text{min}})^{-3}), & x \gg k_{\text{min}}^{-1} \end{cases} \end{aligned} \quad (3.10)$$

and behaves thus qualitatively similarly to (3.8). Clustering is relevant at a scale R if

$$\bar{\xi}_{\text{PBH}}(R) \gg 1 \quad (3.11)$$

given that the volume is at least as large as $1/\bar{n}_{\text{PBH}}$ such that it is expected to contain some PBHs, that is $\langle N(R) \rangle \gg 1$.

3.1 The local bias

To fully calculate the correlation function, we need now to estimate the linear bias b_1 . To do so, we can use the peak-background split picture in which the perturbations are divided into short- (peak) and long-wavelength (background) modes [111]. The first is treated as a stochastic, and the second as a classical variable. Using the condition $\mathcal{C} > \mathcal{C}_c$ as a proxy for the discrete PBH distribution, the probability $P(\mathcal{C} > \mathcal{C}_c)$ allows us to define the non-Poisson component of the PBH fluctuation in the presence of the long mode of the Gaussian component of the curvature perturbation $\zeta_g^l(\vec{x})$:

$$\delta_{\text{PBH}}(\vec{x}) = \frac{P(\mathcal{C} > \mathcal{C}_c | \zeta_g^l(\vec{x}))}{P(\mathcal{C} > \mathcal{C}_c)} - 1 \simeq \left. \frac{\partial \ln P(\mathcal{C} > \mathcal{C}_c | \zeta_g^l(\vec{x}))}{\partial \zeta_g^l(\vec{x})} \right|_{\zeta_g^l=0} \zeta_g^l(\vec{x}). \quad (3.12)$$

On expanding Eq.(2.7) in terms of the short and long mode² (where now we have made explicit the position \vec{x} where the PBH is formed), we obtain

$$\mathcal{C}_1(\vec{x}, r_m) = -\frac{4}{3} r_m \zeta_g^{s'}(r_m) F_s' - \frac{4}{3} r_m \zeta_g^{s'}(r_m) F_s'' \zeta_g^l(\vec{x}) = \mathcal{C}_1^s(r_m) \left(1 + \frac{F_s''}{F_s'} \zeta_g^l(\vec{x}) \right), \quad (3.13)$$

with

$$\mathcal{C}_1^s(r_m) = F_s' \mathcal{C}_g^s(r_m) \quad \text{and} \quad \mathcal{C}_g^s(r_m) = -\frac{4}{3} r_m \zeta_g^{s'}(r_m), \quad (3.14)$$

and the primed quantities F_s' etc. are derivatives of F evaluated at $\zeta_g^l = 0$. Note that we have neglected the radial derivative of the long mode. Likewise,

$$\mathcal{C}(\vec{x}, r_m) = \mathcal{C}_1(\vec{x}, r_m) - \frac{3}{8} \mathcal{C}_1^2(\vec{x}, r_m) \simeq \mathcal{C}_1^s(r_m) \left(1 + \frac{F_s''}{F_s'} \zeta_g^l(\vec{x}) \right) - \frac{3}{8} \mathcal{C}_1^{s2}(r_m) \left(1 + 2 \frac{F_s''}{F_s'} \zeta_g^l(\vec{x}) \right). \quad (3.15)$$

²We go beyond Ref. [112], which only considered the local expansion (2.5) up to cubic order and adopted peak theory.

We can redefine the stochastic field

$$\mathcal{C}_g^s(r_m) \rightarrow \left(1 + \frac{F_s''}{F_s'} \zeta_g^l(\vec{x})\right) \mathcal{C}_g^s(r_m), \quad (3.16)$$

in a way that the presence of the long mode is incorporated in the correlators

$$\begin{aligned} \sigma_c^2 &\rightarrow \left(1 + 2 \frac{F_s''}{F_s'} \zeta_g^l(\vec{x})\right) \sigma_c^2, \\ \sigma_{\text{cr}}^2 &\rightarrow \left(1 + \frac{F_s''}{F_s'} \zeta_g^l(\vec{x})\right) \sigma_{\text{cr}}^2, \\ \sigma_r^2 &\rightarrow \sigma_r^2. \end{aligned} \quad (3.17)$$

As a result,

$$\delta_{\text{PBH}}(\vec{x}) \simeq \left(2 \frac{\partial \ln P(\mathcal{C} > \mathcal{C}_c)}{\partial \ln \sigma_c^2} + \frac{\partial \ln P(\mathcal{C} > \mathcal{C}_c)}{\partial \ln \sigma_{\text{cr}}^2}\right) \frac{F_s''}{F_s'} \zeta_g^l(\vec{x}), \quad (3.18)$$

where F_s''/F_s' is a function of \mathcal{C}_c . Following Ref. [106], we compute the bias factor b_1 according to

$$b_1 = \frac{1}{P(\mathcal{C} > \mathcal{C}_c)} \int_{\mathcal{D}} \frac{(\mathcal{C}_g^2 \sigma_r^2 - \mathcal{C}_g \zeta_g \sigma_{\text{cr}}^2 - \sigma_c^2 \sigma_r^2 + \sigma_{\text{cr}}^4) \exp\left(\frac{\mathcal{C}_g^2 \sigma_r^2 - 2\mathcal{C}_g \zeta_g \sigma_{\text{cr}}^2 + \zeta_g^2 \sigma_c^2}{2\sigma_{\text{cr}}^4 - 2\sigma_c^2 \sigma_r^2}\right) \frac{F_s''}{F_s'} d\mathcal{C}_g d\zeta_g}{2\pi \sigma_c \sigma_r \sqrt{1 - \frac{\sigma_{\text{cr}}^4}{\sigma_c^2 \sigma_r^2}} (\sigma_c^2 \sigma_r^2 - \sigma_{\text{cr}}^4)} \frac{F_s''}{F_s'} d\mathcal{C}_g d\zeta_g, \quad (3.19)$$

where the domain of integration \mathcal{D} is defined in Eq. (2.19). The bias factor can be interpreted as an average over the bivariate Gaussian distribution in the domain \mathcal{D} ,

$$b_1 = \left\langle \left(\frac{\mathcal{C}_g^2 \sigma_r^2 - \mathcal{C}_g \zeta_g \sigma_{\text{cr}}^2}{\sigma_c^2 \sigma_r^2 - \sigma_{\text{cr}}^4} - 1 \right) \left(\frac{F_s''}{F_s'} \right) \right\rangle. \quad (3.20)$$

Before entering the main part of our analysis, it is instructive to pause and compare our result with existing literature.

Assuming a quadratic power series expansion for the primordial NGs, as in Eq. (2.5), in Ref. [113] the authors derive a bias factor linearly proportional to f_{NL} . By contrast, our approach predicts

$$b_1 \sim \left\langle f(\zeta_g, \mathcal{C}_g) \frac{f_{\text{NL}}}{1 + \frac{6}{5} f_{\text{NL}} \zeta_g} \right\rangle. \quad (3.21)$$

This discrepancy arises because, in Ref. [113], the authors consider the thresholded regions of the curvature perturbation field ζ to define the collapse and formation of PBHs. However, as previously discussed, the appropriate field for defining the threshold condition for collapse is the compaction function \mathcal{C} . The non-linear relationship between these two quantities³ leads to the difference in the results.

In Fig. 1, the bias factor b_1 is shown as a function of the non-Gaussian parameters f_{NL} (left panel) and r_{dec} (right panel) for power spectra with two different widths.

³The impact of such non-linearities on PBH abundance was first investigated in Refs. [114, 115].

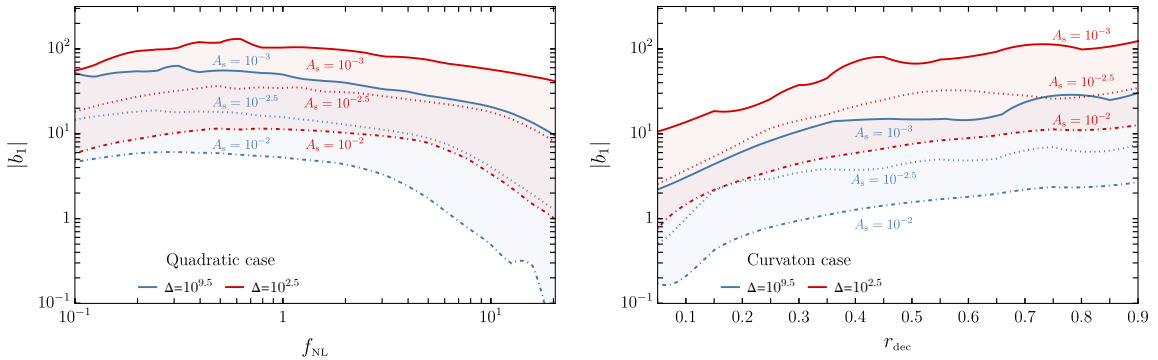


Figure 1. The bias factor b_1 as a function of the NGs parameters f_{NL} (left panel) and r_{dec} (right panel) for several choices of the amplitude A_s , assuming a broad (blue lines) and narrow (red lines) power spectrum.

3.2 Some explicit scenarios

We adopt the broad power spectrum parametrized in Eq. (3.7), without detailing the specific inflationary model for its origin. To be as model-independent as possible, for primordial NGs, we consider the quadratic ansatz (see (2.5)) and the curvaton scenario (see Eq. (A1)). As shown in Ref. [57], in both cases, it is possible to realize a broad spectrum, which connects PBHs in the asteroid mass range to those relevant for the LVK merger events. Details about the non-Gaussian relation for the curvaton models are reported in Appendix A. We do not consider the USR case since for the broad spectrum the primordial NGs are negligible [33]. We stress that the values used for the quadratic ansatz considered in this work cover a wide range of common models for PBH production. We neglect cases with huge primordial NGs that can violate the perturbativity and are not easily realizable in literature. These cases considered here are summarized in Table 1.

| Cases | Δ | A_s | NG | f_{PBH} | b_1 |
|----------|------------|--------------------|-----------------------------------|--------------------------|-------|
| Q_{S1} | $10^{2.5}$ | $\simeq 10^{-2.2}$ | Quadratic $f_{\text{NL}} = 0.42$ | $\simeq 5 \cdot 10^{-4}$ | 18 |
| Q_{S2} | $10^{2.5}$ | $\simeq 10^{-3.2}$ | Quadratic $f_{\text{NL}} = 10.75$ | $\simeq 3 \cdot 10^{-3}$ | 80 |
| Q_{L1} | $10^{9.5}$ | $\simeq 10^{-2.4}$ | Quadratic $f_{\text{NL}} = 0.42$ | $\simeq 1$ | 15 |
| Q_{L2} | $10^{9.5}$ | $\simeq 10^{-3.5}$ | Quadratic $f_{\text{NL}} = 10.75$ | $\simeq 1$ | 63 |
| C_{S1} | $10^{2.5}$ | $\simeq 10^{-2.0}$ | Curvaton $r_{\text{dec}} = 0.5$ | $\simeq 3 \cdot 10^{-4}$ | 8 |
| C_{S2} | $10^{2.5}$ | $\simeq 10^{-2.5}$ | Curvaton $r_{\text{dec}} = 0.1$ | $\simeq 3 \cdot 10^{-4}$ | 5 |
| C_{L1} | $10^{9.5}$ | $\simeq 10^{-2.0}$ | Curvaton $r_{\text{dec}} = 0.5$ | $\simeq 1$ | 1.8 |
| C_{L2} | $10^{9.5}$ | $\simeq 10^{-2.6}$ | Curvaton $r_{\text{dec}} = 0.1$ | $\simeq 1$ | 1.5 |

Table 1. Cases considered in this paper. We imposed for each case $k_{\text{min}} = 10^5 \text{ Mpc}^{-1}$.

In Fig. 2, the corresponding mass distributions f_{PBH} for the various cases are shown. The

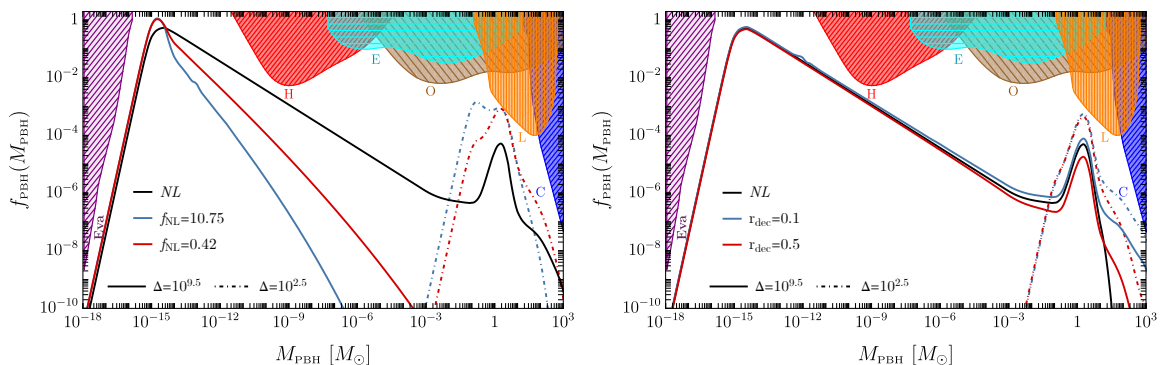


Figure 2. Two classes of mass functions resulting from power spectra with $k_{\min} = 10^5 \text{ Mpc}^{-1}$ and $\Delta = 10^{9.5}$ (solid lines) or $\Delta = 10^{2.5}$ (dotted lines). On the left panel, we consider quadratic NGs with different values of f_{NL} , while on the right panel, we consider the curvaton scenarios with NGs controlled by r_{dec} . The amplitude of the power spectrum has been re-scaled for each value of r_{dec} such that PBHs do not overproduce dark matter. The only non-linear (NL) broad case (black solid line) has been obtained fixing $A_s = 10^{-2.07}$. The most stringent experimental constraints are shown (see description in the text).

following most constraining bounds are reported⁴: **E**vaporation constraints (see also [117–119]): EDGES [120], CMB [121], INTEGRAL [122, 123], 511 keV [124, 125], Voyager [126], EGRB [127]); microlensing constraints from the Hyper-Supreme Cam (**H**), Ref. [128]; microlensing constraints from OGLE, Refs. [129–131]; constraints from modification of the CMB (**C**) spectrum due to accreting PBHs, Ref. [132, 133]; direct constraints on PBH-PBH mergers with LIGO, Refs. [134] (see also [102, 135–143]).

At this stage, several important observations can be made.

- (i) As seen in Fig.,1, reducing the amplitude A_s while keeping the NG parameter fixed increases the bias b_1 . However, in contrast to earlier works, we find that b_1 is not proportional to f_{NL} , as corroborated by the theoretical discussion in the previous section. For a fixed amplitude and the same amount of quadratic NGs, the two ansatz yield identical values for b_1 . However, since the quadratic cases require smaller power spectrum amplitudes to achieve the same abundance, this ansatz results in greater bias.
- (ii) Increasing the degree of NGs (i.e., decreasing the value of r_{dec} from 0.5 to 0.1) leads to an amplification of the mass distribution $f_{\text{PBH}}(M_{\text{PBH}})$. Consequently, to achieve the same amount of dark matter in the form of PBHs, the amplitude A_s must be reduced. This trend is similar to that observed with a positive f_{NL} . Comparing the amplitudes of the power spectra in Tab. 1, we see that, when the full NG relation Eq. (A1) is used instead of the quadratic approximation for f_{NL} Eq. (A4), the quadratic power series enhances PBH production. As a result, to fix the abundance, the amplitudes need to be further decreased, which is consistent with the findings of Ref. [90].

⁴Fig. 2 showing constraints for monochromatic PBH mass spectra, which are not directly comparable to the mass functions shown and rather provide a qualitative comparison. Following Ref. [116], we checked that the constraints are not violated for the benchmark scenarios in Table. 1.

(iii) The right panel of Fig. 2 demonstrates that, in the case of a broad power spectrum, the mass function exhibits two peaks: one at smaller masses associated with the maximum scale of the power spectrum, k_{\max} , and another in the range of solar-mass PBHs.⁵ When modes corresponding to these masses re-enter the cosmological horizon during the QCD phase transitions, the softening of the equation of state enhances PBH formation [102, 103, 144–146]. As shown in the left panel of the same figure, the secondary peak in the quadratic NG case is negligible compared to the curvaton case. This can be understood from the analysis in Ref. [90]. For broad power spectra, in the absence of significant primordial NGs, the mass fraction β is scale-invariant, and for a constant threshold \mathcal{C}_c , the mass distribution f_{PBH} scales as $\propto M_{\text{PBH}}^{-3/2}$ [147]. Primordial NGs break the M_{H} -independence of β , and the effect depends on the type and magnitude of the NGs, as shown in Fig. 2. In curvaton models with large primordial NGs, PBH formation in the solar-mass range is enhanced, while the classical $\propto M_{\text{PBH}}^{-3/2}$ distribution is recovered at intermediate masses. In contrast, the quadratic ansatz tends to underproduce solar-mass PBHs relative to the non-linear case. Furthermore, for intermediate masses, the slope of the mass distribution (in modulus) increases with the amount of quadratic NGs.

4 Clustering and merger length scales

Suppose that PBHs are born clustered inside a sphere of radius $\sim k_{\min}^{-1}$. In radiation domination, the cluster does not evolve to form a halo as long as [105, 148]

$$f_{\text{PBH}} \bar{\xi}_{\text{PBH}}(k_{\min}^{-1}) \lesssim 1. \quad (4.1)$$

This is because, in the mass range under consideration, the f_{PBH} -dependent PBH abundance is too small relative to the dark matter density, and the overdensity of the cluster is insufficient to overcome the radiation pressure and produce a bound matter-dominated region following gravitational collapse [148].

Once the matter-dominated period begins, PBH perturbations decouple from the Hubble flow, collapse, and virialize to form halos with a virial density about 200 times the background density at the time of virialization. The formation of PBH halos is hierarchical like in a standard CDM cosmology: the small mass PBH halos that form first are the progenitors of the more massive PBH halos that virialize at later times. If clustering is relevant up to a given scale $\sim k_{\min}^{-1}$, one expects that a typical PBH halo will contain an average number of PBH equal to $\langle N \rangle \sim \bar{n}_{\text{PBH}} V(k_{\min})(1 + \bar{\xi}_{\text{PBH}}(k_{\min}^{-1}))$.

During the radiation era, PBH binaries form along with the first PBH clusters. These binaries are likely to be highly eccentric since they acquire their angular momentum through tidal torques exerted by surrounding inhomogeneities. In clustered PBH scenarios, these early binaries are more prone to collisions with neighbouring PBHs or to absorption in the early clusters that tend to have frequent collisions between the PBHs. Even mild disturbances can extend the coalescence times of eccentric PBH binaries by several orders of magnitude, which strongly suppresses their merger rate. When this occurs, the PBH merger rate is driven by the formation of PBH binaries via the early three-body channel. This produces less eccentric

⁵The softening of the equation of state near the QCD transitions is expected to slightly influence the evolution of sub-horizon modes. Since this effect is mitigated by the window function, which also smooths out sub-horizon modes, it is neglected here.

binaries and yields merger rates that are less affected by binary-BH interactions [137, 149]. As a result, the population of PBH binaries as well as their merger rate would be quite different from that expected in the absence of initial clustering, which is often assumed in the literature.

To gauge whether clustering impacts the merger of two PBHs, let us consider the typical scales involved in a merger. It is straightforward to verify that the gravitational interaction between two nearby, isolated PBHs drives their dynamical evolution if their average mass exceeds the amount of matter enclosed in a comoving sphere of radius equal to their separation. This situation can arise in the radiation-dominated era owing to the different time dependencies of the two competing effects that influence the PBHs' separation: their mutual gravitational attraction and the pull of cosmic expansion. More precisely, assuming that all PBHs have the same mass M , two neighbouring PBHs will detach from the Hubble flow during the radiation-dominated era if their comoving separation satisfies [12, 136, 150]

$$R < R_{\max} \lesssim (f_{\text{PBH}}/\bar{n}_{\text{PBH}})^{1/3} \simeq 0.31 \left(\frac{M}{M_{\odot}}\right)^{1/3} \text{ kpc}. \quad (4.2)$$

As the radius of the Hubble patch under consideration at the time of binary formation is much bigger than $1/k_{\min}$, the contribution of the clustering to the average PBH number density within that patch is negligible. Therefore, the above condition also holds when clustering is present [151].

In addition, there exists a minimum separation, R_{\min} , below which a binary with any orbital parameter would have already merged. This corresponds to PBH configurations that result in circular orbits. Specifically, considering the current merger rate ($t_0 \approx 14 \text{ Gyr}$), we find

$$R_{\min} \sim 9.5 \cdot 10^{-3} \left(\frac{M}{M_{\odot}}\right)^{7/16} \text{ kpc}. \quad (4.3)$$

Clustering is therefore relevant in volumes of radius R if the condition

$$\bar{\xi}_{\text{PBH}}(R) \gg 1 \text{ for } R_{\min} \lesssim R \lesssim R_{\max} \quad (4.4)$$

is satisfied. We remark that the magnitude of the effect on binary formation depends on the formation channel. For instance, in the (approximately) Poisson scenarios, the most relevant scale for the 2-body binary formation channel is about $\mathcal{O}(10R_{\min})$ due to the high initial eccentricity ($j \equiv \sqrt{1-e^2} = \mathcal{O}(10^{-2})$) of these binaries [136, 149].

The main point of the paper now comes into focus. To set the maximum scale $\sim k_{\min}^{-1}$ below which clustering is not negligible, we must carefully examine the CMB constraints on the power spectrum of the curvature perturbation. As a matter of fact, CMB observations strongly constrain $P_{\zeta}(k)$ at scales $10^{-4} \text{ Mpc}^{-1} \lesssim k \lesssim 1 \text{ Mpc}^{-1}$ [153, 154]. At smaller scales other constraints must be applied, coming from the fact that at redshifts $z \lesssim 10^6$ energy injections into the primordial plasma cause persisting spectral distortions in the CMB [63–67]. These distortions are divided into chemical potential μ -type distortions created higher z and Compton y -type distortions created at $z \lesssim 5 \times 10^4$. For a given curvature power spectrum $\mathcal{P}_{\zeta}(k)$ the spectral distortions are [65, 66]

$$X = \int_{k_{\text{m}}}^{\infty} \frac{dk}{k} \mathcal{P}_{\zeta}(k) W_X(k), \quad (4.5)$$

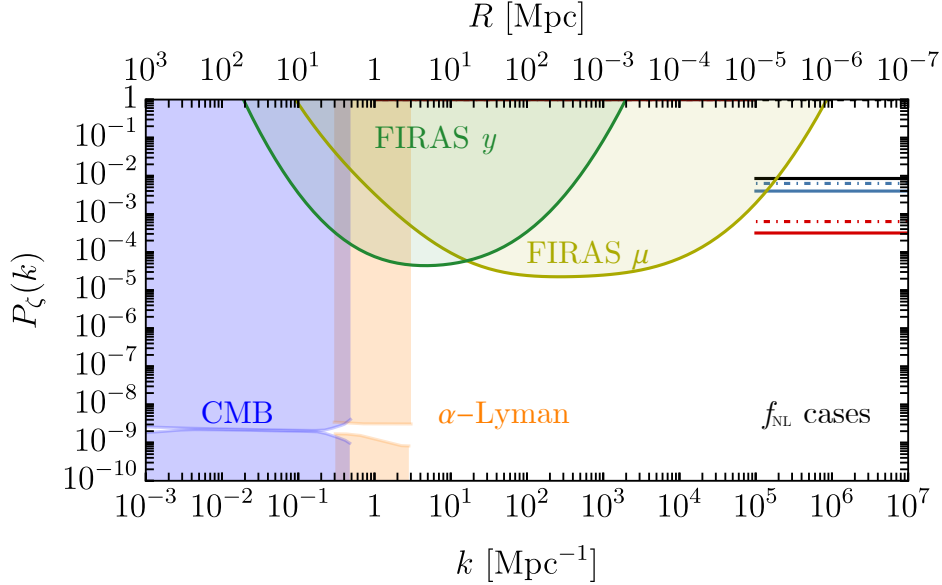


Figure 3. Constraints on the amplitude of the power spectrum, assuming negligible primordial NGs, from the FIRAS experiments [63–68, 152], CMB [153], α -Lyman [154] for a log-normal power spectrum with width $\sigma = 1$ that mimics the broad power spectra assumed in this work. For clarity the f_{NL} cases reported in Tab.1 are shown.

with $X = \mu, y$, while $k_{\text{m}} = 1 \text{ Mpc}^{-1}$ and the window functions⁶ can be approximated by

$$W_{\mu}(k) \simeq 2.2 \left[e^{-\frac{(\hat{k}/1360)^2}{1+(\hat{k}/260)^{0.6}+\hat{k}/340}} - e^{-(\hat{k}/32)^2} \right], \quad W_y(k) \simeq 0.4 e^{-(\hat{k}/32)^2}, \quad (4.6)$$

with $\hat{k} = k / (\text{Mpc}^{-1})$. The COBE FIRAS measurements constrain the μ distortions such as $\mu \leq 4.7 \times 10^{-5}$ [67] and $y \leq 1.5 \times 10^{-5}$ at the 95% confidence level [63].

As shown in Fig. 3, requiring sufficiently large curvature perturbations, i.e. $A_s \gtrsim 10^{-3}$, to obtain a sizeable abundance of PBHs at the relevant scales, the FIRAS constraints set the limit [68]

$$k_{\text{min}}^{-1} \lesssim R_{\text{FIRAS}} \simeq 10^{-2} \text{ kpc} \quad (4.7)$$

on broad spectra, which is already a factor $\mathcal{O}(10)$ smaller than R_{max} . The question, then, is: can $\bar{\xi}_{\text{PBH}}(R)$ be much larger than unity in the range $R_{\text{min}} \lesssim R \lesssim 10^{-2} \text{ kpc} \simeq R_{\text{FIRAS}}$?

In Fig. 4, we show how the required $\bar{\xi}_{\text{PBH}}(R)$ changes with R for the different benchmark cases reported in Tab. 1 and for two masses, $M = 0.1 M_{\odot}$ and $M = 1 M_{\odot}$. For PBHs heavier than the Sun, we have $R_{\text{FIRAS}} < R_{\text{min}}$, making the argument even stronger. As it is clear from this figure, in the relevant range of scales and for masses of interest for LVK [7, 8, 10], i.e. $M \gtrsim 0.1 M_{\odot}$, the initial spatial clustering is irrelevant, given the strong constraints on the PBH abundance in such mass range and the fact that the merger depends on the combination $f_{\text{PBH}}(1 + \bar{\xi}_{\text{PBH}})$ [149].

However, if one considers SGWBs induced by PBH mergers and galactic PBH binaries, LVK O5 and ET can in principle be sensitive to lower masses and probe PBH scenarios

⁶Including NGs modifies only the tails of the constraints and not the most constrained region [155].

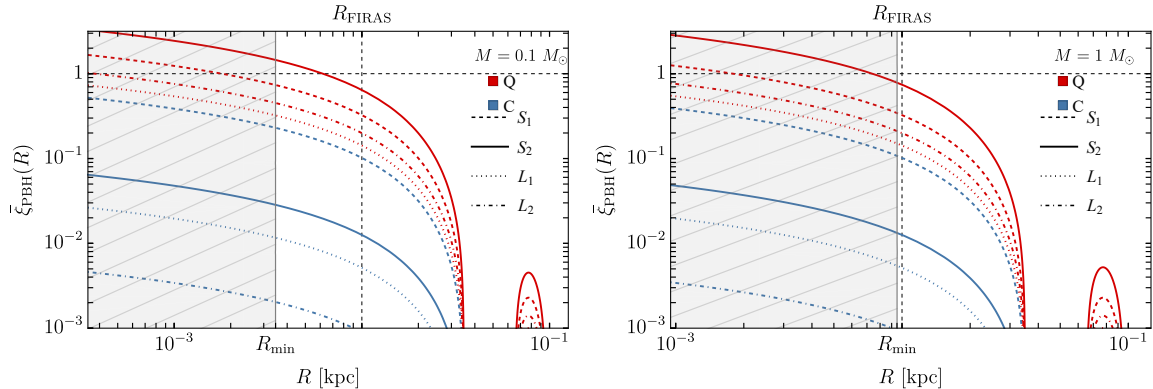


Figure 4. The correlation length $\bar{\xi}_{PBH}(R)$ in the relevant range of scales for the benchmark cases reported in Tab. 1. We focus on the masses for the binary, respectively $M = 0.1 M_\odot$ (left panel) and $M = 1 M_\odot$ (right panel). The shaded region indicates scales which do not affect binaries merging today. This figure assumes $k_{min}^{-1} = R_{FIRAS}$.

involving PBHs as light as $10^{-2} M_\odot$ and $10^{-6} M_\odot$, respectively [156] (see also [157] for LISA). We find that for $M = 10^{-6} M_\odot$, where f_{PBH} may acquire values as large as about 0.1, the volume average correlation is only bounded by $\bar{\xi}_{PBH}(R) \lesssim \mathcal{O}(10)$, which suggests that clustering might play a role for lighter PBHs accessible by future GW experiments. In particular, as non-negligible correlations can be present at scales relevant for binary formation, PBH binary formation and the resulting merger rate may be enhanced. If, furthermore, the correlations are less prominent at scales associated with the formation of $N \gg 2$ PBH clusters, the subsequent clustering evolution will approximately follow the usual Poisson case, and so will also the disruption of PBH binaries within clusters formed during matter domination.

5 Conclusions

Discovering the presence of PBHs is one of the main targets of the new era of gravitational wave astronomy. Assessing the merger rate of PBH binaries has become therefore imperative. In this paper, we have made a simple, but relevant remark, that the initial clustering of PBH in the mass range relevant for LVK does not affect the standard estimate of the PBH binary merger rate [149] which assumes an initial PBH Poisson distribution. Our conclusions apply to realistic models in which PBHs are formed via the collapse of sizeable fluctuations generated during inflation. They are also robust in the sense that we have assumed a broad power spectrum for the curvature perturbation, thus maximally enhancing the possible clustering.

PBH may also form in alternative scenarios [158]. For the case of phase transitions (PTs) [159–162], for example, the correlation length is limited by causality as PTs cannot produce super-Hubble correlations at formation time. The relevant scales that can significantly affect PBH binaries are much larger. So, we do not expect this scenario to avoid our conclusion. We leave the investigation of alternative cases for future work.

In this paper, we have focused on PBH masses reachable by the LVK collaboration. As it is clear from the mass scalings, our no-go argument does not apply to lighter PBH masses, whose binaries could lead to merger rates potentially detectable by ET [163, 164] by ultra-high frequency detector [156, 165, 166]. We leave this analysis for future work.

Acknowledgements

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Appendix A: Primordial Non-Gaussianities in explicit models

When presenting results inspired by the curvaton model, we will focus on primordial NG (derived analytically within the sudden-decay approximation [59])

$$\zeta = \ln [X(r_{\text{dec}}, \zeta_g)], \quad (\text{A1})$$

with

$$X \equiv \frac{\sqrt{K} \left(1 + \sqrt{AK^{-\frac{3}{2}} - 1} \right)}{(3 + r_{\text{dec}})^{\frac{1}{3}}}, \quad (\text{A2a})$$

$$K \equiv \frac{1}{2} \left((3 + r_{\text{dec}})^{\frac{1}{3}} (r_{\text{dec}} - 1) P^{-\frac{1}{3}} + P^{\frac{1}{3}} \right), \quad (\text{A2b})$$

$$P \equiv A^2 + \sqrt{A^4 + (3 + r_{\text{dec}})(1 - r_{\text{dec}})^3}, \quad (\text{A2c})$$

$$A \equiv \left(1 + \frac{3\zeta_g}{2r_{\text{dec}}} \right)^2 r_{\text{dec}}. \quad (\text{A2d})$$

The parameter r_{dec} is the weighted fraction of the curvaton energy density ρ_ϕ to the total energy density at the time of curvaton decay, defined by

$$r_{\text{dec}} \equiv \frac{3\rho_\phi}{3\rho_\phi + 4\rho_\gamma} \Big|_{\text{curvaton decay}}, \quad (\text{A3})$$

where ρ_γ is the energy density stored in radiation after reheating. Thus, r_{dec} depends on the physical assumptions about the physics of the curvaton within a given model. If we expand Eq. (A1) to the second order in ζ_g we find [167]

$$f_{\text{NL}} = \frac{5}{3} \left(\frac{3}{4r_{\text{dec}}} - 1 - \frac{r_{\text{dec}}}{2} \right). \quad (\text{A4})$$

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