ASITP

INSTITUTE OF THEORETICAL PHYSICS ACADEMIA SINICA

AS-ITP-95-27
October 1995

Nuclear Moment of Inertia and Blocking Effect

J. Y. Zeng, Y. A. Lei
and E. G. Zhao

P.O.Box 2735, Beijing 100080, The People's Republic of China

Telefax : (86)-1-2562587 Telephone : 2568348
Telex : 22040 BAOAS CN Cable : 6158

Nuclear Moment of Inertia and Blocking Effect

J. Y. Zeng[a,b], Y. A. Lei[b], E. G. Zhao[b]

[a]Department of Physics, Peking University, Beijing, China

and

[b]Institute of Theoretical Physics, Beijing, China

Abstract

In this paper are addressed three long-standing problems encountered in the BCS theory of nuclear moments of inertia: (1) Excessive reduction of nuclear moment of inertia due to pairing interaction. (2) Odd-even difference in the nuclear moment of inertia and the "identical bands". (3) Additivity of nuclear moments of inertia. It is shown that all the three problems are intimately connected with the proper treatment of the blocking effects and can be solved by the particle-number-conserving treatment, in which the blocking effects are taken into account exactly.

There exist three long-standing problems in the BCS theory of nuclear moment of inertia.

1. Excessive reduction of nuclear moment of inertia due to paring interaction.

It is well-known that the calculated nuclear moment of inertia in the CSM (cranked shell model) neglecting the residual interaction is near the rigid-body value and much larger than the experimental ones [1,2]. Bohr, Mottelson and Pines [3] suggested that the pairing interaction may be responsible for the observed reduction of nuclear moments of inertia compared to that of a rigid rotor. Soon after, the BCS method and the concept of quasi-particle excitation were used to treat the nuclear pairing, and a significant reduction of moments of inertia due to pairing correlation was successfully confirmed [4]. However, the BCS theoretical moments of inertia of the ground bands in rare-earth and actinide even-even nuclei are turned to be systematically smaller than the experimental ones by a factor of 10—40 %; i.e., a systematic excessive reduction of nuclear moments of inertia was found [5]. Many efforts to reduce the discrepancy between theory and experiment have been unsuccessful. For recent reviews, see refs. [6—8].

2. Odd-even difference in moments of inertia and the "identical bands".

According to the BCS approximation for treating nuclear pairing, the moments of inertia associated with one-quasiparticle states in odd mass nuclei should be larger than those of the ground state configuration of adjacent even-even nuclei by a factor of ~ (15—20) % [2]. The striking discovery (1990) of almost identical SD (superdeformed) bands in some adjacent nuclei [9,10] has attracted the attention of many theoretical and experimental nuclear physicists. Shortly afterwards, it was recognized that identical bands are also present in ND (normally deformed) pairs of even- and odd-mass nuclei at low spins [11,12]; i.e., the occurrence of identical bands is not necessarily related to superdeformation phenomena or high-spin states. However, the strong pairing interaction in ND nuclei at low spin has been well established. Therefore, it was asserted [11,12] that the occurrence of identical bands in ND pairs of even- and odd-mass nuclei at low spin poses a serious challenge to the mean field (BCS) approximation.

However, it should be mentioned that Bohr and Mottelson [2] have pointed out that there exist large fluctuations in the odd-even differences of nuclear moments of inertia. Particularly, the moment of inertia of an odd-A nucleus whose unpaired nucleon occupies a high-j intruder orbit is systematically much larger than those of the ground band moments of inertia of neighboring even-even nuclei. For example, the bandhead moment of inertia of the ground band [642]5/2 of ^{161}Dy extracted from the two lowest observed levels is $2J = 159.4\ \hbar^2\mathrm{MeV}^{-1}$, which is over twice as large as those of the ground bands of ^{160}Dy ($2J = 69.1\ \hbar^2\mathrm{MeV}^{-1}$) and ^{162}Dy ($2J = 74.4\ \hbar^2\mathrm{MeV}^{-1}$). In sharp contrast to this, the moment of inertia of the [402]5/2 band in ^{171}Lu is $2J = 70.6\ \hbar^2\mathrm{MeV}^{-1}$, which is almost equal to that of the ground band of ^{170}Yb ($2J = 71.2\ \hbar^2\mathrm{MeV}^{-1}$). In fact, careful examination shows that there exists no clear line of demarcation between the so-called "identical bands" and non-identical bands. The analysis of some typical (most β-stable) odd-Z and odd-N rare-earth nuclei are as follows ($\delta J = J(A) - J_0(A-1)$, A odd):

Odd-Z rare-earth nuclei

rotational bands	[523]7/2 $(h_{11/2})$	[514]9/2 $(h_{11/2})$	[413]5/2 $(g_{7/2})$	[411]3/2 $(d_{5/2})$	[411]1/2 $(d_{3/2})$	[404]7/2 $(g_{7/2})$	[402]5/2 $(d_{5/2})$
odd-Z nuclei	$^{161}_{67}$Ho	$^{171}_{71}$Lu*	$^{157}_{63}$Eu	$^{159}_{65}$Tb	$^{167}_{69}$Tm	$^{171}_{71}$Lu	$^{171}_{71}$Lu*
$\delta J/J$	0.31	0.24	0.16	0.14	0.03	0.037	-0.008

Odd-N rare-earth nuclei

rotational	[642]5/2	[633]7/2	[624]9/2	[523]5/2	[521]3/2	[514]7/2	[512]5/2	[521]1/2
bands	$(i_{13/2})$	$(i_{13/2})$	$(i_{13/2})$	$(f_{7/2})$	$(h_{9/2})$	$(f_{7/2})$	$(h_{9/2})$	$(p_{3/2})$
odd-N nuclei	$^{161}Dy_{95}$	$^{167}Er_{99}$	$^{170}Hf_{107}$	$^{165}Er_{97}$	$^{157}Gd_{93}$	$^{177}Hf_{105}$	$^{173}Yb_{103}$	$^{171}Yb_{101}$
$\delta J/J$	1.31	0.52	0.39	0.38	0.36	0.17	0.17	0.06

How can we understand such large fluctuations?

3. Additivity of nuclear moments of inertia.

Since the seventies, a lot of multi-quasiparticle excitation bands were established in the ND rare-earth and actinide nuclei. Let J_0 and $J_{\mu\nu}$ denote the moments of inertia of the quasiparticle vacuum $|0\rangle\rangle$ and two-quasiparticle state $\alpha_\mu^+\alpha_\nu^+|0\rangle\rangle$ in even-even nuclei, and J_μ and $J_{\mu\nu\sigma}$ denote those of the one-quasiparticle state $\alpha_\mu^+|0\rangle\rangle$ and three quasiparticle state, $\alpha_\mu^+\alpha_\nu^+\alpha_\sigma^+|0\rangle\rangle$ in odd-A nuclei. According to the BCS approximation (residual quasiparticle interaction being neglected), the additivity of the nuclear moments of inertia can be derived [13] :

$$J_{\mu\nu} - J_0 = (J_\mu - J_0) + (J_\nu - J_0), \tag{1}$$

or equivalently, the ratio

$$R = \frac{(J_\mu - J_0) + (J_\nu - J_0)}{J_{\mu\nu} - J_0} = 1. \tag{2}$$

Similarly,

$$R = \frac{(J_\mu - J_0) + (J_\nu - J_0) + (J_\sigma - J_0)}{J_{\mu\nu\sigma} - J_0} = 1. \tag{3}$$

However, all the R values extracted from experimental rotational spectra show that $R > 1$, which implies that the additivity of nuclear moments of inertia does not hold. For example, the bandhead moment of inertia of ^{170}Yb is $2J_0 = 71.2\ \hbar^2\mathrm{MeV}^{-1}$, and for its two-quasiparticle bands $K^\pi = 6^-$ ([633]7/2⊗[512]5/2) [14]

6^-	7^-	8^-	9^-	
1815.4	1964.9	2098.9	2253.9	(keV)

the bandhead moment of inertia is 105.8 $\hbar^2\mathrm{MeV}^{-1}$ (signature $\alpha = 0$), and 117.6 $\hbar^2\mathrm{MeV}^{-1}$ ($\alpha = 1$), and their average is 111.7 $\hbar^2\mathrm{MeV}^{-1}$. For the neighboring odd-N nucleus ^{171}Yb, the ground band [633]7/2, $2J = 122.1\ \hbar^2\mathrm{MeV}^{-1}$ and the excited band [512]5/2, $2J = 82.4\ \hbar^2\mathrm{MeV}^{-1}$. Hence, we get the ratio $R_{\mathrm{expt}} = 1.53$. How can we understand such a large deviation from the additivity of nuclear moments of inertia?

General considerations show that the BCS theory is very suitable for a system of a large number of particles. However, the number of nucleons ($\sim 10^2$), particularly the number of valence nucleons (~ 10), which dominate the properties of the low-lying excited states, is very limited. Therefore, the BCS statements concerning the nuclear features (e.g. the various odd-even differences) which depend sensitively on the particle number need careful re-examination. Of course, the defect of number nonconservation in the BCS treatment may be partly remedied by various types of particle-number projection, and indeed, some improved agreements with experiment were obtained. However, the most serious weakness of the BCS approximation is that it is not able to treat the blocking-effects properly. Rowe [16] pointed out that *while the blocking effects are straightforward, it is very difficult to treat them in the BCS formalism, because they introduce different quasiparticle bases for different blocked levels.*

All the three problems mentioned above, particularly the various odd-even differences (mass or binding energy, low-lying excited spectrum, moments of inertia, etc.) are intimately connected with the proper treatment of the blocking effect, which is especially important for low spin states. Calculations show that all the three difficulties mentioned above can be overcome in the PNC treatment for the CSM Hamiltonian, in which the blocking effects are taken into account exactly. For the details of calculation, see refs. [6] and [8], and the earlier papers [17] and [18]. Here we only sketch some key points of the PNC treatment for the eigenvalue problem of the CSM Hamiltonian of an axially symmetric nucleus,

$$H_{\mathrm{CSM}} = H_{\mathrm{sp}} + H_{\mathrm{C}} + H_{\mathrm{P}} = H_0 + H_{\mathrm{P}} \tag{4}$$

$$H_0 = H_{\mathrm{sp}} + H_{\mathrm{C}}, \quad H_{\mathrm{C}} = -\omega J_z \tag{5}$$

where $H_{\mathrm{sp}} = \sum_i h_{\mathrm{sp}}(i)$, $h_{\mathrm{sp}}(i)$ is the single-particle Nilsson Hamiltonian, $H_{\mathrm{C}} = -\omega J_z$ is the Coriolis interaction in the rotating frame, and H_{P} the pairing interaction. $H_0 = H_{\mathrm{sp}} + H_{\mathrm{C}} = \sum_i h_0(i)$, $h_0(i) = h_{\mathrm{sp}}(i) + \omega j_x(i)$ is a one-body operator. In the PNC calculation, the usually adopted single-particle-level truncation is replaced by the CMPC (cranked many-particle configuration) truncation; i.e., H_{CSM} is diagonalized in a sufficiently large CMPC space to obtain the sufficiently accurate eigenenergies and eigenfunctions of the low-lying excited states of H_{CSM}. The bases of the CMPC space are the eigenstates of H_0, characterized by the good quantum numbers N_0 (particle number), parity π, signature $r = e^{-i\pi\alpha}$, and E_i (CMPC energy, the eigenvalues of H_0). In the diagonalization of H_{CSM}, the CMPC energy truncation, $E_i - E_0 < E_c$, is adopted, where E_0 is the energy of the lowest CMPC, and E_c is sufficiently large. In addition to the CMPC energy truncation, the seniority truncation ($\Delta v < 6$), and angular momentum projection truncation ($\Delta K < 6$) and CMPC mixing strength truncation are also very effective. Calculations show that for the yrast and yrare bands at not too high rotational frequency, the number of main CMPC's (e.g. weight $> 10^{-3}$), are rather limited, and all of them have been involved in our calculations, hence the calculated results are reliable and accurate enough. In our calculations of bandhead moments of inertia, $E_c \sim 1\hbar\omega_0 \sim$ (7–8) MeV is adopted, and in this case, over twenty Nilsson orbitals near the Fermi surface are involved.

In our calculations the Nilsson parameters (ϵ_2, ϵ_4, κ, μ, ω_0) are taken from the Lund systematics [19,20] and no change is made to improve the calculated moments of inertia. The pairing interaction strength is determined from the experimental odd-even binding-energy difference; e.g. the neutron pairing interaction strength G_n is extracted by

$$\begin{aligned} P_n &= \frac{1}{2}[B_{\rm expt}(Z,N) + B_{\rm expt}(Z,N+2)] - B_{\rm expt}(Z,N+1), \quad (Z,\ N \text{ even}) \\ &= E_g(Z,N+1) - \frac{1}{2}[E_g(Z,N) + E_g(Z,N+2)] \end{aligned} \tag{6}$$

where E_g's are the calculated ground state energies, which depend on the value of G_n.

Assume the yrast eigenstate of $H_{\rm CSM}$ is expressed as

$$|\psi\rangle = \sum_i C_i |i\rangle \tag{7}$$

which is a coherent superposition of many CMPC's. The angular momentum alignment of $|\psi\rangle$ is

$$\langle\psi|J_x|\psi\rangle = \sum_i |C_i|^2 \langle i|J_x|j\rangle + 2\sum_{i<j} C_i^* C_j \langle i|J_x|j\rangle. \tag{8}$$

Considering J_x being a one-body operator, $\langle i|J_x|j\rangle$ ($i \neq j$) does not vanish only when $|i\rangle$ and $|j\rangle$ differ by one particle occupation. After certain permutation of creation operators, $|i\rangle$ and $|j\rangle$ are brought in to the forms

$$|i\rangle = (-)^{M_{i\mu}}|\mu\cdots\rangle, \quad |j\rangle = (-)^{M_{j\nu}}|\nu\cdots\rangle,$$

where the ellipses stand for the same particle occupation, and $(-)^{M_{i\mu}} = \pm 1$, $(-)^{M_{j\nu}} = \pm 1$ according to the permutation is even or odd. Then the kinematic moment of inertia of the

state $|\psi\rangle$ can be expressed conveniently in terms of the single-particle picture as follows:

$$J = \langle J_x\rangle/\omega = \sum_\mu J_{\mu\mu} + \sum_{\mu<\nu} J_{\mu\nu} \tag{9}$$

$$\begin{aligned} J_{\mu\mu} &= \frac{1}{\omega}\langle\mu|j_x|\mu\rangle \sum_i |C_i|^2 P_{i\mu} = \frac{1}{\omega}\langle\mu|j_x|\mu\rangle n_\mu \\ J_{\mu\nu} &= \frac{2}{\omega}\langle\mu|j_x|\nu\rangle \sum_{i<j} (-)^{M_{i\mu}+M_{j\nu}} C_i^* C_j, \quad \mu \neq \nu) \end{aligned} \tag{10}$$

where $n_\mu = \sum_i |C_i|^2 P_{i\mu}$ is the particle occupation probability of the cranked Nilsson orbital $|\mu\rangle$ in the state $|\psi\rangle$, and $P_{i\mu} = 1$ if $|\mu\rangle$ is occupied in $|i\rangle$ and $P_{i\mu} = 0$ otherwise. If the pairing interaction is missing, only one CMPC appears in $|\psi\rangle$ and all the interference terms $J_{\mu\nu}$ vanish, and the calculated moment of inertia is near the rigid-body value. When the pairing interaction is taken into account, while the diagonal part ($\sum_\mu J_{\mu\mu}$) changes only a little, the reduction of the moment of inertia originate mainly from the destructive interference ($\sum_{\mu<\nu} J_{\mu\nu} < 0$) due to the *anti-alignment effect of pairing interaction*. This is a pure quantum mechanical effect. The off-diagonal part ($\sum_{\mu<\nu} J_{\mu\nu}$) depends sensitively on the features and distribution of the cranked Nilsson orbitals near the Fermi surface. For μ and ν very far from the Fermi surface, $J_{\mu\nu}$ would be negligibly small. In fact, only when both μ and ν are near the Fermi surface would $J_{\mu\nu}$ be of importance. Calculations show that for normal deformed rare-earth nuclei, no contribution comes from $N \leq 3$ proton major shells and $N \leq 4$ neutron major shells, and the contribution from the $N \geq 6$ proton shell and $N \geq 7$ neutron shell are very small. For normally deformed even-even nuclei, due to the anti-alignment effect of pairing interaction, the destructive interference will reduce the moment of inertia, usually by a factor of about 1/2. The calculated bandhead moments of inertia of twenty-seven well-deformed rare-earth even-even nuclei are shown in Fig. 1. It is seen that the observed moments of inertia are reproduced rather well (except for a few cases) and no systematic excessive reduction of moment of inertia compared to that of a rigid rotor is found.

For odd-A nuclei, if a single-particle ν_0 is occupied by an odd nucleon, the pairing correlation is reduced (blocking effect). Calculation shows that $J_{\mu\nu_0}$ becomes positive (rather than negative in even-even nuclei) with magnitude depending sensitively on the energetic location and Coriolis response of the blocked level ν_0, hence the calculated moments of inertia show large variation with the blocked level. When ν_0 is a high-j intruder orbit near the Fermi surface, $J_{\mu\nu_0}$ would be very large, which leads to a very large odd-even differ-

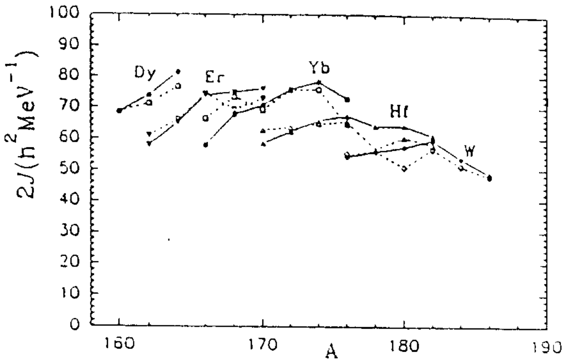

Fig. 1. Moments of inertia of the ground bands of even-even rare-earth nuclei (160 < A < 186). The experimental results for each isotope chain are connected by solid lines, and the corresponding calculated ones are connected by dashed lines.

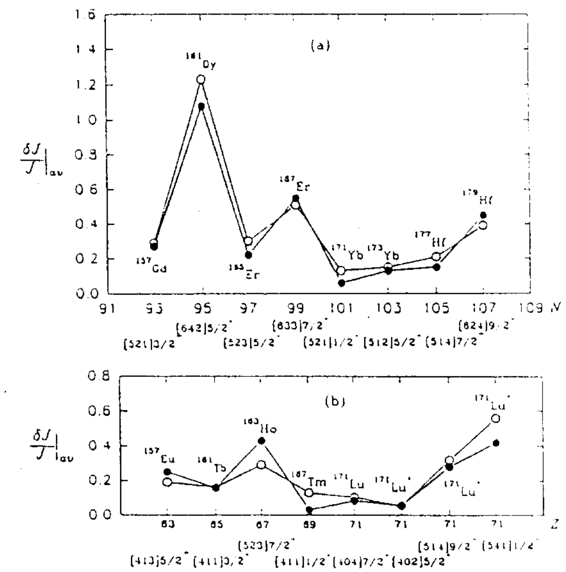

Fig. 2. Relative odd-even differences in the moments of inertia

$$\frac{\delta J}{J}\bigg|_{\rm av} = \left(J(A) - \frac{1}{2}[J_0(A+1) + J_0(A-1)]\right) \Big/ \left(\frac{1}{2}[J_0(A+1) + J_0(A-1)]\right)$$

of some typical (most β-stable) rare-earth nuclei versus the particle numbers and the corresponding Nilsson levels blocked by the odd particles. The experimental and calculated $\delta J/J|_{\rm av}$ are denoted by open and solid circles, respectively. (a) Odd-N nuclei. (b) Odd-Z nuclei.

ence in moments of inertia. This is just the reason why the observed moment of inertia of an odd-A nucleus whose unpaired nucleon occupies the neutron orbital $i_{13/2}$ ([642]5/2, [633]7/2, [624]9/2), or the proton orbital $h_{9/2}$ ([523]7/2, etc.) is so large. In contrast, when ν_0 is a low-j and high-Ω orbital (e.g. [402]5/2, [404]7/2, proton orbitals), or very far from the Fermi surface, $J_{\mu\nu_0}$ would almost vanish, hence δJ would be very small, and "identical bands" may appear. In other intermediate cases, the calculated odd-even difference in moments of inertia $\delta J/J \sim$ (10–20) %. The calculated odd-even differences in the moments of inertia of some typical (most β-stable) rare-earth nuclei are displayed in Fig. 2. It is seen that the general trend of the experimental variation of $\delta J/J|_{\rm av}$ is reproduced rather well by the PNC calculation. Considering that there are no adjustable parameters involved in the PNC calculation, these result seem encouraging. Particularly, in Fig. 2a, there exist three peaks of $\delta J/J|_{\rm av}$, corresponding to the blocked neutron orbitals [642]5/2, [633]7/2 and [624]9/2, respectively, which originate from the high-j intruder spherical orbital $i_{13/2}$ having a strong Coriolis response.

Finally, the observed systematic deviation from the additivity of nuclear moments of inertia can also be understood from the PNC calculation. Preliminary calculation shows that for the example mentioned above, the calculated R ratio is

$$R_{\rm calc} = \frac{J(^{171}\mathrm{Yb},[633]7/2) - J_0(^{170}\mathrm{Yb}) + J(^{171}\mathrm{Yb},[512]5/2) - J_0(^{170}\mathrm{Yb})}{J(^{170}\mathrm{Yb},([633]7/2 + [512]5/2)) - J_0(^{170}\mathrm{Yb})} = 1.50$$

which is close to the experimental results $R_{\rm expt} = 1.53$. The large deviation of the R ratio from 1 implies the residual quasiparticle interactions due to the blocking effects should be considered seriously.

In summary, the three long-standing problems concern with the moments of inertia of the quasi-particle vacuum and the relations between the moments of inertia associated with different quasiparticle configurations. All these problems are closely connected with the blocking effects, which is very difficult to be considered properly in the usual BCS formalism. Therefore, it is not surprised that the BCS approximation offers no satisfactory explanation for these problems.

Historically, the odd-even difference in various nuclear properties have been the interest subjects of nuclear structure theory, including the odd-even differences in mass or binding energy, Coulomb-energy, low-lying excitation spectra, moment of inertia, bandcrossing frequency, pair transfer, α-decay, etc. In fact, all systematic odd-even differences are mainly due to the blocking effects on pairing correlation.

By the way, we would like to mention that careful investigation [21] shows that there exist systematic odd-even difference in the moments of inertia even in the superdeformed nuclei in the $A \sim 190$ region (see Fig. 3), which indicates that pairing interaction still remains in superdeformed nuclei. Preliminary calculations [22] shows that the pairing interaction strength in superdeformed nuclei is much smaller than that in normally deformed nuclei

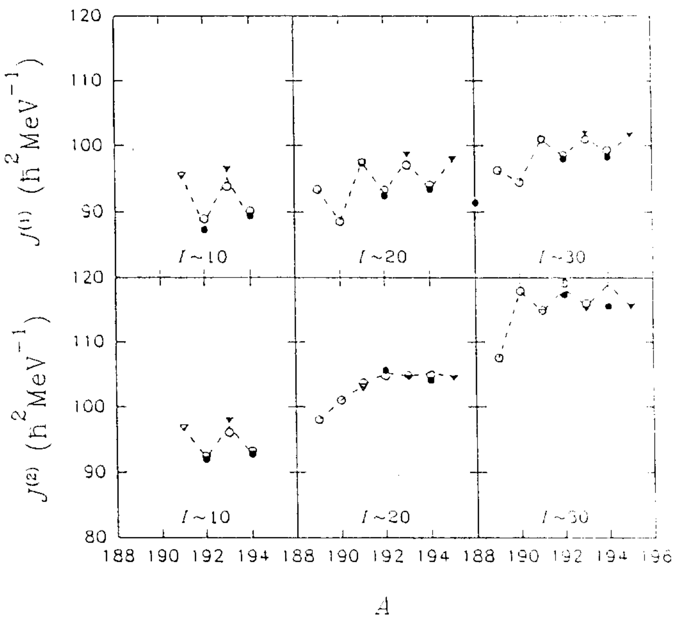

Fig. 3. The odd-even differences in $J^{(1)}$ and $J^{(2)}$ of the SD bands in the $A \sim 190$ region at the spin values $I \sim 10$, 20, and 30. $J^{(1)}$ and $J^{(2)}$ are extracted from the experimental transition energy using $J^{(1)}(I-1) = (2I-1)\hbar^2/E_\gamma(I \to I-2)$ and $J^{(2)}(I) = 4\hbar^2/E_\gamma(I+2 \to I) - E_\gamma(I \to I-2)$. The spin assignments are taken from ref. [23-25]. ∇—Au, ▼—Tl, O—Hg, ●—Pb.

Here we would like to add a few words about the odd-even difference in the bandcrossing frequency. It was noted by Garrett et al. [26] that in most rare-earth nuclei,

$$\delta\omega_c = \omega_c\ (\text{even} - \text{even}) - \omega_c\ (\text{odd } A) \sim 40\ \text{keV},$$

but a few anomalous ($\delta\omega_c \ll 40$ keV, even $\delta\omega_c \sim 0$) were also recognized, which was interpreted as due to the quadrupole pairing [27] or the blocking effect [28]. Afterwards, even more anomalous $\delta\omega_c < 0$ was found for the proton [541]1/2 bands in the isotopic chain of $^{167-175}$Ta. For example, it was found by the Tandem I13 group in the Chinese institute of Atomic Energy (Beijing) that [29] for the ^{169}Ta [541]1/2 band, $\delta\omega_c \sim -58$ keV. Several approaches to interpret these anomalous bandcrossing shifts have been suggested. In refs. [30,31] the observed anomalous delays of the crossing frequencies in odd-Z nuclei $^{167-195}$Ta were explained partly by means of the deformation driving effect. The strong shape-driving force generated when a high-j and low-Ω orbit is occupied with a particle may lead to dramatic change of deformation and thus change the pairing field of a secondary effect. In ref. [32], it is considered that the n-p interaction seems necessary to account for these anomalous delays. In ref. [33], the decoupling term in the rotational spectrum was considered. These anomalous delays were also investigated in ref. [34], in which the quadrupole pairing and the angular momentum projection were considered. It seems that the anomalous band-crossing shift of the proton [541]1/2 band still remains unresolved and need further investigation.

1. D. R. Inglis, *Phys. Rev.* 96, (1994) 1059.
2. A. Bohr, B. R. Mottelson, *Nuclear Structure* (Benjamin, 1975), vol. 2, p. 82, pp. 311-318.
3. A. Bohr, B. R. Mottelson, and D. Pines, *Phys. Rev.* 110, 936 (1958).
4. S. T. Belyaev, *Mat. Fys. Medd. Dan. Vid. Selsk.* **31**, No. 11 (1959);
5. S. G. Nilsson and O. Prior, *Mat. Fys. Medd. Dan. Vid. Selsk.* **32**, No. 16 (1961);
6. N. H. Allah and M. Fellah, *Phys. Rev.* **C43**, (1991) 2648.
7. M. Hasegawa and S. Tazaki, *Phys. Rev.* **C47**, (1993) 188.
8. J. Y. Zeng, T. H. Jin and Z. J. Zhao, *Phys. Rev.* **C50**, (1994) 1388.
9. T. Byrski et al., *Phys. Rev. Lett.* **64**, (1990) 1650.
10. F. S. Stephens et al., *Phys. Rev. Lett.* **64**, (1990) 2623.

11. C. Baktash, J. D. Garrett, D. F. Winchell, and A. Smith, *Phys. Rev. Lett.* 69, (1992) 1500.

12. J. Y. Zhang and L. L. Riedinger, *Phys. Rev. Lett.* 69, 3448 (1992).

13. J. Y. Zeng and T. S. Cheng, *Nucl. Phys.* **A405** (1983) 1.

14. D. C. Sood, D. M. Headly, and R. K. Sheline, *At. Data Nucl. Data Tables*, 47 (1991) 89.

15. D. J. Rowe, *Nuclear Collective Motion*, (Methuen, London, 1970), p. 194.

16. J. Y. Zeng, Y. A. Lei, T. H. Jin and Z. J. Zhao, *Phys. Rev.* C50, (1994) 1388.

17. C. S. Wu and J. Y. Zeng, *Phys. Rev.* **C41**, (1990), 1822.

18. C. S. Wu and J. Y. Zeng, *Phys. Rev.* C39, (1989) 666.

19. R. Bengtsson, S. Frauendorf, and F. R. May, *At. Data Nucl. Data Tables* 35, (1986) 15.

20. S. G. Nilsson et al., Nucl. Phys. **A131** (1969) 1.

21. J. Y. Zeng, Y. A. Lei and E. G. Zhao, *Science in China* (A), 38 (1995) 730.

22. T. H. Jin, Z. J. Zhao, and J. Y. Zeng, to be published in *Science in China*.

23. J. A. Becker et al., *Phys. Rev.* C46 (1992) 889; Nucl. Phys. A520 (1990) 187c.

24. C. S. Wu, et al., *Phys. Rev.* C45 (1992) 261.

25. R. Piepenbring and K. V. Protasov, *Z. Phys.* **A345** (1993) 7.

26. J. D. Garrett et al., *Phys. Rev. Lett.* 47, (1981) 75.

27. J. D. Garrett et al., *Phys. Lett.* B118, (1982) 297.

28. J. Y. Zeng et al, *Nucl. Phys.* **A411**, (1983) 49.

29. S. G. Li et al., *Nucl. Phys.* **A555**, (1993) 435.

30. C. X. Yang et al., *Chin. J. Nucl. Phys.* **16**, (1994) 217.

31. Y. S. Chen et al., *Chin. J. Nucl. Phys.* **16**, (1994) 201.

32. C. X. Yang et al., *Chin. J. Nucl. Phys.* **16**, (1994) 223.

33. C. S. Wu, *Phys. Rev.* C51, (1995) 1819.

34. Y. Sun, S. Wen and D. H. Feng, *Phys. Rev. Lett.* **72**, (1994) 3483.