3B

ASITP

INSTITUTE OF THEORETICAL PHYSICS ACADEMIA SINICA

AS-ITP-95-26
October 1995

$\Delta I = 4$ Bifurcation and the sdg Interacting Boson Model

Yu-xin Liu, Hong-zhou Sun
and En-guang Zhao

P.O.Box 2735, Beijing 100080, The People's Republic of China

Telefax : (86)-1-2562587 Telephone : 2568348

Telex : 22040 BAOAS CN Cable : 6158

$\Delta I = 4$ Bifurcation and the sdg Interacting Boson Model

Yu-xin Liu[a,b,d], Hong-zhou Sun[c,d] and En-guang Zhao[a,d]

[a] CCAST(World Laboratory), P. O. Box 8730, Beijing 100080, China
[b] Department of Physics, Peking University, Beijing 100871, China
[c] Department of Physics, Tsinghua University, Beijing 100084, China
[d] Institute of Theoretical Physics, Academia Sinica, P. O. Box 2735, Beijing 100080, China

Abstract

We show that the superdeformed nuclear states can be described in the framework of the interacting boson model with the g-bosons being taken into account in this letter. The superdeformed rotational bands with $\Delta I = 4$ bifurcation can be reproduced in the $SU(5)$ limits of the sdg IBM. The perturbation causing the $\Delta I = 4$ bifurcation to emerge in the $\Delta I = 2$ superdeformed rotational band is shown to possess the $SU(5)$ symmetry.

PACS Numbers: 21.10.Re, 21.60.Fw, 23.20.Lv

In the spectroscopy of superdeformed nuclei, rotational level sequences have been observed in which the $\Delta I = 2$ rotational band is perturbed and split into two branches[1,2]. Because in each of the branches the energy levels differ in angular momentum by four, this phenomenon is called $\Delta I = 4$ bifurcation, or $\Delta I = 2$ staggering, since the states differing by two in angular momentum are staggered in energy. The occurrence of such a staggering in the rotational energy suggests that the strongly prolate spheroidal deformation has been perturbed by hexadecupole deformation[3] (for example, the deformation parameter β_4 of ^{149}Gd may be up to 0.08[4]). In other word, a perturbation holding symmetry Y_{44} with respect to the symmetry axis of the nucleus emerges in the superdeformed nuclear states[5]. Thus, the experiments are regarded as evidences that superdeformed nuclei have C_{4v} symmetry[5-7] or intrinsic vortical motion[8]. In the framework of interacting boson model (IBM)[9], the nucleus with quadrupole and hexadecupole deformations consists of s-, d- and g-bosons. The approach to describe the nucleus including s-, d- and g-bosons is usually referred as sdg interacting boson model (sdg IBM)[10,11] With the sdg IBM we will show in this letter that the perturbation may probably possess much higher symmetry $SU(5)$ in the sdg IBM.

In sdg interacting boson model, the collective nuclear states (with quadrupole and hexadecupole deformations) are generated as states of a system with N s-, d-, g-bosons. Since the total single boson space is 15 dimensional, the symmetry group is $U(15)$, and the states of N-bosons belong to the totally symmetric irreducible representation (irrep) $[N]_{15}$ of the $U(15)$. Moreover, it has been shown that the sdg IBM has strong coupling dynamical symmetries $SU(3)$, $SU(5)$, $SU(6)$, $O(15)$ and weak coupling dynamical symmetries $U_{sd}(6) \otimes U_g(9)$, $U_{dg}(14)$, $U_d(5) \otimes U_{sg}(10)$. Numerical calculations show that the sdg IBM is quite successful in describing nuclear states with large deformation and $E4$ transitions[12]. On the other hand, with the projective coherent state scheme[9] being exploited, Devi and collaborators showed[12,13] that the potential energy surface of the nucleus with dynamical symmetry $U_{sd}(6) \otimes U_g(9)$, $U_{dg}(14)$, $U_d(5) \otimes U_{sg}(10)$, $SU(3)$ or $O(15)$ has one minimum , which is just the same as that in the sd IBM. However, the energy surface of a nucleus with $SU(6)$ symmetry has three degenerate minima and that of the $SU(5)$ symmetry has two minima that are displaced in energy (see Fig. 5 of Ref.[12] and Figs. 1b–4b of Ref [13]). It indicates that the sdg IBM admits shape coexistence and shape phase transformation which can be driven by angular momentum[14]. Considering this fact and the view that superdeformed states are generated in the second minimum of the potential energy surface[15], we know that the superdeformed nuclear states can be described with the $SU(5)$ limit of the sdg IBM.

As a nucleus has the $SU(5)$ symmetry in the sdg IBM, the states of the nucleus can be classified by the irreps of the group chain

$$U(15) \supset SU(5) \supset SO(5) \supset SO(3). \tag{1}$$

The wave function can then be written as

$$|\,\psi(I)\,\rangle = |\,\underset{U(15)}{[N]_{15}}\ \underset{SU(5)}{[n_1,n_2,n_3,n_4]_5}\ \beta\ \underset{SO(5)}{(\tau_1,\tau_2)_5}\ \alpha\ \underset{SO(3)}{I}\ \rangle, \tag{2}$$

where β is the additional quantum number to distinguish the same $(\tau_1,\tau_2)_5$ belonging to the same $[n_1,n_2,n_3,n_4]_5$, and α is the additional quantum number to differ the same I belonging to the same $(\tau_1,\tau_2)_5$. They are the integers satisfying the relations $1 \le \beta \le \beta_{max}$, $1 \le \alpha \le \alpha_{max}$ respectively, in which β_{max} is the multiplicity of irrep $(\tau_1,\tau_2)_5$ belonging to the irrep $[n_1,n_2,n_3 n_4]_5$, and α_{max} is the multiplicity of I belonging to the irrep $(\tau_1,\tau_2)_5$. All the irreps and the multiplicities can be determined from the branching rules of the irrep reductions which have been discussed by Sun *et al.*[11]. For a given $[N]_{15}$, the irrep $[n_1,n_2,n_3,n_4]_5$ can be all the possible $[2N-2p-4q-6r-6s, 2(p+q+r), 2(q+r), 2r]_5$ with restriction $2p+3q+4r+5s \le N$. For a given $[n_1,n_2,n_3,n_4]_5$, the irrep $(\tau_1,\tau_2)_5$ can be determined with the Young tableaux technique[16] or Schur function method[17], for instance,

$$[n,0,0,0]_5 = (n,0) \oplus (n-2,0) \oplus (n-4,0) \oplus \cdots \oplus \begin{cases} (0,0)\ (\text{for } n = \text{even}), \\ (1,0)\ (\text{for } n = \text{odd}). \end{cases} \tag{3}$$

$$[n_1,n_2,0,0]_5 = \begin{cases} F(n_1,n_2) \oplus F(n_1-2,n_2) \oplus \cdots \oplus F(n_2+2,n_2) \\ \quad \oplus [n_2,n_2], \qquad\qquad (\text{for } n_1-n_2 = \text{even}), \\ F(n_1,n_2) \oplus F(n_1-2,n_2) \oplus \cdots \oplus F(n_2+3,n_2) \\ \quad \oplus F(n_2+1,n_2), \qquad (\text{for } n_1-n_2 = \text{odd}), \end{cases} \tag{4}$$

where

$$[n,n] = \sum_{i,j} \oplus (n-2i, n-2i-2j),$$

$$F(n_1,n_2) = \sum_{i,j} \oplus (n_1 - i, n_2 - i - 2j).$$

For a given $(\tau_1,\tau_2)_5$, the reduction of $SO(5) \supset O(3)$ can be obtained with the multi-partition technique[18]. For example, for a given $(\tau_1,0)_5$, the corresponding I can be $2\tau_1$, $2\tau_1-2$, $2\tau_1-3$, $\cdots\cdots$. For a given $(\tau_1,\tau_2)_5$, the corresponding I can be $2\tau_1+\tau_2$, $2\tau_1+\tau_2-1$, $2\tau_1+\tau_2-2$, $\cdots\cdots$ or $2\tau_1+\tau_2$, $2\tau_1+\tau_2-2$, $\cdots\cdots$.

The interaction Hamiltonian of the bosons in a nucleus with the $SU(5)$ symmetry can be written as

$$H = E_0 + AC_{2SU(5)} + BC_{2SO(5)} + CC_{2SO(3)}, \tag{5}$$

in which C_{2g} is the quadratic Casimir operator of the group g. The energy of the state $|N[n_1,n_2,n_3,n_4]_5(\tau_1,\tau_2)_5 I\rangle$ can be given as

$$\begin{aligned} E(I) = E_0 &+ A[n_1(n_1+4) + n_2(n_2+2) + n_3^2 + n_4(n_4-2) - \tfrac{1}{5}(n_1+n_2+n_3+n_4)^2] \\ &+ B[\tau_1(\tau_1+3) + \tau_2(\tau_2+1)] + CI(I+1), \end{aligned} \tag{6}$$

where A determines the energy difference between the lowest $I=0$ states with different irrep $[n_1,n_2,n_3,n_4]_5$. B decides the energy difference between the different lowest I states with different $(\tau_1,\tau_2)_5$ but the same $[n_1,n_2,n_3,n_4]_5$. C gives the energy difference between the states with different I but the same $(\tau_1,\tau_2)_5$ and $[n_1,n_2,n_3,n_4]_5$. In order to keep the $I=0$ state with irrep $[2N,0,0,0]_5$ (N is the total number of the bosons) and $(\tau_1,\tau_2)_5 = (0,0)$ being the ground state, the parameters should be taken as $A<0$, $B>0$, $C>0$. Thus, the energy bands generated by the totally symmetric irrep $[2N,0,0,0]$ are usually lower than those generated by the nontotally symmetric irreps $[2N-2,2,0,0]$ and others. On the other hand, the branch rules of the irrep reduction show that the totally symmetric irrep $[2N,0,0,0]$ of the $SU(5)$ generates the energy bands with level sequences $\{0, 4, 8, \cdots, 4N\}$, $\{2, 6, 10, \cdots, 4N-2\}$, $\cdots\cdots$ respectively (see Fig. 2a of Ref [13]). We know then that the irrep $[2N,0,0,0]$ of $SU(5)$ can reproduce the low-lying energy bands with level sequence I_0, I_0+4, I_0+8, $\cdots$, but can not generate the superdeformed energy band with level sequence I_0, I_0+2, I_0+4, $\cdots$. For the irrep $[2N-2,2,0,0]$ of $SU(5)$, since the irreps of $SO(5)$ can be $(2N-2,2)$, $(2N-4,2)$, $\cdots$, $(2,2)$ and $(2N-2,0)$, $(2N-4,0)$, $\cdots$, $(2,0)$, $(0,0)$, of which the corresponding largest angular momenta are $4N-2$, $4N-6$, $4N-10$, $\cdots$, 10, 6 and $4N-4$, $4N-8$, $4N-12$, $\cdots$, 8, 4, 0 respectively. Two closely placed energy bands with level sequences I_0, I_0+4, I_0+8, $\cdots$, and I_0+2, I_0+6, I_0+10, $\cdots$ come to naturally, and couple to one band with level sequence I_0, I_0+2, I_0+4, $\cdots$, of which the states differing by 2 in angular momentum are staggering in energy. Figure 1 is an example of the energy spectrum built on the irrep $[2N-2,2,0,0]$ of the $SU(5)$. From the irrep reduction rule in the group chain $SO(5) \supset SO(3)$ and figure 1 we know that many bands with $\Delta I = 2$ staggering in energy can appear in one system. Therefore, one nucleus can have more than one superdeformed rotational bands with $\Delta I = 4$ bifurcation, such as the ones reported in Ref. [2].

To show the $\Delta I = 4$ bifurcation more explicitly, we discuss the energy differences ΔE_γ between two consecutive γ-ray transitions of the energy band generated by the irrep $[2N-2,2,0,0]$ of the $SU(5)$ as a function of angular momentum and rotational frequency respectively after subtraction of a smooth reference $\Delta E_\gamma^{ref}(I)$. Since $\Delta E_\gamma(I)$ is defined as $\Delta E_\gamma(I) = E_\gamma(I+2) - E_\gamma(I)$ and the $\Delta E_\gamma^{ref}(I)$ is given as $\Delta E_\gamma^{ref}(I) = [\Delta E_\gamma(I-2) + 2\Delta E_\gamma(I) + \Delta E_\gamma(I+2)]/4$, we have $\Delta E_\gamma(I) - \Delta E_\gamma^{ref}(I) = [E_\gamma(I-2) - 3E_\gamma(I) + 3E_\gamma(I+2) - E_\gamma(I+4)]/4$. Figures 2 and 3 illustrate the changing feature of the $\Delta E_\gamma(I) - \Delta E_\gamma^{ref}(I)$ versus the angular momentum I and that against the rotational frequency $\hbar\omega = \frac{1}{2}E_\gamma(I)$ respectively with the parameters in eqs. (5) and (6) are taken as $E_0 = 0$, $A = 0$, $B = 5$ KeV, $C = 0.1$ KeV for a nucleus with 18 s-, d-, g-bosons. In figures 4 and 5 we show the two kind variation features with parameters $E_0 = 0$, $A = 0$, $B = 0.01$ KeV, $C = 6.25$ KeV for the same nucleus. Since the parameters E_0 and A determine only the energy of the lowest $I = 0$ state belonging to $[2N-2, 2, 0, 0]$ with respect to the ground state, we take $E_0 = 0$ and $A = 0$. And the choice of parameters in this way does not change the band structure being considered.

As the parameters are taken as $B = 5$ KeV, $C = 0.1$ KeV, i.e., $B \gg C$, the Hamiltonian (5) has the $SU(5)$ symmetry. Figure 2 shows that the $\Delta I = 2$ staggering in energy generated by the $SU(5)$ symmetry is very obvious (for negative energy differences $\Delta E_\gamma(I) - \Delta E_\gamma^{ref}(I)$, the amplitude is in the same order of magnitude as the $E_\gamma(I)$; for the positive energy differences, the amplitude is in the same order of magnitude as the neighboring transition energy $E_\gamma(I-2)$ or $E_\gamma(I+2)$) and the staggering amplitude gets large and large as the angular momentum I increases. Figure 3 indicates that energy differences $\Delta E_\gamma(I) - \Delta E_\gamma^{ref}(I)$ versus the rotational frequency are separated into two isolated branches for the two $\Delta I = 4$ level sequences. For one $\Delta I = 4$ level sequence the $\Delta E_\gamma(I) - \Delta E_\gamma^{ref}(I)$ is always positive and the rotational frequency of the state I is very small (less than 0.03 MeV). For another $\Delta I = 4$ level sequence $\Delta E_\gamma(I) - \Delta E_\gamma^{ref}(I)$ is definitely negative and the rotational frequency is in the usual situation (from 0.03 MeV to 0.32 MeV). Since the difference between the rotational frequencies of the states with angular momentum I, $I+2$ is quite large, the positive $\Delta E_\gamma(I) - \Delta E_\gamma^{ref}(I)$ and the negative $\Delta E_\gamma(I) - \Delta E_\gamma^{ref}(I)$ does not appear alternately but split into two branches. It suggests that if the energy of state I in a band with level sequence I_0, I_0+4, I_0+8, $\cdots\cdots$ is close to the energy of the state $I+2$ in the band with level sequence I_0+2, I_0+6, $\cdots\cdots$, the $\Delta I = 2$ staggering of the $\Delta E_\gamma(I) - \Delta E_\gamma^{ref}(I)$ washes out and two segregated branches emerge. It shows also that to describe well the observed $\Delta I = 4$ bifurcation theoretically, the staggering of energy differences as a function of the rotational frequency is a more reliable characteristic than that versus the angular momentum.

When the parameters are chosen as $B = 0.01$ KeV, $C = 6.25$ KeV, i.e., $B \ll C$, the interacting Hamiltonian is expressed as an axial rotational interaction and a perturbation with the $SU(5)$ symmetry. The calculated results of the energy differences between two consecutive γ-ray transitions $\Delta E_\gamma(I) - \Delta E_\gamma^{ref}(I)$ as a function of the angular momentum and that as a function of the rotational frequency are illustrated in figures 4 and 5 respectively. The figures show that the energy differences ΔE_γ between two consecutive γ-ray transitions after subtraction of a smooth reference are really very small even though they do not agree with the experimentally data precisely (we are not attempt to fit the data

in this letter at all). Meanwhile the variation characteristic of $\Delta E_\gamma(I) - \Delta E_\gamma^{ref}(I)$ as a function of rotational frequency represents the observed changing feature of the staggering versus the rotational frequency, and is consistent with that as a function of angular momentum. Comparing figures 2, 3 with figures 4, 5 respectively, we know that, as the interaction with the $SU(5)$ symmetry is handled as a perturbation, the $\Delta I = 2$ staggering in γ-ray energy differences can be described better than in the case that the interaction is taken as a dominant. It suggests that, even though the $SU(5)$ symmetry is imperative in generating energy levels with $\Delta I = 4$ sequence, the interaction generating the superdeformed nuclear states with $\Delta I = 4$ bifurcation is not governed by the $SU(5)$ symmetry, but still regulated by the rotational interaction. Nevertheless, the perturbation causing the $\Delta I = 2$ rotational band to split into $\Delta I = 4$ bifurcation may possess $SU(5)$ symmetry. We have also investigated the changing feature of the energy differences ΔE_γ introduced as the manifestation of the $\Delta I = 4$ bifurcation by Cedrewall[2]. The same result as shown above is obtained.

In summary, we have shown in this letter that the superdeformed nuclear states can be described in the framework of the interacting boson model as the g-bosons are taken into account. In experiment, the representation to show the $\Delta I = 4$ bifurcation is the staggering in energy differences ΔE_γ between two consecutive γ-ray transitions after subtraction of a smooth reference[1,2] as a function of the rotational frequency since the angular momenta are not assigned. In theoretical description, although the angular momentum can be assigned, the changing feature of the energy differences ΔE_γ as a function of angular momentum is not consistent with that versus the rotational frequency when the difference of rotational frequencies between the two $\Delta I = 4$ branches is large. Then, to describe the observed $\Delta I = 2$ staggering well, one should consider the same changing characteristic as in experiment as fully as possible. Otherwise, we have not attempted to fit the experimental data in the sd$\acute{g}$ IBM in this letter. However, preliminary calculation indicates that the general feature of the staggering in energy differences ΔE_γ in a $\Delta I = 2$ superdeformed rotational band can be described in the sdg IBM as the Hamiltonian is taken as a rotational interaction plus a perturbation with $SU(5)$ symmetry. In this scheme, one nucleus can naturally have more than one superdeformed rotational bands with $\Delta I = 4$ bifurcation. We then come to a conclusion that the perturbative interaction making the $\Delta I = 2$ superdeformed rotational band split into $\Delta I = 4$ bifurcation may possess $SU(5)$ symmetry.

This work is supported by the National Natural Science foundation of China. This work has benefited from stimulating discussions with Professor F. Iachello. Helpful discussions with Professor Qi-zhi Han are also acknowledged with thanks.

References

[1] S. Flibotte *et al.*, Phys. Rev. Lett. 71 (1993), 4299.

[2] B. Cederwall et *al.* Phys. Rev. Lett. 72 (1994), 3150.

[3] L. K. Peker, Phys. Rev. Lett. 50 (1983), 1749.

[4] W. Nazarewicz, R. Wyss and A. Johnson, Nucl. Phys. **A503** (1989), 285.

[5] I. Hamamoto and B. R. Mottelson, Phys. Lett. **333B** (1994), 294.

[6] A. O. Macchiavelli *et al.*, Phys. Rev. C51 (1995), R1.

[7] I. M. Pavichenkov and S. Flibotte, Phys. Rev. C51(1995), R460.

[8] I. N. Mikhailov and P. Quentin, Phys. Rev. Lett. 74 (1995), 3336.

[9] F.Iachello and A Arima, *The Interacting Boson Model* (Cambridge: Cambridge University Press, 1987).

[10] A. Bohr and B. R. Mottelson, Phys. Scr. 22 (1980), 468; 25 (1982), 28.

[11] H. Z. Sun, M. Moshinsky, A. Frank and P. van Isacker, kinam 5 (1983) 135; V. K. B. Kota, H. De Meyer, J. Vander Jeugt and G. Vanden Berghe, J. Math. Phys. 28 (1987), 1644.

[12] for review see, for example, Y. D. Devi and V. K. B. Kota, Pramana J. Phys. 39 (1992), 413.

[13] Y. D. Devi and V. K. B. Kota, Z. Phys. A337 (1990), 15.

[14] S. Kuyucak, V. S. Lac and I. Morrison, Phys. Lett. **263B** (1991), 146.

[15] V. M. Strutinsky, Nucl. Phys. A95 (1967), 420; A122 (1968), 1.

[16] M. Hamermesh, *Group Theory and its Application to Physical Problems* (Reading MA: Addison Wesley, 1962).

[17] B. G. Wybourne, Classical Groups for Physicists(New York: Wiley, 1974).

[18] J. J. Wang and H. Z. Sun, High Energy Phys. Nucl. Phys. 14(1990), 842; H. Z. Sun, Q. Z. Han and S. T. Hsieh. J. Phys. A28 (1995), 18.

Figure Captions:

Figure 1. A part of the energy spectrum generated by the irrep $[8,2,0,0]$ of the $SU(5)$(the parameter are taken as $E_0 = A = 0$, $B = 25$ KeV, $C = 5$ KeV. The labels at the left side of the levels are the corresponding irrep of the $SO(5)$ group).

Figure 2. The energy differences ΔE_γ between two consecutive γ-ray transitions in the energy band generated by the irrep $[2N-2,2,0,0]$ of the $SU(5)$ as a function of angular momentum after subtraction of a smooth reference $\Delta E_\gamma^{ref}(I)$. The nucleus is taken as the one with 18 s-, d- and g-bosns and the parameters are taken as $E_0 = 0$, $A = 0$, $B = 5$ KeV, $C = 0.1$ KeV.

Figure 3. The energy differences ΔE_γ between two consecutive γ-ray transitions in the energy band belonging to the irrep $[2N-2,2,0,0]$ of the $SU(5)$ as a function of rotational frequency after subtraction of a smooth reference $\Delta E_\gamma^{ref}(I)$. The nucleus is also taken as the one with 18 s-, d- and g-bosons and the parameters are taken as $E_0 = 0$, $A = 0$, $B = 5$ KeV, $C = 0.1$ KeV.

Figure 4. The same as figure 2 but for parameters $E_0 = 0$, $A = 0$, $B = 0.01$ KeV, $C = 6.25$ KeV.

Figure 5. The same as figure 3 but for parameters $E_0 = 0$, $A = 0$, $B = 0.01$ KeV, $C = 6.25$ KeV.

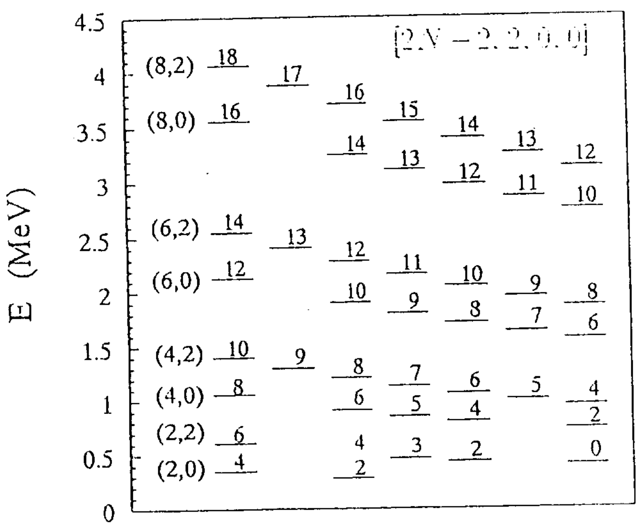

Fig. 1.

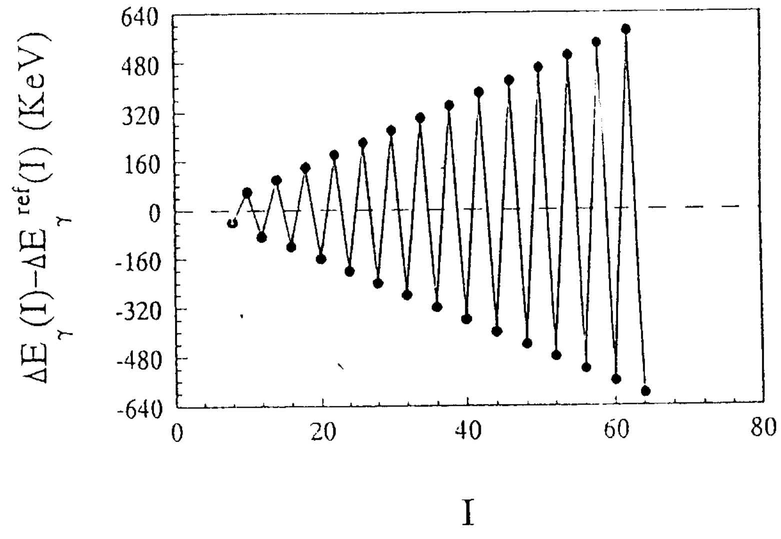

Fig. 2.

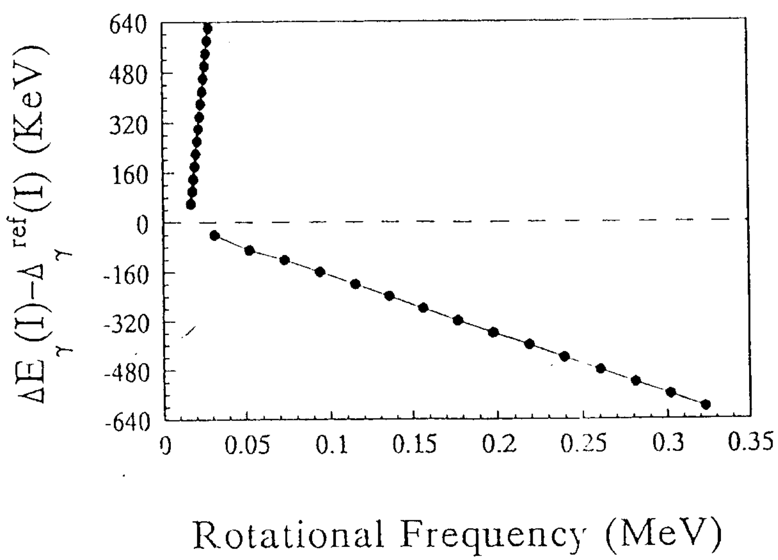

Fig. 3.

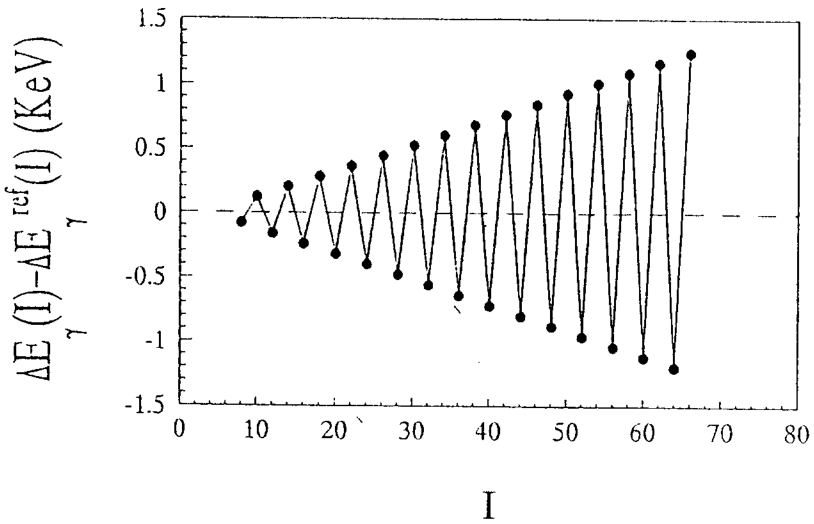
1.5
1
0.5
0
-0.5
-1
-1.5
0
10
20
30
40
50
60
70
80
I
ΔE_γ(I)-ΔE_γ^ref(I) (KeV)

Fig. 4.

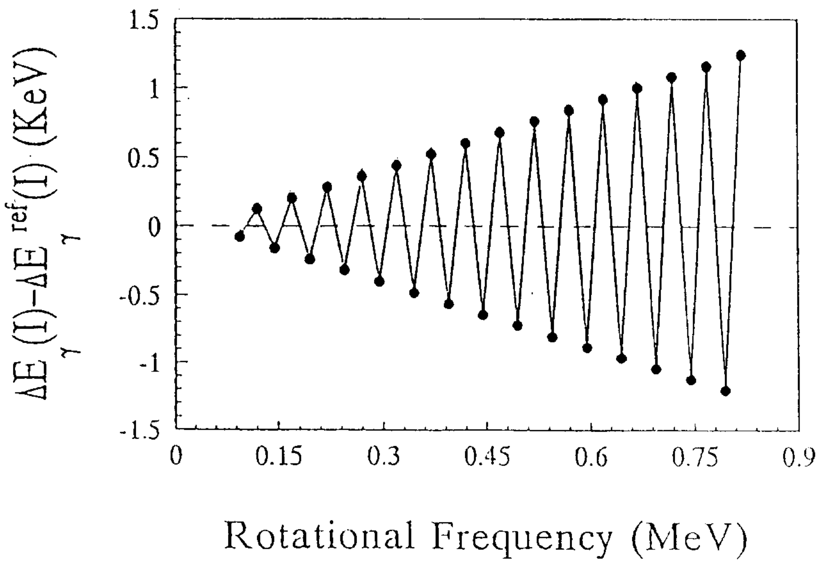
1.5
1
0.5
0
-0.5
-1
-1.5
0
0.15
0.3
0.45
0.6
0.75
0.9
Rotational Frequency (MeV)
ΔE_γ(I)-ΔE_γ^ref(I) (KeV)

Fig. 5.