CERN-TH-2024-112

Flavour Deconstructing the Composite Higgs

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Abstract: We present a flavour non-universal extension of the Standard Model combined with the idea of Higgs compositeness. At the TeV scale, the gauge groups $SU(2)_R$ and $U(1)_{B-L}$ are assumed to act in a non-universal manner on light- and third-generation fermions, while the Higgs emerges as a pseudo Nambu-Goldstone boson of the spontaneous global symmetry breaking $Sp(4) \to SU(2)_L \times SU(2)_R^{[3]}$, attributed to new strong dynamics. The flavour deconstruction means the couplings of the light families to the composite sector (and therefore the pNGB Higgs) are suppressed by powers of a heavy mass scale (from which the Higgs is nevertheless shielded by compositeness), explaining the flavour puzzle. We present a detailed analysis of the radiatively generated Higgs potential, showing how this intrinsically-flavoured framework has the ingredients to justify the unavoidable tuning in the Higgs potential necessary to separate electroweak and composite scales. This happens for large enough values of the $SU(2)_R^{[3]}$ gauge coupling and light enough flavoured gauge bosons resulting from the deconstruction, whose phenomenology is also investigated. The model is compatible with current experimental bounds and predicts new states at the TeV scale, which are within the reach of near future experimental searches.

Contents

1 Introduction

The Higgs sector of the Standard Model (SM) faces two significant structural problems. The first one, the *flavour puzzle*, is the unexplained hierarchical structure of the Yukawa couplings, which span five orders of magnitude. To address this issue, as well as other deficits of the SM, it is natural to posit the existence of new heavy dynamics. This assumption, in turn, inevitably leads to the second structural problem of the Higgs sector: the *electroweak hierarchy problem* or the instability of the quadratic term in the Higgs potential caused by the presence of heavy degrees of freedom.

Traditional model-building approaches tend to separate the solution to these two problems. This is achieved assuming flavour-blind dynamics close to the electroweak (EW) scale to screen the Higgs sector from heavy dynamics, and flavour-full dynamics at much higher energies to generate the flavour hierarchies. From a general quantum field theory perspective, this is certainly a viable option: the Yukawa couplings are marginal operators whose origin could be attributed to very high energy scales. This separation leads to the so-called Minimal Flavour Violating (MFV) paradigm [1], which has the important phenomenological advantage of minimising corrections to the precisely tested sector of rare flavour-changing neutral-current processes (FCNCs). However, the advantage of MFV in the pre-LHC era has turned into a disadvantage nowadays [2, 3]: strong bounds from direct searches on flavour-universal new physics make it less effective for solving the EW hierarchy problem. The high-energy searches at the LHC are sending a clear signal: to minimise the fine-tuning in the Higgs sector, flavour non-universal New Physics (NP) is needed at the TeV scale, which in turn suggests the flavour puzzle and electroweak hierarchy problem are inexorably related and must be addressed simultaneously [2]. In this paper, we combine the hypothesis of a composite Higgs, in the spirit of the minimal composite Higgs model originally proposed in [4, 5], with the idea of flavour deconstruction, along the lines developed recently in [2, 6]. As we will demonstrate, combining these two hypotheses provides significant benefits, as it allows new-physics coupled dominantly to the third generation, responsible for the stability of the Higgs sector, to lie in the few TeV domain.

In composite models the Higgs is assumed to be the pseudo Nambu-Goldstone boson (pNGB) of a new strongly interacting sector, much like the pions in QCD (see [7] for a comprehensive review). Being a composite state, the Higgs is protected from quantum corrections induced by dynamics occurring at energies above the compositeness scale. Moreover, the shift symmetry associated to Goldstone bosons allows a scale separation between the Higgs mass term and the confinement scale of the new strong dynamics [8]. This scale separation cannot be large, and the Higgs mass term (and the electroweak scale) is radiatively generated by the unavoidable breaking of the global symmetry of the composite sector induced by the Yukawa couplings and coupling to the SM gauge fields. Present constraints from EW observables and Higgs physics, as well as the absence of direct signals of new dynamics, imply that the scale associated to the new dynamics must lie above about 1 TeV. This *little hierarchy* between the EW and the composite scales would still be acceptable to minimise fine tuning in the Higgs potential.

However, a compositeness scale in the TeV range is ruled out by FCNC data if the flavour structure of the model is generic. In this class of models, the SM fermions are the result of a linear mixing between elementary fields and composite states (partial compositeness) that delivers the effective Yukawa couplings. An extensive and updated phenomenological analysis of this mechanism has been presented in [9], where it has been show that global flavour symmetries are a key ingredient to allow a small scale separation between the elementary and composite dynamics. Not surprisingly, the most efficient way to achieve this goal is via global $U(2)^n$ symmetries acting on the light fermions only, as originally proposed in [10–13]. This is where the mechanism of flavour deconstruction becomes relevant, as it provides a natural origin for these otherwise *ad hoc* flavour symmetries. Namely, they emerge as an accidental low-energy property of a manifestly flavour non-universal gauge group in the ultraviolet (UV).

The hypothesis of flavour non-universal gauge interactions as the origin of the flavour hierarchies is quite old and has been pursued in different contexts (see *e.g.* [14–17]). The last few years have witnessed a renewed phenomenological interest in this approach [18–27] largely motivated by the the so-called B-anomalies. While the significance of these deviations from the SM remains the subject of ongoing debate (see [28] for a recent update), the corresponding model-building activity has provided interesting explicit examples of models featuring TeV-scale dynamics, coupled mainly to the third generation and fully compatible with present observations, that can be motivated to address the flavour puzzle.

The general ingredient of flavour deconstruction is the breaking of a flavour non-universal gauge group of the type $G_A \times G_B$ into its flavour-diagonal and maximal subgroup G_{A+B}.[1] If G_A acts only on the light families (and G_B on the third one) we naturally achieve the accidental $U(2)^n$ symmetries acting on the light fermion families, which are the key ingredient to minimise the tight bounds on new physics dictated by FCNCs. The class of deconstructed model we are interested in in this paper was identified in [2] as being an especially natural candidate, and further developed in [6]. It is based on the deconstruction of $SU(2)_R$ and $U(1)_{B-L}$, where $SU(2)_L$ is kept flavour-universal. This deconstruction pattern leads to a parametric structure for the Yukawa couplings of the type

$$Y_{u,d,e} \sim \begin{pmatrix} \epsilon_R & \epsilon_L \\ \epsilon_R\epsilon_L & 1 \end{pmatrix} \tag{1.1}$$

where, for simplicity, we do not distinguish first and second generations. The parameters $\epsilon_{L,R}$ appearing in (1.1) are ratios of the vacuum expectation values (VEVs) of the scalar (link) fields responsible for the spontaneous symmetry breaking $G_A \times G_B \to G_{A+B}$, over the mass of appropriate heavy fermions. The presumed smallness of these ratios is the origin of the flavour hierarchies. If one considered only flavour hierarchies and ignored the B-anomalies, nothing anchors the overall scale of these new degrees of freedom. However, the situation changes if we aim at addressing also the electroweak hierarchy problem. As advocated in [2, 32–34], the overall scale of the deconstructed gauge dynamics should be anchored by its impact on the Higgs sector, which is unavoidable since (at least some) the link fields are charged under the EW symmetry group, and hence couple directly to the Higgs. While this argument was put forward in [2, 32–34] using only semi-quantitative finite-naturalness arguments, in this paper we provide a quantitative concrete analysis implementing flavour deconstruction in the context of the minimal composite Higgs model.

Given the deconstruction pattern sketched above, where the Higgs and third-generation fermions are charged under the family-specific $SU(2)_R^{[3]}$ symmetry, the minimal embedding

[1] That the flavour-diagonal subgroup is left unbroken is completely generic when G is semi-simple [29], since a lemma of Goursat [30, 31] implies there are no other non-trivial subgroups isomorphic to G.

of the Higgs as a pNGB is realised assuming a global $Sp(4)$ symmetry in the composite sector, spontaneously broken to $SU(2)_L \times SU(2)_R^{[3]}$. While many features of the flavour-universal framework go through unchanged, two important differences arise. First, the freedom in choosing the value of the $SU(2)_R^{[3]}$ gauge coupling provides a new ingredient to achieve the little hierarchy (*i.e.* a small ratio between the EW scale and the composite one). This is obtained via an accidental cancellation between fermion and gauge contributions in the Higgs potential, which are predicted to have opposite sign. Second, this same cancellation requires the masses of the heavy gauge bosons resulting from deconstruction (in particular the $W_R^\pm$ and Z_R bosons) to be in the few TeV range, well below any of the composite states except for the Higgs and the top partners. As we shall see, this expectation is fully consistent with present data for large enough values of the $SU(2)_R^{[3]}$ coupling, which is also what the tuning in the potential asks for. Merging partial compositeness and flavour deconstruction, we achieve a framework that is more predictive than when considering the two hypotheses separately and is fully compatible with present observations.

This work is not the first attempt to merge the idea of a composite Higgs sector and flavour non-universal gauge interactions. Interesting alternative proposals, based on different non-universal gauge groups and/or different symmetries of the strong sector, have been presented in [21, 23, 35–39]. Our choice, which is dictated by minimality on both fronts (*i.e.* both local and global symmetries), is particularly well-suited to explore analytically the interplay of the two hypotheses in the generation of the Higgs potential. We are indeed able to compute the latter in a simple analytic form, with a minimal set of assumptions about the strong dynamics.

The paper is organised as follows. In Section 2, we introduce the UV gauge group and the field content of the model; we describe the two-step symmetry-breaking pattern and the main features of the construction. Section 3 provides a detailed analysis of the composite sector: we introduce the formalism to describe the pNGB dynamics, the mechanism of partial compositeness, and present a detailed analysis of the different contributions to the Higgs potential. Finally, the phenomenological implications of the model are present in Section 4, with particular focus on the effects generated by the massive gauge bosons resulting from flavour deconstruction, which are the distinctive feature of this setup. A summary of the main findings and an outlook on future directions are presented in Section 5.

2 Definition and main features of the model

In this section we introduce the main ingredients of the model and summarise its main features. We start by introducing the field content and gauge group, and then summarise the chain of symmetry-breaking steps occurring at different energy scales.

2.1 Field content and gauge group

As anticipated, we assume a strong sector that gives rise to the minimal global symmetry breaking pattern to deliver a pNGB Higgs, namely

$$\mathcal{G} \equiv Sp(4) \xrightarrow{\Lambda_{\rm HC}} SU(2)_L \times SU(2)_R^{[3]} \equiv \mathcal{H}\,. \tag{2.1}$$

We assume the entire group $\mathcal{H}$ to be gauged. Here and in the following, the upper index on a gauge group factor (as in $SU(2)_R^{[3]}$) denotes a flavour non-universal gauge symmetry acting only on the given family of chiral SM-like fermions. The spontaneous symmetry breaking (SSB) breaking (2.1) occurs at a scale

$$\Lambda_{\rm HC} \equiv 4\pi f_{\rm HC} \gg v. \tag{2.2}$$

We envisage some strongly coupled gauge sector that triggers this transition dynamically, with some *hypercolour* gauge group $G_{\rm HC}$ (that we do not specify) underlying the global symmetry $\mathcal{G}$ in the UV.[2] Note that, at the Lie algebra level, we have $\mathfrak{sp}(4) \cong \mathfrak{so}(5)$ and $\mathfrak{su}(2) \oplus \mathfrak{su}(2) \cong \mathfrak{so}(4)$, so this is usually referred to as the $SO(5)/SO(4)$ breaking pattern, which is a slight abuse of notation. Interestingly, we do not have the option of imposing an additional semi-direct product between $\mathcal{H}$ and $\mathbb{Z}_2$ (that acts by exchanging the two $SU(2)$ factors [43]), simply because that would be inconsistent with the representation of the scalar link field Σ_R needed for the flavour deconstruction.[3]

Including also the part of the gauge symmetry acting on the light families, the relevant symmetry group above the scale $\Lambda_{\rm HC}$ has the form $G_{\rm HC} \times G_{\rm elem}$, where

$$G_{\rm elem} = SU(3)_c \times SU(2)_L \times SU(2)_R^{[3]} \times U(1)_{B-L}^{[3]} \times U(1)_Y^{[12]}\,. \tag{2.3}$$

The flavour deconstruction of $B-L$ is, as we shall see, motivated by the SM flavour puzzle.

The matter content of the model can be decomposed into two main sectors: the elementary and the composite one. The elementary sector describes fundamental fields which are not charged under $G_{\rm HC}$ and are well defined also above the scale $\Lambda_{\rm HC}$. Its structure is composed by (see Table 1):

- **Light-family chiral fermions:** The first and second generation fermions are charged under $SU(2)_L \times U(1)_Y^{[12]}$: the charges are exactly as in the SM case, but for the replacement of the universal hypercharge with $U(1)_Y^{[12]}$. The $SU(2)_L$ doublets have a mass mixing with appropriate vector-like fermions of the composite sector.

- **Third-family chiral fermions:** These fields are charged under $SU(2)_L \times SU(2)_R^{[3]} \times U(1)_{B-L}^{[3]}$: they consist of two $SU(2)_L$ doublets (q_L and ℓ_L) and two $SU(2)_R^{[3]}$ doublets (q_R and ℓ_R), all equally charged under $U(1)_{B-L}^{[3]}$. They all have a mass mixing with appropriate vector-like fermions of the composite sector.

[2] As is well-known, this minimal $\mathfrak{so}(5) \to \mathfrak{so}(4)$ breaking pattern does not arise straightforwardly from a chiral condensate in a QCD-like hypercolour theory – in contrast to less minimal options such as the $\mathfrak{su}(4) \to \mathfrak{sp}(4)$ breaking pattern that delivers the Higgs plus an extra gauge singlet pNGB [40], which can naturally emerge from an Sp gauge theory. Nonetheless with a little more model-building, the minimal option can be realised from a fundamental gauge theory. For example, by including explicit $\mathfrak{su}(4)$ breaking interactions in the $\mathfrak{su}(4)/\mathfrak{sp}(4)$ model, one can lift the extra singlet and give an effective description with the minimal coset structure, as proposed in [41] (see also [42]).

[3] Such a $\mathbb{Z}_2$ exchange symmetry was considered desirable in the pre-LHC era of composite Higgs model building, to evade the LEP-II $Z \to b\bar{b}$ constraints that are strong when the scale separation $v/f_{\rm HC}$ (and hence the requisite tuning) is small [44], as was then viable. Now that $f_{\rm HC}$ is constrained to be at least the TeV scale by LHC data, the LEP-II constraints from $Z \to b\bar{b}$ do not play as big a role.

Elementary fields		$U(1)^{[3]}_{B-L}$	$U(1)^{[12]}_{Y}$	$SU(2)_L$	$SU(2)^{[3]}_R$
chiral	$q_L^{[12]}$	0	1/6	**2**	**1**
light quarks	$u_R^{[12]}$	0	2/3	**1**	**1**
	$d_R^{[12]}$	0	−1/3	**1**	**1**
chiral	$q_L^{[3]}$	1/6	0	**2**	**1**
3rd gen. quarks	$q_R^{[3]}$	1/6	0	**1**	**2**
vector-like	F_L^q	1/6	0	**2**	**1**
quarks	F_R^q	0	1/6	**1**	**2**
scalar	Σ_R	0	1/2	**1**	**2**
link fields	Ω_q	−1/6	1/6	**1**	**1**
	Ω_ℓ	1/2	−1/2	**1**	**1**

Table 1: Matter content of the elementary sector. For simplicity, among the fermions only the quarks are shown. Note our non-standard choice of normalisation for B and L charges in the $U(1)^{[3]}_{B-L}$ group, which are chosen to coincide with their appearance in the SM hypercharge. The final link field Ω_ℓ is not strictly needed in the model; its presence (motivated perhaps by a desire for quark-lepton unification) means the charged lepton Yukawa parametrically mirrors the quark Yukawa matrices, namely matching (1.1).

- **Third-family vector-like fermions:** For each third-family chiral fermion an elementary vector-like fermion with the same transformation properties under $SU(2)_L \times SU(2)^{[3]}_R$ is introduced. When integrated out, these fields generate the suppressed entries of the Yukawa couplings involving light families, via appropriate higher-dimensional operators.

- **Scalar link fields:** The breaking of the non-universal gauge group to the universal SM group occurs via the scalar fields Σ_R and $\Omega_{q,\ell}$, with transformation properties as shown in Table 1.

The composite sector is well defined only below the scale $\Lambda_{\rm HC}$: it describes bound states of fields charged under the hypercolour gauge group which, by construction, form complete representations of $\mathcal{H}$. Among them, the states playing a key role in the construction are:

- **The Higgs field**, namely the pNGB of the global symmetry breaking $\mathcal{G} \to \mathcal{H}$, that will itself trigger EW symmetry breaking upon acquiring its own vacuum expectation value.

- **The lightest vector resonances**, namely the lightest composite spin-1 states transforming under $\mathcal{H}$ as the corresponding gauge bosons. Their mass, m_ρ, is assumed to be below the scale $\Lambda_{\rm HC}$.

- **The top partners**, namely composite spin-$\frac{1}{2}$ states with mass $M_T = O(1) \times f_{\rm HC}$ that, when integrated out, provide the largest contribution to the effective top-quark Yukawa coupling.

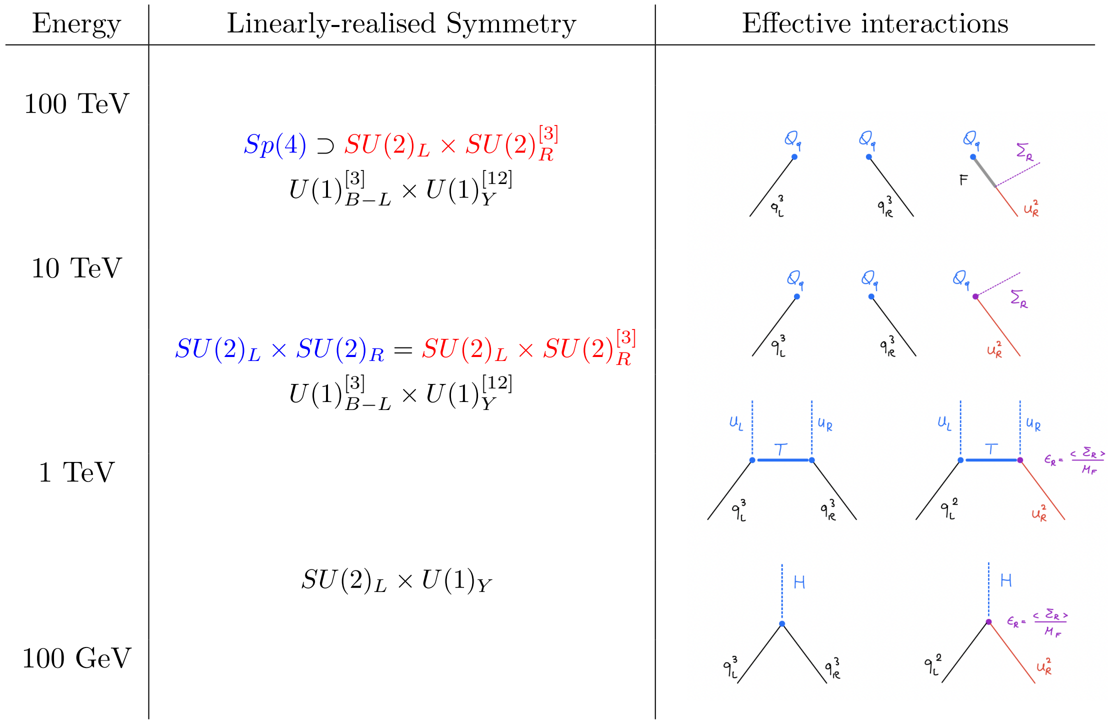

Figure 1: Schematic representation of the symmetry breaking chain. In the second column, the terms in blue denote global symmetries of the strong sector, while the corresponding gauged subgroups are indicated in red. The interaction terms on the right are those contributing to third- and second-family quark Yukawa couplings ($u_{L,R}$ denote Goldstone boson fields, see §3).

2.2 Symmetry-breaking pattern

The breaking to the flavour universal electroweak gauge group is realised by the VEVs of the elementary scalar link fields Σ_R and $\Omega_{q,\ell}$:

$$SU(2)_R^{[3]} \times U(1)_{T_R^3}^{[12]} \xrightarrow{\langle \Sigma_R \rangle} U(1)_{T_R^{[3]}} \tag{2.4}$$

$$U(1)_{B-L}^{[3]} \times U(1)_{B-L}^{[12]} \xrightarrow{\langle \Omega_{q,\ell} \rangle} U(1)_{B-L} \,. \tag{2.5}$$

Recall that (third-family) hypercharge is given by $Y^{[3]} = T_{R^3}^{[3]} + (B-L)^{[3]}$. This symmetry-breaking pattern leads to 4 massive gauge bosons: two Z', associated to the neutral $B-L$ and T_R^3 generators, and a pair of $W_R^{\pm}$. The masses and couplings of these heavy gauge bosons will be given in §4 when we discuss phenomenology.

In parallel to the symmetry breaking induced by the link fields in Eqs. (2.4)–(2.5), the global symmetry breaking $\mathcal{G} \to \mathcal{H}$ occurring in the hypercolor sector provides masses to all the composite states but for the pNGB (*i.e.* the SM-like Higgs). As we shall see, the massive Z' and the $W_R^{\pm}$ are predicted to be lighter than all the massive composite states.

The symmetry breaking chain and the degrees of freedom relevant at different energy scales, illustrated in Fig. 1, can be summarised in four main steps as listed below.

I. The flavour structure is seeded at high energies $\gtrsim$ 100 TeV or so) by interactions involving the elementary chiral fermions and appropriate fermionic operators in the strong sector, denoted $\mathcal{O}_f$ ($f = q, \ell$). We remain agnostic on the latter except for their transformation properties under $G_{\rm elem}$. The non-inversal gauge symmetry dictates that only third-generation chiral and VL fermions have a non-suppressed linear mixing with the $\mathcal{O}_f$. A key observation at this stage is that appropriate products of VL fermions and link fields have the same gauge transformation properties as the light chiral fermions. In particular, in the quark sector we have

$$F_R^q \times \Sigma_R \sim u_R^{[12]} , \qquad F_R^q \times \Sigma_R^c \sim d_R^{[12]} , \qquad F_L^q \times \Omega_q \sim q_L^{[12]} . \tag{2.6}$$

where $\Sigma_R^c = i\sigma_2 \Sigma_R^*$. This allow us to generate a non-local coupling of the light chiral fermions to the $\mathcal{O}_f$ via the exchange of VL fermions (see Fig. 1).

II. Below about 100 TeV, and above the confinement scale $\Lambda_{\rm HC}$, the elementary VL fermions can be integrated out. This leads to the appearence of higher-dimensional operators involving the light chiral fermions and the strong sector. When, at lower scales, the link fields acquire a VEV, these higher-dimensional operators result in the effective suppression factors

$$\epsilon_R = \frac{v_R}{M_F} \equiv \frac{\sqrt{2}|\langle \Sigma_R \rangle|}{M_F} , \qquad \epsilon_L = \frac{v_\Omega}{M_F} \equiv \frac{\sqrt{2}\langle \Omega_q \rangle}{M_F} . \tag{2.7}$$

Setting $\epsilon_R = O(m_c/m_t) = O(10^{-2})$ and $\epsilon_L = O(|V_{cb}|) = O(10^{-1})$ we obtain the ingredients to achieve, naturally, the hierarchical structure of the Yukawa couplings.

III. Below $\Lambda_{\rm HC} = O(10)$ TeV and above $f_{\rm HC} = O(1)$ TeV the strong sector leads to the global $\mathcal{G} \to \mathcal{H}$ breaking that deliver the SM-like Higgs filed as pNGB. At the same time, the two point function $\langle 0|T\{\bar{\mathcal{O}}_f(x)\mathcal{O}_f(0)\}|0\rangle$ is assumed to be dominated (at low momentum transfer) by the exchange of a few light spin-1/2 composite states. For the operators coupled to third-generation quarks these composite states are the so-called top partners.

IV. Around and below the scale $f_{\rm HC}$, having integrated out the top partners and frozen the link fields to their VEVs, we end up with an effective SM-like Yukawa interaction and an effective Higgs potential. The latter is radiatively generated by couplings/dynamics that explicit break $\mathcal{G}$, namely: i) the mass-mixing in the fermion sector, ii) the gauging of $\mathcal{H}$, iii) the VEV of Σ_R.

The first two steps related to the flavour hierarchies are conceptually very similar to what was discussed in refs. [2, 6], in the context of an elementary Higgs sector. The main difference is that the couplings of the fermions to the Higgs are replaced by couplings to the strong sector. This has interesting implications for the Higgs hierarchy problem: since above $\Lambda_{\rm HC}$ there is no Higgs field, the heavy elementary VL fermions do not destabilise the Higgs mass term. In other words, the merging of composite dynamics with flavour deconstruction removes any quadratic sensitivity of m_H^2 to the highest mass scale ($M_F \sim$

100 TeV) in this theory of flavour (as might otherwise be naïvely estimated in a theory with a fundamental Higgs, à la 'finite naturalness' [45], by computing loops involving the heavy fermion as done in [2]).

In the next section we discuss in detail the steps III and IV, which are specific to the composite Higgs model. As we shall see, the explicit construction of the Higgs potential, and the requirement of a light Higgs mass, forces us to choose $\langle \Sigma_R \rangle = O(f_{\rm HC}) \ll \Lambda_{\rm HC}$. This, in turn, implies that the lightest exotic states of this setup are the massive Z' and $W_R^\pm$ bosons, plus the top partners, following from the SSB in Eqs. (2.4)–(2.5). Phenomenological constraints and implications of these fields are discussed in §4.

3 Composite dynamics

3.1 Notation and conventions for the Goldstone boson fields

In our convention, $Sp(4) \subset SU(4)$ is the 10-dimensional group of 4×4 special unitary matrices $\{U\}$ that moreover satisfy $U^T \Omega U = \Omega$, where the symplectic form Ω, in our basis, is given by the following antisymmetric matrix:

$$\Omega = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix} .$$

$Sp(4)$ Algebra and basis

The Lie algebra $\mathfrak{sp}(4)$ and its representations are probably familiar to most readers, thanks to the Lie algebra isomorphism $\mathfrak{sp}(4) \cong \mathfrak{so}(5)$. The corresponding Lie group isomorphism is $Sp(4) \cong \mathrm{Spin}(5)$, where Spin(5) is the double cover of $SO(5)$ that admits spinor representations. We choose a particular basis for the fundamental 4-dimensional representation of the $Sp(4)$ group, where the 10 generators take the form

$$T_L^a = \frac{1}{2}\begin{pmatrix} \sigma^a & 0 \\ 0 & 0 \end{pmatrix} , \qquad T_R^a = \frac{1}{2}\begin{pmatrix} 0 & 0 \\ 0 & \sigma^a \end{pmatrix} , \qquad T_X^a = \frac{1}{2\sqrt{2}}\begin{pmatrix} 0 & \bar{\sigma}^a \\ \bar{\sigma}^{a\dagger} & 0 \end{pmatrix} . \tag{3.1}$$

Here σ^a denote the Pauli matrices and $\bar{\sigma}^a = \{i\sigma^a, \mathbb{1}_2\}$, with $\mathbb{1}_2$ being the 2×2 identity matrix.[4] The $T_{L,R}$ correspond to the $\mathfrak{su}(2)_{L,R}$ sub-algebras of $\mathfrak{sp}(4)$ that we assume to be unbroken, while the T_X^a are the broken generators. The generators are normalised such that $\mathrm{Tr}(T^a T^b) = \frac{1}{2}\delta^{ab}$.

The Goldstone boson matrix

We define the coset element as

$$U(x) \equiv \exp\left(i\sqrt{2}\phi_a(x) T_X^a / F\right) , \tag{3.2}$$

[4] Lest there is confusion, we use the symbol 'a' to denote a generic Lie algebra index (usually summed on); *e.g.* in the case of T_L^a, a runs from 1 to 3, while for the broken generators T_X^a, a runs from 1 to 4.

where $\phi_a(x)$ are the Goldstone bosons and T_X^a the broken generators as given above. The $U(x)$ field transforms under $Sp(4)$ as

$$U(x) \to \hat{g} \cdot U(x) \cdot \hat{h}[\phi_a; g]^{-1}\,, \tag{3.3}$$

where $\hat{g}$ is an element of the $Sp(4)$ group and $\hat{h}[\phi_a; g]$ is an element of the unbroken subgroup $SU(2)_L \times SU(2)_R^{[3]}$. Choosing the representation (3.1) for the $Sp(4)$ generators leads to

$$U[\phi] = \begin{pmatrix} \cos(\frac{h}{2F})\,\mathbb{1}_2 & i\sin(\frac{h}{2F})\frac{\Phi}{|\Phi|} \\ i\sin(\frac{h}{2F})\frac{\Phi^\dagger}{|\Phi|} & \cos(\frac{h}{2F})\,\mathbb{1}_2 \end{pmatrix} \equiv \begin{pmatrix} \cos(\frac{h}{2F})\,\mathbb{1}_2 & i\sin(\frac{h}{2F})\frac{\Phi}{|\Phi|} \\ i\sin(\frac{h}{2F})\frac{\Phi^\dagger}{|\Phi|} & \cos(\frac{h}{2F})\,\mathbb{1}_2 \end{pmatrix} \tag{3.4}$$

where we have defined the 2×2 matrix

$$\Phi \equiv \begin{pmatrix} i\phi_3 + \phi_4 & i\phi_1 + \phi_2 \\ i\phi_1 - \phi_2 & \phi_4 - i\phi_3 \end{pmatrix} \tag{3.5}$$

and the scalar field

$$h(x) = |\Phi| \equiv \sqrt{\mathrm{Det}(\Phi)} = \sqrt{\phi_1(x)^2 + \phi_2(x)^2 + \phi_3(x)^2 + \phi_4(x)^2}\,. \tag{3.6}$$

Given the factorised structure of the unbroken group, it is convenient to introduce two orthogonal Goldstone bosons *vectors*, denoted $u_{L,R}[\phi]$ and defined as

$$u_L[\phi] \equiv U \cdot \mathbb{P}_L = \begin{pmatrix} \cos(\frac{h}{2F})\,\mathbb{1}_2 \\ i\sin(\frac{h}{2F})\frac{\Phi^\dagger}{|\Phi|} \end{pmatrix}\,, \qquad u_R[\phi] \equiv U \cdot \mathbb{P}_R = \begin{pmatrix} i\sin(\frac{h}{2F})\frac{\Phi}{|\Phi|} \\ \cos(\frac{h}{2F})\,\mathbb{1}_2 \end{pmatrix}\,, \tag{3.7}$$

where the 4×2 left and right projectors are $\mathbb{P}_L \equiv \begin{pmatrix} \mathbb{1}_2 & 0 \end{pmatrix}^T$ and $\mathbb{P}_R \equiv \begin{pmatrix} 0 & \mathbb{1}_2 \end{pmatrix}^T$. Using these vectors the full $U[\phi]$ matrix can be written as $U[\phi] = (u_L[\phi], u_R[\phi])$. The transformation properties of $u_{L,R}[\phi]$ under $Sp(4)$ assume the following convenient form

$$u_L \to \hat{g} u_L \hat{h}_L^\dagger\,, \qquad u_R \to \hat{g} u_R \hat{h}_R^\dagger\,, \tag{3.8}$$

where, for simplicity, we have omitted to indicate the dependence of the $\hat{h}_{L/R}$ elements on the Goldstone bosons. Note also that

$$u_L^\dagger u_R = u_R^\dagger u_L = 0\,, \qquad u_R^\dagger u_R = u_L^\dagger u_L = \mathbb{1}_2\,. \tag{3.9}$$

3.1.1 Lowest-order Goldstone boson Lagrangian

Using these definitions, the canonically normalised Lagrangian at leading order in the derivative expansion, including external gauge fields of $SU(2)_L \times SU(2)_R^{[3]}$, can be written as

$$\mathcal{L}_U^{(2)} = \frac{F^2}{2}\,\mathrm{Tr}_{[4]}\left[(\hat{D}^\mu U)^\dagger \hat{D}_\mu U\right]\,, \tag{3.10}$$

where

$$\hat{D}_\mu U = \partial_\mu U - iU\hat{\Gamma}_\mu - ig_L\left[\hat{A}_\mu^L, U\right] - ig_R\left[\hat{A}_\mu^R, U\right]\,. \tag{3.11}$$

Following the formalism of Callan, Coleman, Wess, and Zumino (CCWZ) [46], we define $\hat{\Gamma}_\mu$ from the decomposition of $U^\dagger \partial_\mu U$ in terms of broken and unbroken generators,

$$U^\dagger \partial_\mu U = i(\Gamma^a_\mu T^a_L + \Gamma^a_\mu T^a_R) + i u^a_\mu T^a_X \equiv i\hat{\Gamma}_\mu + i\hat{u}_\mu, \tag{3.12}$$

such that under *global* $Sp(4)$ transformations

$$\begin{aligned} \hat{\Gamma}_\mu &\to \hat{h}[\phi_a; g]\Gamma_\mu \hat{h}[\phi_a; g]^\dagger - i\hat{h}[\phi_a; g]\,\partial_\mu \hat{h}^\dagger[\phi_a; g]\,, \\ \hat{u}_\mu &\to \hat{h}[\phi_a; g]\hat{u}_\mu \hat{h}[\phi_a; g]^\dagger\,. \end{aligned} \tag{3.13}$$

On the other hand, $\hat{A}^{L,R}_\mu$ and $g_{L(R)}$ denote gauge fields and related couplings ensuring *local* invariance under $SU(2)_L \times SU(2)^{[3]}_R$. Note that $\hat{\Gamma}_\mu$ is a function of the Goldstone bosons, but its expansion in powers of ϕ_a starts at second order, hence it plays no role in the expansion of $\mathcal{L}^{(2)}_U$ up to quadratic terms in H (see appendix A for more details). As we shall see, the constant F appearing in the normalisation of $\mathcal{L}^{(2)}_U$ and in the expansion of $U[\phi]$ in (3.4) is related to the order parameter of the strong sector $f_{\rm HC}$, defined as in [8], via $F = f_{\rm HC}/2$.

From (3.10), expanding up to second order in Φ, which is equivalent to considering the $F \to \infty$ limit, we can derive the dimension-4 part of the effective Lagrangian describing the dynamics of the field Φ, which transforms as a $(\mathbf{2}, \mathbf{2})$ under $SU(2)_L \times SU(2)^{[3]}_R$:

$$\mathcal{L}^{(2)}_H = \frac{1}{4}\,\mathrm{Tr}_{[2]}\,(D^\mu \Phi)^\dagger\,(D_\mu \Phi)\ . \tag{3.14}$$

In this case the covariant derivative reads

$$D_\mu \Phi = \partial_\mu \Phi - i g_L \hat{A}^L_\mu \Phi + i g_R \Phi \hat{A}^R_\mu \qquad \hat{A}^{L(R)}_\mu = \frac{1}{2}\sigma^a A^a_{L(R),\mu}\ . \tag{3.15}$$

At this point it is clear that in the limit $F \to \infty$ we can identify Φ with the SM Higgs field (in a bi-doublet notation), and $h(x)$ with its radial component:

$$\Phi(x)|_{\rm unit.-gauge} = h(x)\,\mathbb{1}_2\,, \tag{3.16}$$

such that the vacuum expectation value of Φ reads

$$\langle \Phi \rangle = \langle h \rangle\,\mathbb{1}_2 \equiv v\,\mathbb{1}_2\,. \tag{3.17}$$

Considering only $\mathcal{L}^{(2)}_H$ in (3.14), the value of v thus defined yields $m^2_W = g^2_L v^2/4$.

Casimirs of $SU(2)_L \times SU(2)^{[3]}_R$ and $SU(2)^{[3]}_R$ to encode explicit breaking

It is convenient to introduce the following quadratic Casimir elements of the unbroken sub-algebra $\mathfrak{su}(2)_L \oplus \mathfrak{su}(2)^{[3]}_R$, written in the fundamental representation of $\mathfrak{sp}(4)$:

$$\Delta_R \equiv \Delta = \begin{pmatrix} 0 & 0 \\ 0 & \mathbb{1}_2 \end{pmatrix}, \qquad \Delta_L \equiv \mathbb{1}_4 - \Delta = \begin{pmatrix} \mathbb{1}_2 & 0 \\ 0 & 0 \end{pmatrix}.$$

Any linear combination of these two matrices (excluding $\Delta_R + \Delta_L = \mathbb{1}_4$) commutes with the unbroken generators, and does not commute with the broken ones:

$$\left[\Delta_{L,R}, T^a_{L,R}\right] = 0 \qquad [\Delta_{L,R}, T^a_X] \neq 0\,. \tag{3.18}$$

In building effective operators, we can treat $\Delta_{L,R}$ as *spurions* of $Sp(4)$, in the sense that these matrices can be used to encode the effects of explicit $Sp(4)$ breaking present in our model.

We remark that $\Delta_{L,R}$ do *not* correspond strictly to the VEVs of some (even spurious) fundamental field transforming in a linear representation of $\mathcal{G}$, but can (in certain cases, such as when Δ appear below to encode the effects of gauging) arise from integrating out more complicated combinations of UV fields. In any case, identifying $\Delta_{L/R}$ as quadratic Casimir elements in the universal enveloping algebra, we can take them formally to transform under $\mathcal{G}$ as

$$\Delta_{(L,R)} \to \hat{g}\Delta_{(L,R)}\hat{g}^{-1}\,. \tag{3.19}$$

This allows us to systematically build operators that break $Sp(4)$ but are invariant under $SU(2)_L \times SU(2)_R^{[3]}$, keeping track of the origin of this explicit symmetry breaking. Since $\Delta_L + \Delta_R = \mathbb{1}_4$, the two spurions are not independent and we can limit ourselves to introduce a single one that we choose to be $\Delta \equiv \Delta_R$.

Since we are interested in describing also the explicit breaking $SU(2)_R^{[3]} \to U(1)_{T_R^3}^{[3]}$, we further introduce the spurion

$$\Delta_\Sigma = \begin{pmatrix} 0 & 0 \\ 0 & \sigma_3 \end{pmatrix}\,, \tag{3.20}$$

with formal $Sp(4)$ transformation properties as in (3.19). By construction, Δ_Σ commutes only with the generators of $SU(2)_L \times U(1)_R^{[3]}$. Operators built in terms of Δ_Σ in the composite sector effectively describe the result of the non-vanishing expectation value of Σ_R in (2.4), after integrating out heavy fields that couple to Σ_R.

3.2 Coupling to fermions

3.2.1 Third-family partial compositeness

As anticipated, we assume that the coupling of the (elementary) third-generation fermions to the composite Higgs is the result of a linear mixing between the elementary fermions and appropriate fermionic operators of the strong sector. For simplicity, we illustrate this mechanism in the quark sector, denoting generically with $\mathcal{O}_q$ the corresponding fermionic operators. To build invariant terms, it is more convenient to use u_L and u_R rather than the U field. The most general linear mixing is described by

$$\mathcal{L} \supset F\left[\bar{q}_L^{[3]} u_L^\dagger \left(\lambda_L^q \mathbb{1} + \tilde{\lambda}_L^q \Delta\right)\mathcal{O}_q + \bar{q}_R^{[3]} u_R^\dagger \left(\lambda_R^q \mathbb{1} + \tilde{\lambda}_R^q \Delta\right)\mathcal{O}_q\right] + \text{h.c.}, \tag{3.21}$$

where $\lambda^q_{L(R)}$ and $\tilde{\lambda}^q_{L(R)}$ are arbitrary complex couplings. The terms proportional to $\lambda^q_{L(R)}$ preserve $Sp(4)$-invariance, while those proportional to $\tilde{\lambda}^q_{L(R)}$ do not: the spurion matrices Δ appear in this context to act as projectors onto relevant fermionic degrees of freedom.

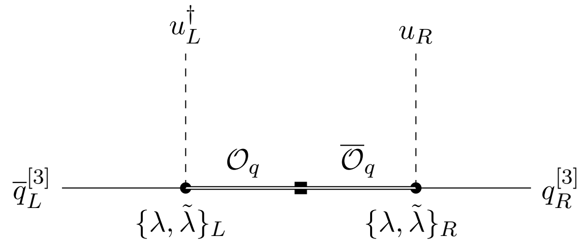

Figure 2: Diagram describing the mechanism responsible for third-generation Yukawa couplings.

The explicit breaking introduced by these latter terms is essential to generate a SM-like effective Yukawa interaction. To ease the notation, in the following we omit the flavour and gauge indices on the elementary fields.

Integrating out the heavy composite states as shown schematically in Fig. 2, and assuming that the spectrum of the composite states is characterized by a single mass scale M_q, the effective Yukawa interaction among elementary fields assumes the form

$$\mathcal{L}_Y^{\rm eff} = \lambda_L^q \lambda_R^{q*} \kappa_{LR}^q \frac{F^2}{M_q}\, \overline{q}_L u_L^\dagger \Delta u_R q_R = \frac{y_q}{\sqrt{2}}\, \overline{q}_L H q_R + O(H^2)\,, \tag{3.22}$$

where

$$|y_q| = \lambda_L^q \lambda_R^{q*} \kappa_{LR}^q \frac{F}{\sqrt{2} M_q}\,, \qquad \kappa_{LR}^q = \left(1 + \tilde{\lambda}_L^q/\lambda_L^q\right)\left(1 + \tilde{\lambda}_R^{q*}/\lambda_R^{q*}\right) - 1\,. \tag{3.23}$$

As can be seen, y_q is non-vanishing if both q_L and q_R mix with the composite states, and at least one of the two mixing terms is not $Sp(4)$ invariant.[5]

The effective Yukawa interaction in Eq. (3.22) is $SU(2)_R^{[3]}$ invariant. The breaking of custodial symmetry in the Yukawa sector, which is necessary to describe the top-bottom mass splitting, can be achieved adding terms proportional to Δ_Σ in Eq. (3.21). The limiting case where only the top quark Yukawa coupling is non-zero can be obtained, in particular, via the replacement

$$\Delta \to \Delta_+ = \frac{1}{2}(\Delta + \Delta_\Sigma) \equiv \begin{pmatrix} 0 & 0 \\ 0 & \sigma_+ \end{pmatrix}\,. \tag{3.24}$$

3.2.2 Fermion contribution to the Higgs potential

In the absence of explicit $Sp(4)$ breaking, the Higgs field would be an exact Goldstone boson and therefore massless. The explicit breaking of $Sp(4)$ induced by the couplings of the elementary fermions to the composite sector, together with the gauging of only a subgroup of $Sp(4)$ (soon to be discussed), transform the Higgs field into a pseudo Goldstone boson with non-vanishing Higgs potential. In this section we evaluate the one-loop contribution to the potential obtained by integrating out the elementary fermions.

[5] In principle, we could also introduce $Sp(4)$-breaking terms in the mass-matrix of the composite states. However, this is a subleading effect with respect to the breaking present in the linear couplings of the elementary fields to the composite sector, hence we neglect it in the following.

Following the Coleman–Weinberg approach, the starting point is to identify the effective vertices with two fermion fields and arbitrary powers of the Higgs field at zero momentum transfer. We parameterise these two-point functions via appropriate form factors which depends only on the momentum of the fermion fields. Taking into account only the spurion terms necessary to generate the top-quark Yukawa coupling, we define

$$\mathcal{L}_{\rm eff} \supset \overline{q}_L \not{p} \left[\Pi_0^{q_L}(p^2)\mathbb{1} + \Pi_1^{t_L}(p^2) u_L^\dagger \Delta_+ u_L\right] q_L + \overline{q}_R \not{p} \left[\Pi_0^{q_R}(p^2)\mathbb{1} + \Pi_1^{t_R}(p^2) u_R^\dagger \Delta_+ u_R\right] q_R + \left\{ \overline{q}_L \left[\mathcal{M}_t(p^2) u_L^\dagger \Delta_+ u_R\right] q_R + \text{h.c.}\right\} . \quad (3.25)$$

Omitting to indicate the momentum dependence of the form factors, and evaluating the explicit dependence from the Higgs field in the unitary gauge, leads to

$$\mathcal{L}_{\rm eff} \supset \bar{t}_L \not{p} \left[\Pi_0^{q_L} + \Pi_1^{t_L} \sin^2\left(\frac{h}{2F}\right)\right] t_L + \bar{t}_R \not{p} \left[\Pi_0^{t_R} - \Pi_1^{t_R} \sin^2\left(\frac{h}{2F}\right)\right] t_R + \left\{ \bar{t}_L \left[\mathcal{M}_t \sin\left(\frac{h}{2F}\right)\cos\left(\frac{h}{2F}\right)\right] t_R + \text{h.c.}\right\} , \quad (3.26)$$

where $\Pi_0^{t_R} = \Pi_0^{q_R} + \Pi_1^{t_R}$. Note that we moved from a doublet notation for the fermions in (3.25) to an explicit indication of the individual components (t_L and t_R) coupled to the Higgs field in the unitary gauge.

The value of the form factors at $p^2 = 0$ can be either fixed by the normalization of the fields or expressed in terms of parameters controlling the elementary-composite fermion mixing. In particular, via a field redefinition we can set

$$\Pi_0^{q_L}(0) = \Pi_0^{t_R}(0) = 1 , \quad (3.27)$$

while the matching with (3.21) leads to

$$\Pi_1^{t_L}(0) = \frac{F^2}{M_T^2}\left[2\lambda_L^t \tilde{\lambda}_L^t + (\tilde{\lambda}_L^t)^2\right] \equiv \frac{F^2}{M_T^2}(\lambda_L^t)^2 \kappa_L^t , \qquad \Pi_1^{t_R}(0) \equiv \frac{F^2}{M_T^2}(\lambda_R^t)^2 \kappa_R^t , \quad (3.28)$$

and

$$|\mathcal{M}_t(0)| = \frac{F^2}{M_T}\lambda_L^t \lambda_R^t \kappa_{LR}^t \equiv y_t \sqrt{2} F . \quad (3.29)$$

Here we have denoted with M_T the effective mass of the composite state(s) mixing with the top quark (the so-called 'top partners'), while y_t denotes the SM-like Yukawa coupling ($y_t = \sqrt{2} m_t / v \approx 1$).

To compute the one-loop potential, we need to resum the series of one-loop diagrams involving arbitrary number of insertions of form factors, as indicated schematically in Fig. 3. The result thus obtained is

$$\Delta V(h)_f = -2N_c \int \frac{d^4 p_E}{(2\pi)^4} \left\{ \log\left[1 + \frac{\Pi_1^{t_L}}{\Pi_0^{q_L}} \sin^2\left(\frac{h}{2F}\right)\right] + \log\left[1 - \frac{\Pi_1^{t_R}}{\Pi_0^{t_R}} \sin^2\left(\frac{h}{2F}\right)\right] + \log\left[1 + \frac{|\mathcal{M}_t|^2 \sin^2\left(\frac{h}{2F}\right)\cos^2\left(\frac{h}{2F}\right)}{p_E^2 \left(\Pi_0^{q_L} + \Pi_1^{q_L}\sin^2\left(\frac{h}{2F}\right)\right)\left(\Pi_0^{q_R} - \Pi_1^{q_R}\sin^2\left(\frac{h}{2F}\right)\right)}\right]\right\} , \quad (3.30)$$

where the pre-factor is determined by the number of colors and the two spin degrees of freedom of the fermion fields. The explicit evaluation of the integral, which requires one to choose a functional dependence for the form factors at $p^2 \neq 0$, is discussed in §3.4.

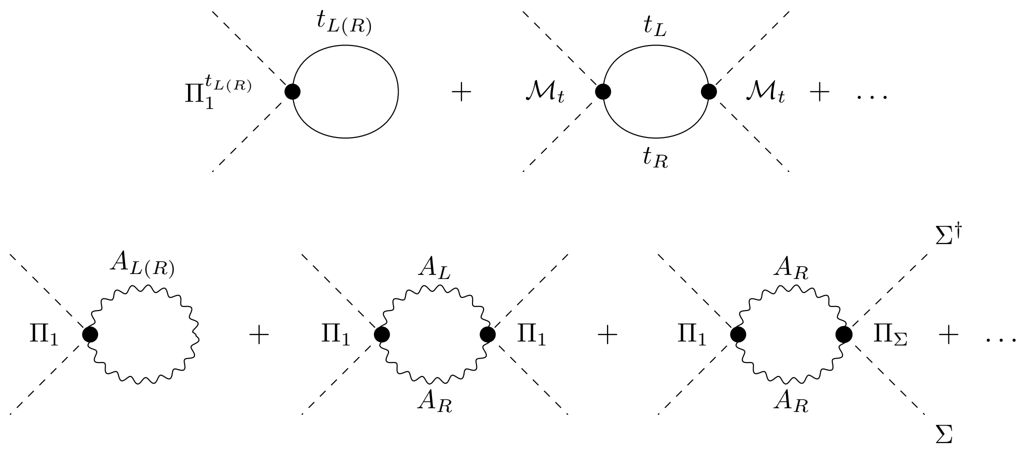

Figure 3: Schematic illustration of the one-loop contributions to the Higgs potential, coming from the explicit $Sp(4)$ breaking required in both the fermion (3.2.2) and gauge (3.3.2) sectors.

3.3 Coupling to gauge bosons

3.3.1 Chiral structure

In this subsection, the flavour deconstruction of the electroweak gauge group will play a crucial role when we calculate the impact of gauging on the pNGB Higgs potential.

As for the fermion sector, in order to compute the contribution of the $SU(2)_L \times SU(2)_R^{[3]}$ gauge fields to the Higgs potential we need to determine the two-point function of two gauge fields and an arbitrary number of Higgs fields. The mass term can be derived directly from the chiral Lagrangian, where the symmetry breaking pattern $Sp(4) \to SU(2)_L \times SU(2)_R^{[3]}$ implies that only one specific combination of A_μ^L and A_μ^R appears. This can be easily seen by expanding the covariant derivative acting on U from (3.10). In the unitary gauge, the leading and universal term quadratic in A_μ^L and A_μ^R is:

$$\mathcal{L}_U^{(2)}\Big|_{\text{unit.-gauge},\ A^2} = \frac{F^2}{2}\left(g_L A^a_{L,\mu} - g_R A^a_{R,\mu}\right)^2 \sin^2\left(\frac{h}{2F}\right) . \tag{3.31}$$

This term accounts for the SM relation between the W-boson mass and the Higgs VEV by considering the limit $h = v$.

In principle, additional terms can be constructed by considering non-minimal operators in the chiral Lagrangian, obtained with appropriate insertions of the Δ matrices. In this context (namely, of encoding the effects of gauging a non-trivial subgroup), the Δ matrices can be thought of as arising from tracing over a quadratic form built from a fundamental spurion field that transforms in the adjoint representation of $\mathcal{G} = Sp(4)$, with non-vanishing components only in directions that pick out the unbroken $\mathcal{H}$ generators. For example, the non-minimal term

$$\mathcal{L}_U^{(2')} = \delta_\pi F^2\ \mathrm{Tr}_{[4]}\left[\Delta(\hat{D}_\mu U^\dagger U)\Delta(U^\dagger \hat{D}^\mu U)\right] \tag{3.32}$$

leads to

$$\mathcal{L}_U^{(2')}\Big|_{\text{unit.-gauge},\ A^2} = \delta_\pi \frac{F^2}{2}\left(g_L A^a_{L,\mu} - g_R A^a_{R,\mu}\right)^2 \sin^4\left(\frac{h}{2F}\right) . \tag{3.33}$$

Again, the same relative sign between $SU(2)_L$ and $SU(2)_R^{[3]}$ gauge bosons appears, but now with a different trigonometric dependence on $h/2F$. The required insertion of the spurions in $\mathcal{L}_U^{(2')}$ implies this interaction term is suppressed compared to $\mathcal{L}_U^{(2)}$ (*i.e.* we expect $|\delta_\pi| \ll 1$). Hence higher powers of trigonometric functions in the two-point function of the gauge fields are expected to be suppressed, even though both terms (3.31) and (3.33) are formally the same order (*i.e.* quadratic) in the gauge couplings $g_{L/R}$ which here act as the spurion couplings.[6]

3.3.2 Gauge boson contribution to the Higgs potential

Taking into account also the kinetic terms of A_μ^L and A_μ^R, the complete two-point function written in terms of form factors for arbitrary momenta of the gauge fields assumes the form

$$\begin{aligned}\mathcal{L}_{\text{eff}}^{(A^2)} = \frac{1}{2}(P_T)^{\mu\nu}\Big\{&\Pi_0\left(q^2\right)\left[A_{L,\mu}^a A_{L,\nu}^a + A_{R,\mu}^a A_{R,\nu}^a\right] \\ +&\left[\Pi_1\left(q^2\right)\sin^2\left(\frac{h}{2F}\right) + \Pi_2\left(q^2\right)\sin^4\left(\frac{h}{2F}\right)\right]\left(g_L A_{L,\mu}^a - g_R A_{R,\mu}^a\right)\left(g_L A_{L,\nu}^a - g_R A_{R,\nu}^a\right)\Big\}\,,\end{aligned} \tag{3.34}$$

where the transverse projector is $(P_T)^{\mu\nu} \equiv \eta_{\mu\nu} - q_\mu q_\nu/q^2$. Matching to the usual Yang-Mills Lagrangian, we identify $\Pi_0(q^2) = -q^2$, up to (irrelevant) higher order terms in powers q^2. From (3.31) and (3.33), we deduce the normalization conditions for Π_1 and Π_2 in the low momentum limit:

$$\Pi_1(0) = F^2\,, \qquad \Pi_2(0) = \delta_\pi F^2\,. \tag{3.35}$$

In the case of the right-handed fields an additional contribution to the two-point function is generated by the kinetic Lagrangian of Σ_R: when this link field acquires a VEV, a mass term for the $A_{R,\mu}^a$ field is generated. In the limit where we neglect the $U(1)_Y^{[12]}$ gauge coupling (see §4.2), this effect is described by

$$\Delta\mathcal{L}_{\text{eff}}^{(A_R^2)} = \frac{1}{4}(P_T)^{\mu\nu} g_R^2 v_\Sigma^2\, A_{R,\mu}^a A_{R,\nu}^a\,. \tag{3.36}$$

This term does contain any coupling to the Higgs field; however, it affects the calculation of the Higgs potential since it modifies the propagator of the right-handed gauge fields. In analogy with (3.34), it turns out to be convenient to define $\Pi_\Sigma = v_\Sigma^2/2$.

Similarly to the fermion sector, to compute the Coleman–Weinberg potential we need to sum all the one-loop bubbles of gauge fields with arbitrary insertions of the form factors (see Fig. 3). The contribution to the potential thus obtained is:

$$\begin{aligned}\Delta V(h)_A = +\frac{9}{2}\int dq_E^4\ &\log\left(1 + g_L^2\frac{\Pi_1\left(q_E^2\right)\sin^2\left(\frac{h}{2F}\right) + \Pi_2\left(q_E^2\right)\sin^4\left(\frac{h}{2F}\right)}{\Pi_0^L\left(q_E^2\right)}\right) \\ &+\log\left(1 + g_R^2\frac{\Pi_1\left(q_E^2\right)\sin^2\left(\frac{h}{2F}\right) + \Pi_2\left(q_E^2\right)\sin^4\left(\frac{h}{2F}\right) + \Pi_\Sigma}{\Pi_0^R\left(q_E^2\right)}\right)\end{aligned} \tag{3.37}$$

[6] The fact that there are two independent functions $\sin^2(h/2F)$ and $\sin^4(h/2F)$ that appear in the Lagrangian at leading (quadratic) order in the gauge couplings corresponds to the group-theoretic fact that there are two independent quadratic Casimir elements in the unbroken subgroup. This counting, as described in Ref. [47], corresponds to the pair of real irreps $(\mathbf{3},\mathbf{1})$ and $(\mathbf{1},\mathbf{3})$ of $\mathcal{H} = SU(2)_L \times SU(2)_R^{[3]}$ that appear in the decomposition of the adjoint of $Sp(4)$ to $\mathcal{H}$ (after discarding the adjoint of $\mathcal{H}$ itself).

$$+\log\left(1+g_Lg_R\frac{4\left(\Pi_1\left(q_E^2\right)\sin^2\left(\frac{h}{2F}\right)+\Pi_2\left(q_E^2\right)\sin^4\left(\frac{h}{2F}\right)\right)^2}{\tilde{\Pi}_0^2\left(h,q_E^2\right)}\right),$$

where

$$\tilde{\Pi}_0\left(h,q^2\right)\equiv\Pi_0\left(q^2\right)+\Pi_1\left(q^2\right)\sin^2\left(\frac{h}{2F}\right)+\Pi_2\left(q^2\right)\sin^4\left(\frac{h}{2F}\right). \tag{3.38}$$

The numerical pre-factor of $\frac{9}{2}$ originates from the fact that there are three Lorentz polarizations for the vector field, that there are three $SU(2)_{L,R}$ degrees of freedom, and there is a relative factor of $\frac{1}{2}$ with respect to the fermion contribution due to the fact that the gauge bosons are real and not complex.

3.4 The Higgs potential

We can now compute explicitly the radiatively induced Higgs potential, combining both the fermion and gauge contributions that we have just discussed.

In both cases, the UV behaviour of the relevant form factors should be modelled to ensure a convergence of the corresponding integrals, such that the the logarithms in (3.30) and (3.37) are well approximated by their expansion to the first or second order (according to the parametric dependence on the couplings). This implies a general decomposition of the type

$$V(h)=\Delta V_f(h)+\Delta V_A(h)\approx c_0-c_1\sin^2\left(\frac{h}{2F}\right)+c_2\sin^4\left(\frac{h}{2F}\right). \tag{3.39}$$

Higher-power trigonometric functions, such as $\sin^6(h/2F)$, can be safely neglected.[7] Before proceeding with the calculation of the fermion and gauge contributions to $c_{1,2}$, note that naïve dimensional analysis implies $c_{1,2}=O(1)\times F^4$, while the physical conditions to be imposed in order to recover the SM potential at leading order are

$$\left.\frac{c_1}{F^4}\right|_{\text{phys.}}=\frac{m_h^2}{F^2}\qquad\text{and}\qquad\left.\frac{c_2}{F^4}\right|_{\text{phys.}}=\frac{2m_h^2}{v^2}\approx\frac{1}{2}. \tag{3.40}$$

Hence c_2 has a natural value. On the other hand, the experimental bounds on F (see §4.1.1 and §4.1.2) require $m_h^2/F^2\lesssim 0.03$, which necessarily implies a sizable tuning in c_1.

To obtain explicit expressions for $c_{1,2}$ in terms of the model parameters, we assume the following simple functional form for the fermion form factors

$$\mathcal{M}_t(q^2)=\mathcal{M}_t(0)\times\frac{M_T^2}{M_T^2-q^2} \tag{3.41}$$

and

$$\frac{\Pi_1^{t_L}(q^2)}{\Pi_1^{t_L}(0)}\frac{\Pi_0^{q_L}(0)}{\Pi_0^{q_L}(q^2)}=\frac{\Pi_1^{t_R}(q^2)}{\Pi_1^{t_R}(0)}\frac{\Pi_0^{q_R}(0)}{\Pi_0^{q_R}(q^2)}=\frac{M_T^2}{M_T^2-q^2}\frac{M_f^2}{M_f^2-q^2}. \tag{3.42}$$

[7] We remark that it has recently been observed that generating the Higgs potential at higher-order, which is radiatively stable if the symmetry breaking spurion is in a large representation, can offer an alternative route to avoiding the v/F tuning [48, 49].

The dependence on M_T follows from the assumption of a single top-partner (of mass M_T) dominating the diagram in Fig. 2 at small q^2. On the other hand, we let $M_f \lesssim 4\pi f_{\rm HC}$ denote the mass of generic heavy spin-1/2 resonances, which act to cut-off the quadratically divergent loop integrals in (3.30). Similarly, in the gauge sector we assume

$$\Pi_{1(2)}(q^2) = \Pi_{1(2)}(0)\frac{M_\rho^2}{M_\rho^2 - q^2}\,, \tag{3.43}$$

where $M_\rho \lesssim 4\pi f_{\rm HC}$ denotes the mass of the spin-1 resonances which tame the quadratically divergent integrals in (3.37). Residual logarithmic divergences in the integrals proportional to M_f^2 and M_ρ^2 are reabsorbed via an $O(1)$ redefinition of these masses.

Under these assumptions, we find the following expressions for the coefficient of the $\sin^2(h/2F)$ term,

$$\frac{c_1}{F^4} = \left(\frac{c_1}{F^4}\right)^{f-\rm quad} + \left(\frac{c_1}{F^4}\right)^{\rm min}\,, \tag{3.44}$$

$$\left(\frac{c_1}{F^4}\right)^{f-\rm quad} = \frac{N_c}{8\pi^2}\left[(\lambda_R^t)^2\kappa_R^t - (\lambda_L^t)^2\kappa_L^t\right]\frac{M_f^2}{F^2}\,, \tag{3.45}$$

$$\left(\frac{c_1}{F^4}\right)^{\rm min} = \frac{N_c y_t^2}{4\pi^2}\frac{M_T^2}{F^2} - \frac{9g_R^2}{32\pi^2}\left(1 - \frac{g_R^2 v_\Sigma^2}{2M_\rho^2}\right)\frac{M_\rho^2}{F^2} + \mathcal{O}(g_L g_R, g_L^2)\,, \tag{3.46}$$

and of the $\sin^4(h/2F)$ term,

$$\frac{c_2}{F^4} = \frac{N_c y_t^2}{4\pi^2}\frac{M_T^2}{F^2} + \frac{9g_R^2}{32\pi^2}\delta_\pi\left(1 - \frac{g_R^2 v_\Sigma^2}{2M_\rho^2}\right)\frac{M_\rho^2}{F^2} - \frac{9g_R^4}{64\pi^2}\log\left(\frac{M_\rho^2}{M_{W_R}^2}\right) + \mathcal{O}(g_L g_R, g_L^2)\,. \tag{3.47}$$

Let's start by analysing the result for c_2. The physical condition in (3.40) can be matched by keeping the fermion contribution only, which is expected to be parametrically larger than the gauge contributions here, in the limit of a light M_T. Note that this contribution is finite, hence it is largely insensitive to the details of the composite model but for the key assumption of light top partners. More precisely, assuming $\delta_\pi \ll 1$ one needs $M_T \approx 2.5F$, which is perfectly consistent with present bounds (see §4). Higher M_T values can be obtained at the expense of a modest tuning of δ_π. The term proportional to g_R^4 can be neglected if $g_R \lesssim 2$.

Concerning c_1, the tuning needed to satisfy the physical condition in (3.40) requires two main ingredients. First, a cancellation of the leading (quadratically divergent) fermion contribution in (3.45). This can be achieved imposing additional symmetries in the fermion mass terms forcing a cancellation between left-handed and right-handed contributions.[8] Doing so, one is left with explaining the *minimal tuning* [50] related to the term in (3.46), where the fermion contribution is the same as in c_2 (*i.e.* the model-independent contribution associated to the top-quark Yukawa coupling). In our setup, a natural ingredient to achieve this tuning is a cancellation between the fermion and gauge contributions, which

[8] Extensive discussions of fermion representations and related symmetries that can be used to achieve this goal can be found in Refs. [50, 51].

are predicted to have opposite sign. This cancellation does not happen with a flavour-universal gauge group since, in that case, the gauge coupling is too small. In order for the cancellation to take place in our setup, the following two conditions need to be satisfied

- A relatively large $g_R \equiv g_{R,3}$, still within the perturbative regime, such that the gauge contribution reaches the same size as the fermion one. More precisely, we need $g_R^2 \times M_\rho^2/(6F)^2 \gtrsim 1$, that for natural values of M_ρ implies $g_{R,3} = O(1) \gg g_{R,12} \approx g_Y^{\text{SM}}$.
- A relatively light $M_{W_R}^2$, so as not to suppress (or even change the sign of) the gauge contribution. As can be inferred from (3.46), the condition required is

$$M_{W_R}^2 = \frac{1}{4} g_R^2 v_\Sigma^2 < \frac{1}{2} M_\rho^2 \,, \tag{3.48}$$

where the first equality will be derived explicitly in §4.2.1. Note that to see the need for this second condition required carefully expanding the logarithm appearing in the Coleman–Weinberg potential to formally higher order before integrating, and relies on the fact that the terms proportional to g_R^2 *vs.* g_R^4 in (3.47) have strictly opposite sign.

4 Phenomenology

The phenomenological implications of the model can be divided into two main categories: those related to the strong dynamics (such as modified Higgs-boson couplings, dynamics of the top partners and heavy resonances, *etc.*) and those related to the flavoured gauge bosons. The former category does not differ significantly from what is discussed in the composite Higgs model literature, hence we shall discuss this category of bounds rather briefly.

On the other hand, we present in more detail the phenomenology of the flavoured gauge bosons, which are specific to the flavour-deconstructed model. As we have shown in the previous section, these exotic fields are predicted to be lighter than all composite states but for the Higgs boson and the top partners.

4.1 Constraints from the strong dynamics

4.1.1 Bounds on F from Higgs couplings

We here derive the bound on the scale F coming from modifications of the Higgs couplings to gauge bosons due to the strong dynamics. The SM Higgs Lagrangian has the form

$$\mathcal{L}_H^{\text{SM}} = (D_\mu H)^\dagger (D^\mu H) - V_{\text{SM}}(H) \,, \qquad D_\mu \equiv \partial_\mu - i g_L A^a_{L,\mu} \frac{\sigma_{a_L}}{2} - i g_Y B_\mu Y \,. \tag{4.1}$$

Here H denotes the Higgs field in the doublet notation, such that

$$H_{\text{unit.-gauge}} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ h^{\text{SM}}_{\text{phys}}(x) + v_{\text{EW}} \end{pmatrix} \,, \tag{4.2}$$

where $v_{\rm EW} = (\sqrt{2}G_F)^{-1/2} \approx 246$ GeV. From the kinetic term, the couplings of the physical Higgs boson to the $SU(2)_L$ SM gauge fields read

$$g^{\rm SM}_{VVh} = g_L^2 \frac{v_{\rm EW}}{4}\,, \qquad g^{\rm SM}_{VVhh} = \frac{g_L^2}{8}\,. \tag{4.3}$$

In our composite model, the kinetic part of the Higgs Lagrangian is given in (3.10). Expanding this Lagrangian in powers of $h(x)$ around its VEV determined by the potential (3.39) leads to

$$\mathcal{L}_h \supset g_L^2 \frac{F^2}{2}\left[\sin^2\left(\frac{v}{2F}\right) + \sin\left(\frac{v}{F}\right)\left(\frac{h}{2F}\right) + \cos\left(\frac{v}{F}\right)\left(\frac{h}{2F}\right)^2 + \cdots\right]\left(A^i_\mu\right)^2\,. \tag{4.4}$$

From the SM relation $m_W = \frac{1}{4}g_L^2 v_{\rm EW}^2$, we deduce

$$v_{\rm EW}^2 \equiv 4F^2 \sin^2\frac{v}{2F}\,. \tag{4.5}$$

Note that, consistently with the discussion in §3.1.1, we recover $v \to v_{\rm EW}$ in the limit $F \to \infty$. Expressing v in terms of $v_{\rm EW}$ we can relate the Higgs-gauge-boson couplings of the composite model to the SM ones, obtaining

$$g_{VVh} = g^{\rm SM}_{VVh}\sqrt{1-\xi}\,, \qquad g_{VVhh} = g^{\rm SM}_{VVhh}\left(1-2\xi\right)\,, \tag{4.6}$$

where, following [52], we have defined

$$\xi = \frac{v_{\rm EW}^2}{4F^2} \equiv \frac{v_{\rm EW}^2}{f_{\rm HC}^2}\,. \tag{4.7}$$

As expected, these results are perfectly consistent with those originally derived in [5] for the so-called minimal composite Higgs model.

Up-to-date bounds from the LHC on the Higgs couplings can be found in [53, 54]. Combining ATLAS and CMS data leads to $g_{VVh}/g^{\rm SM}_{VVh} > 0.97$ (95% CL), which implies

$$\xi < 0.06 \ (95\%\,\text{CL}) \qquad \text{or} \qquad f_{\rm HC} > 1.0 \text{ TeV} \ (95\%\,\text{CL})\,. \tag{4.8}$$

4.1.2 Top partners and heavier resonances

Light top partners are a general prediction of composite Higgs models and extensive searches have been performed at the LHC for them. A selection of recent results can be found in [55–58]. The bounds based on pair production, which are the most stringent except in a few specific model-dependent constructions, imply

$$M_T \gtrsim 1.5 \text{ TeV}\,. \tag{4.9}$$

In order to fulfil the relation $M_T \approx 2.5F$ from the Higgs potential (see §3.4), this implies $f_{\rm HC} \gtrsim 1.2$ TeV, delivering a slightly stronger limit than the one set by the Higgs couplings in (4.8).

As far as the direct searches for heavy resonances are concerned, specifically on the vector resonances which are expected to be the lightest ones, present LHC limits on W' and

Z' bosons [59, 60] imply $M_\rho \gtrsim 5$ TeV. More stringent constraints are derived from indirect searches via electroweak observables. The leading (tree-level) effects of heavy composite resonances in EW precision observables (such as the S parameter or the m_W/m_Z ratio) are of the order of

$$\delta_{\rm EW} = g_{L,R}^2 \frac{v^2}{M_\rho^2} \equiv \xi \frac{g_{L,R}^2}{g_\rho^2} \,, \tag{4.10}$$

where we have defined $g_\rho \equiv f_{\rm HC}/M_\rho$. Present bounds are satisfied for $\delta_{\rm EW} \lesssim 10^{-3}$, which can be obtained for a large enough g_ρ, well within the upper bound $g_\rho^{\rm max} \approx 4\pi$ set by perturbativity.

Aiming at the lowest possible value of $f_{\rm HC}$ in order to minimize the tuning in the Higgs potential, a reference benchmark point/region for the composite sector is provided by[9]

$$f_{\rm HC} \approx 1.5 \text{ TeV}, \qquad M_T \approx 1.8 \div 2.0 \text{ TeV}, \qquad g_\rho \approx 5 \div 6 \tag{4.11}$$

This choice implies a 3% tuning in the Higgs potential ($\xi \approx 0.03$) with $O(1\%)$ corrections to the Higgs couplings. Corrections to the EW precision observables are slightly below the per-mille level, and $M_\rho \approx 8 \div 10$ TeV.

4.2 Flavoured gauge bosons

We now turn to the phenomenology associated with the flavour deconstruction, which gives rise to heavy gauge bosons. As described above, the flavour structure is generated thanks to the minimal symmetry breaking pattern

$$U(1)_Y^{[12]} \times U(1)_{B-L}^{[3]} \times SU(2)_R^{[3]} \to U(1)_Y \,, \tag{4.12}$$

triggered by two link fields getting VEVs: $\Sigma \sim \left(\frac{1}{2}, 0, \mathbf{2}\right)$, which gets the VEV $\langle\Sigma\rangle = \frac{1}{\sqrt{2}}(0\ v_\Sigma)^T$, and $\Omega_q \sim \left(\frac{1}{6}, -\frac{1}{6}, \mathbf{1}\right)$, which acquires the VEV $\langle \Omega_q \rangle = \frac{1}{\sqrt{2}} v_\Omega$.[10]

4.2.1 Gauge boson masses

To obtain the spectrum of the heavy gauge bosons, the relevant Lagrangian is

$$\mathcal{L} \supset \left| \partial_\mu \Sigma + i \frac{g_{R,3}\sigma^a}{2} W_{a\mu}^{[3]} \Sigma + i \frac{g_{Y,12}}{2} B_\mu^{[12]} \Sigma \right|^2 + \left| \partial_\mu \Omega_q + i \frac{g_{Y,12}}{6} B_\mu^{[12]} \Omega_q - i \frac{g_{3,B-L}}{6} X^{[3]} \Omega_q \right|^2 ,$$

which contains the following mass terms after subbing in the VEVs:

$$\begin{aligned} \mathcal{L} \supset \frac{1}{8} v_\Sigma^2 \left[g_{R,3}^2 (W_1^{[3]\,2} + W_2^{[3]\,2}) + (g_{Y,12} B^{[12]} - g_{R,3} W_3^{[3]})^2 \right] \\ + \frac{1}{72} v_\Omega^2 (g_{Y,12} B^{[12]} - g_{B-L,3} X^{[3]})^2 \end{aligned} \tag{4.13}$$

[9] The benchmark range for g_ρ closely replicates a QCD-like spectrum.

[10] For simplicity, unless otherwise stated, in this section we disregard the possibility of the additional link field $\Omega_\ell \sim \left(-\frac{1}{2}, \frac{1}{2}, \mathbf{1}\right)$.

The three neutral gauge bosons all mix. The mass mixing matrix is

$$\mathcal{L} \supset \frac{1}{8}(B^{[12]}\ X^{[3]}\ W_3^{[3]}) \begin{pmatrix} g_{Y,12}^2(v_\Sigma^2 + \frac{1}{9}v_\Omega^2) & -\frac{1}{9}g_{Y,12}g_{B-L,3}v_\Omega^2 & -g_{Y,12}g_{R,3}v_\Sigma^2 \\ -\frac{1}{9}g_{Y,12}g_{B-L,3}v_\Omega^2 & \frac{1}{9}g_{B-L,3}^2 v_\Omega^2 & 0 \\ -g_{Y,12}g_{R,3}v_\Sigma^2 & 0 & g_{R,3}^2 v_\Sigma^2 \end{pmatrix} \begin{pmatrix} B^{[12]} \\ X^{[3]} \\ W_3^{[3]} \end{pmatrix} . \tag{4.14}$$

This matrix has zero determinant, so there remains a massless neutral gauge boson, corresponding to the unbroken SM $U(1)_Y$ subgroup. This massless field corresponds to the linear combination

$$B \propto \frac{1}{g_{Y,12}}B^{[12]} + \frac{1}{g_{B-L,3}}X^{[3]} + \frac{1}{g_{R,3}}W_3^{[3]} , \qquad m_B = 0 , \tag{4.15}$$

which, sure enough, has flavour-universal couplings to the SM fermions (see below).

The masses of the two heavy neutral gauge bosons are quite complicated expressions of the couplings and the VEVs, in general. However, motivated by our considerations of the Higgs potential (plus collider phenomenology), we are most interested in the régime

$$g_{Y,12} \ll g_{R,3},\ g_{B-L,3} , \tag{4.16}$$

i.e. with smaller gauge couplings to the light generations than to the third one. In this limit, the off-diagonal terms responsible for mixing are all suppressed, and one can determine the two heavy mass eigenvalues perturbatively in the small ratio of couplings:

$$M_{Z_V}^2 \approx \frac{1}{36}(g_{B-L,3}^2 + g_{Y,12}^2)v_\Omega^2 + \mathcal{O}(g_{Y,12}^4) , \qquad M_{Z_R}^2 \approx \frac{1}{4}(g_{R,3}^2 + g_{Y,12}^2)v_\Sigma^2 + \mathcal{O}(g_{Y,12}^4) . \tag{4.17}$$

Finally, looking back to (4.13), there are massive eigenstates of the unbroken $U(1)_Y$ with charge ± 1 (which can be read off from the non-abelian $SU(2)_R$ algebra, specifically the non-zero commutator between the unbroken σ_3 direction and the combinations $\sigma_1 \mp i\sigma_2$ corresponding to $W^\pm$):

$$W_R^\pm = \frac{1}{\sqrt{2}}(\tilde{W}_1 \mp i\tilde{W}_2) , \qquad M_{W_R}^2 = \frac{1}{4}g_{R,3}^2 v_\Sigma^2 . \tag{4.18}$$

To leading order, the mass ratio for the Z_R and W_R is

$$\left(\frac{M_{Z_R}}{M_{W_R}}\right)^2 = \frac{g_{R,3}^2 + g_{Y,12}^2}{g_{R,3}^2} \geq 1 , \tag{4.19}$$

mirroring the formulae for the Z to W mass ratio in the SM.

4.2.2 Gauge boson couplings to SM fields

We now turn to the couplings of the mass eigenstate gauge bosons. We first derive the couplings to SM fermions, then to the Higgs.

Couplings to fermions

Firstly, the massless gauge field B couples to the flavour-universal SM hypercharge, $Y = Y^{[12]} + (B-L)^{[3]} + T^{[3]}_{R^3}$, that is left unbroken by this symmetry breaking pattern irrespective of the values of the VEVs or the gauge couplings.[11] The corresponding gauge coupling g' then satisfies the following matching condition

$$\frac{1}{g'^2} = \frac{1}{g^2_{Y,12}} + \frac{1}{g^2_{B-L,3}} + \frac{1}{g^2_{R,3}} \,. \tag{4.20}$$

One can enforce this condition by parametrizing the UV gauge couplings via

$$g' = g_{Y,12}\cos\theta = g_{B-L,3}\sin\theta\cos\phi = g_{R,3}\sin\theta\sin\phi \,. \tag{4.21}$$

This let us trade the three couplings $\{g_{Y,12}, g_{B-L,3}, g_{R,3}\}$ for two polar angles $\{\theta, \phi\}$. The inverse coordinate transformation is

$$\theta = \arctan\left(\frac{g_{Y,12}}{g_{B-L,3}g_{R,3}}\sqrt{g^2_{B-L,3} + g^2_{R,3}}\right) , \qquad \phi = \arctan\left(\frac{g_{B-L,3}}{g_{R,3}}\right) . \tag{4.22}$$

Other useful relations are:

$$\frac{g_{Y,12}}{g_{R,3}} = \tan\theta\sin\phi, \qquad \frac{g_{Y,12}}{g_{B-L,3}} = \tan\theta\cos\phi, \tag{4.23}$$

In these coordinates, the limit $g_{Y,12} \ll g_{R,3}$, $g_{B-L,3}$ that we are interested in translates to the small angle limit

$$\theta \ll 1, \qquad \phi \sim 1 \,. \tag{4.24}$$

The two heavy mass eigenstates coincide with the gauge eigenstates up to small corrections: we have $Z_V \approx X^{[3]} - \frac{g_{Y,12}}{g_{B-L,3}} B^{[12]} + \mathcal{O}(g_Y^2) W_3^{[3]}$ and $Z_R \approx W_3^{[3]} - \frac{g_{Y,12}}{g_{R,3}} B^{[12]} + \mathcal{O}(g_Y^2) X^{[3]}$. The notation Z_V and Z_R reminds us that the former neutral boson couples nearly vector-like to fermions, while the latter is nearly right-handed. To deduce the couplings to SM fields, we invert these relations:

$$B^{[12]} \approx B - t_\theta c_\phi Z_V - t_\theta s_\phi Z_R \,, \tag{4.25}$$

$$X^{[3]} \approx Z_V + t_\theta c_\phi B \,, \tag{4.26}$$

$$W_3^{[3]} \approx Z_R + t_\theta s_\phi B \,, \tag{4.27}$$

where $s_\theta := \sin\theta$ *etc.* We can thence deduce the couplings of both Z' bosons to SM fermions, which take the form

$$\mathcal{L} \supset Z_{V\mu} J^\mu_{V,\psi} + Z_{R\mu} J^\mu_{R,\psi} \,, \tag{4.28}$$

where the relevant fermionic currents are found to be

$$J^\mu_{V,\psi} = g_{B-L,3} J^{[3]\mu}_{B-L} - \frac{g^2_{Y,12}}{g_{B-L,3}} J^{[12]\mu}_Y = \frac{g'}{s_\theta c_\phi}\left(J^{[3]\mu}_{B-L} - t^2_\theta c^2_\phi J^{[12]\mu}_Y\right) , \tag{4.29}$$

[11] This reflects a general group theory statement that one only ever breaks to the diagonal subgroup, under certain conditions that are here satisfied – see Refs. [29, 34].

$$J^{\mu}_{R,\psi} = g_{R,3} J_R^{[3]\mu} - \frac{g^2_{Y,12}}{g_{R,3}} J_Y^{[12]\mu} = \frac{g'}{s_\theta s_\phi} \left(J_R^{[3]\mu} - t_\theta^2 s_\phi^2 J_Y^{[12]\mu} \right) . \tag{4.30}$$

We notice from these formulae that one cannot take the coupling to light generations to zero ($\theta \to 0$) without the coupling to the third family simultaneously diverging.[12] This is the case for any flavour deconstruction setup: one cannot totally decouple the heavy gauge bosons from the light SM generations.

Finally, there is the coupling of the charged current gauge bosons $W_R^{\pm}$ that come from breaking the off-diagonal generators in $SU(2)_R^{[3]}$. This gives a charged current interaction with third generation right-handed quarks:

$$\mathcal{L} \supset -\frac{1}{\sqrt{2}} g_{R,3} W^-_{R\mu} \overline{d}^3_R \gamma^\mu u^3_R + \text{h.c.} \tag{4.31}$$

There is no leptonic coupling because, even if there are right-handed neutrinos, they are assumed heavy and already integrated out. Note this also decouples the high-p_T bound because we have no semi-leptonic operators coming from integrating out the W_R that modify $\ell\ell$ or $\ell\nu$ final states.

Couplings to the Higgs

The massive gauge bosons generated by the $SU(2)_R^{[3]}$ breaking couple also to the Higgs fields. To derive these couplings, it is convenient to use the bi-doublet notation Φ for the Higgs field that we introduced above in Eq. (3.5). This is related to the Higgs $SU(2)_L$ complex doublet H via

$$\Phi = \begin{pmatrix} H^c & H \end{pmatrix}, \qquad H^c = i\sigma_2 H^* . \tag{4.32}$$

The covariant derivative acting on Φ was given above in Eq. (3.15), by which $SU(2)_L$ ($SU(2)_R^{[3]}$) gauge bosons act straightforwardly by left (right) matrix multiplication.

To obtain the linear couplings of the heavy gauge fields to the Higgs doublet H (which is all we need for tree-level matching to the SMEFT at dimension-6), we expand the kinetic part of the action $\mathcal{L} = \frac{1}{4}\text{Tr}\left[D_\mu \Phi^\dagger D_\mu \Phi\right]$, and isolate the piece that is linear in the A_R gauge field. This current is

$$L \supset \frac{1}{4}\text{Tr}\left[i g_{R,3} \partial_\mu \Phi^\dagger \left(\Phi . \frac{\sigma^a}{2} A^a_{R\mu} \right) - i g_{R,3} \left(\frac{\sigma^a}{2} A^a_{R\mu} . \Phi^\dagger \right) \partial_\mu \Phi \right] . \tag{4.33}$$

After the symmetry breaking described above, we can deduce the couplings to the heavy charged current bosons by rotating to the ($W_R^{\pm}, Z_R \approx A_R^3$) basis. The charged-current interaction then assumes the form

$$L \supset \frac{g_{R,3}}{2\sqrt{2}} W^-_{R\mu} \Big[- \phi_1 \partial_\mu \phi_4 + \phi_2 \partial_\mu \phi_3 - \phi_3 \partial_\mu \phi_2 + \phi_4 \partial_\mu \phi_1 \tag{4.34}$$

[12] There is also, in principle, a small deviation in the light family couplings due to the mixing between SM fermions and the heavy vector-like fermions $F^q_{R,L}$ that are integrated out at the high scale to generate the Yukawa hierarchies. The effects of such mixing has been taken into account in previous phenomenological studies of flavour deconstruction *e.g.* [61], but for our purposes it is good enough to ignore these small mixing effects. Their inclusion would change the high-p_T bounds derived below by a small amount.

$$
\begin{aligned}
&\qquad + i(\phi_1\partial_\mu\phi_3 + \phi_2\partial_\mu\phi_4 - \phi_3\partial_\mu\phi_1 - \phi_4\partial_\mu\phi_2)\Big] + \text{h.c.} \\
&= -\frac{g_{R,3}}{2\sqrt{2}} W^-_{R\mu} i D_\mu H^T i\sigma_2 H + \text{h.c.} \qquad (4.35)
\end{aligned}
$$

while the neutral-current coupling is

$$
L \supset \frac{1}{2} g_{R,3} Z_R^\mu (\phi_1\partial_\mu\phi_2 - \phi_2\partial_\mu\phi_1 - \phi_3\partial_\mu\phi_4 + \phi_4\partial_\mu\phi_3) \qquad (4.36)
$$

$$
= \frac{g_{R,3}}{4} Z_R^\mu (iH^\dagger D_\mu H + \text{h.c.}) \;\equiv\; Z_R^\mu J_{R,H\,\mu}\,, \qquad (4.37)
$$

where the last equality defines $J^\mu_{R,H}$.

4.2.3 Matching to the SMEFT

Having derived the couplings of the heavy vector bosons, we can now integrate out at tree-level and match to the dimension-6 SMEFT to estimate the most important phenomenological consequences. We do so using the tree-level matching dictionary of Ref. [62].

When the two neutral gauge bosons are integrated out, we can write down the SMEFT Lagrangian that results in terms of the currents derived above:

$$
\mathcal{L} \supset -\frac{J^\mu_{V,\psi} J_{V,\psi\,\mu}}{2M^2_{Z_V}} - \frac{J^\mu_{R,\psi} J_{R,\psi\,\mu}}{2M^2_{Z_R}} - \frac{J^\mu_{R,\psi} J_{R,H\,\mu}}{M^2_{Z_R}} - \frac{J^\mu_{R,H} J_{R,H\,\mu}}{2M^2_{Z_R}} \qquad (4.38)
$$

The first term comes from the Z_V boson, which couples only to fermions and hence produces only 4-fermion operators. The latter three terms come from integrating out the Z_R boson, containing 4-fermion operators with various chiral structures (second term), mixed fermion-boson operators of the form $\mathcal{O}_{Hf} \sim (H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{f}\gamma^\mu f)$ (third term), and pure bosonic operators $\mathcal{O}_{HD} \sim (H^\dagger D_\mu H)^2$ and $\mathcal{O}_{H\Box} \sim |H|^2\Box|H|^2$ (fourth term). This accounts for all tree-level dimension-6 operators produced by integrating out the two heavy neutral gauge bosons. All these operators will lead to important experimental constraints, which we discuss in the following subsections.

Integrating out the charged current $W_R^\pm$ bosons gives additional operators. Starting with four-fermion operators, only the SMEFT operators $\mathcal{O}^{(1)}_{ud}$ and $\mathcal{O}^{(8)}_{ud}$ are generated given the coupling (4.31), with suppression factors for all light-family flavour components. These are subject only to very weak experimental bounds, and will play no role in the phenomenological analysis that follows. Concerning mixed fermion-boson operators, different structures are generated by the charged current (compared to neutral one). Firstly, we get modified Yukawa operators $\mathcal{O}_{eH} \sim |H|^2 \bar{l} H e$, $\mathcal{O}_{dH}$ and $\mathcal{O}_{uH}$ (defined similarly). These operators are generated only via the Higgs interaction, and so their Wilson coefficients are suppressed by SM Yukawa couplings:

$$
C_{eH,\,dH,\,uH} \sim y_{\tau,\,b,\,t} \frac{g^2_{R,3}}{16 M^2_{W_R}}\,. \qquad (4.39)
$$

These operators modify the $H \to \tau\tau$ and $H \to bb$ signal strengths; however, the suppression by y_τ (y_b) in (4.39) implies the corresponding bounds are weak. Integrating out the W_R also generates the operator $\mathcal{O}_{Hud}^{ij} \sim (H^\dagger i D_\mu H)\overline{u}_R^i \gamma^\mu d_R^j$, with unsuppressed Wilson coefficient for the $i = j = 3$ component: this operator has a significant impact in flavour physics (see §4.2.4). Lastly, the W_R also gives contributions to pure bosonic operators, in particular an additional contribution to $\mathcal{O}_{HD}$, which modifies the SM prediction for m_W (§4.2.5). Both these effects are discussed in detail below.

4.2.4 Flavour observables

The heavy gauge bosons in this theory have flavour non-universal couplings to the SM fermions, but respect an accidental $U(2)^5$ global flavour symmetries acting on the light families, with a minimal breaking structure linked to the subleading entries in the Yukawa couplings. This symmetry allow them to be close to the TeV scale without contravening the tight flavour bounds for flavour-changing processes in the 1-2 sector (such as those from kaon mixing).

Nonetheless, there are important bounds coming from flavour-changing processes involving bottom quarks, which we here survey:

- **Bound on W_R mass from $B \to X_s\gamma$.** As anticipated, the W_R boson generates a tree-level contribution to $O_{Hud} \sim (H^\dagger i D_\mu H)\overline{u}_R \gamma^\mu d_R$, with unsuppressed coupling for third generation quarks,

$$C_{Htb} = -\frac{g_{W_R}^H g_{W_R}^{b_R t_R}}{M_{W_R}^2} = \frac{1}{v_\Sigma^2}\,. \tag{4.40}$$

  This operator is severely constrained by its one-loop contribution to $B \to X_s\gamma$. Using the results of [3], we deduce

$$v_\Sigma \geq 3.2 \text{ TeV}\,. \tag{4.41}$$

  Note that this effect is independent of the flavour orientation of the Yukawa couplings, since the flavour mixing come from the CKM matrix (or the flavour-off-diagonal left-handed coupling of the W field in the mass-eigenstate basis). The diagonalization of the Yukawa couplings induce, in principle, also non-vanishing contributions to the O_{Hud} operator with light quarks. However, the corresponding couplings are very suppressed by the negligible amount of flavour mixing in the right-handed sector, hence can be safely neglected.

- **Bounds on Z' masses from FCNC processes.** The smallness of flavour mixing in the right-handed sector implies the Z_R boson is weakly constrained from FCNC processes. The situation is different for the Z_V boson, where left-handed flavour mixing (from the diagonalisation of the Yukawa couplings) can induce sizeable FCNC couplings. The corresponding bounds on v_Ω, dominated by B_s-mixing, can be deduced from the recent analysis in Ref. [6]. They reach

$$v_\Omega \gtrsim 2.7 \text{ TeV} \tag{4.42}$$

  in the case of pure up-alignment, but could be reduced with respect to this value by assuming some degree of down-alignment.

- **Flavour constraints on the composite states.** On general grounds, the composite states (top-partners, vector resonances, and higher-mass states) also mediate flavour-changing interactions. First, it is worth noting that the minimal breaking of the flavour symmetry prevents tree-level contributions to FCNCs suppressed only by the light top partner mass. As far as other effects are concerned, an extensive analysis in various classes of composite models has been presented recently in Ref. [9]. The case there denoted 'partial Right Universality' (pRU) is quite close to our framework in this respect, since our deconstructed gauge symmetry structure enshrines the pRU global symmetry pattern as accidental. From the analysis of Ref. [9] we deduce that the bounds from flavour observables are all satisfied if g_ρ is large enough to satisfy the EW bounds (see §4.1.2).

4.2.5 Higgs and electroweak observables

The heavy gauge bosons coming from the $SU(2)_R^{[3]}$ breaking couple to the Higgs, giving rise to mixed fermion-Higgs operators and pure Higgs operators at tree-level (see §4.2.3). These operators give sizeable effects in electroweak precision observables (EWPO), principally the Z-pole measurements made at LEP-II as well as measurements of the W mass. We here pinpoint the most important observables and estimate the strength of constraints to guide future studies. We do not undertake a comprehensive fit to data in this work.

W-boson mass

The heavy $W_R^\pm$ and Z_R bosons leads to a shift (δm_W) in the prediction for the W mass. The leading effect is related to their tree-level contribution to the Wilson coefficient C_{HD}:[13]

$$C_{HD} = \left(\frac{|g^H_{W_R}|^2}{M^2_{W_R}} - \frac{2|g^H_{Z_R}|^2}{M^2_{Z_R}} \right) = \frac{g^2_{R,3}}{8} \left(\frac{1}{M^2_{W_R}} - \frac{1}{M^2_{Z_R}} \right) . \tag{4.43}$$

Notice that (4.43) vanishes in the 'custodial' limit $M_{W_R} = M_{Z_R}$. This would occur in principle for pure $SU(2)_R^{[3]} \to \{0\}$ breaking, i.e. without the linking which results in the Z_R mixing with the light-family-hypercharge boson (that shifts M_{Z_R} without affecing M_{W_R}). Accounting for this mixing, we have the following non-zero C_{HD} at tree-level

$$C_{HD} = \frac{1}{2v_\Sigma^2} \left[1 - \left(1 + \frac{g^2_{Y,12}}{g^2_{R,3}} \right)^{-1} \right] \approx \frac{1}{2v_\Sigma^2} \frac{g^2_{Y,12}}{g^2_{R,3}} . \tag{4.44}$$

To a good accuracy, the shift in the W-mass is given by [63] $\delta m_W^2/m_W^2 \approx \frac{1-s_w^2}{1-2s_w^2} C_{HD} v^2/2$, where $\delta m_W^2 := (m_W^2)_{\text{SM}} - (m_W^2)_{\text{SMEFT}}$ and s_w is the sine of the Weinberg angle. Using also $\delta m_W^2/m_W^2 \approx 2\delta m_W/m_W$, we find

$$\delta m_W \approx \frac{1 - s_w^2}{1 - 2s_w^2} \frac{v_{\text{EW}}^2}{8v_\Sigma^2} \frac{g^2_{Y,12}}{g^2_{R,3}} m_W = \left(\frac{9 \text{ TeV}}{v_\Sigma} \right)^2 \frac{g^2_{Y,12}}{g^2_{R,3}} \times (10 \text{ MeV}) . \tag{4.45}$$

[13] In principle, there is also a contribution from the four-lepton operator C_{ll}^{1221}, induced by integrating out the Z' bosons, which arises because this operator modifies the extraction of G_F from the muon decay. However, this operator is lepton-flavour-violating and is induced by a rotation in the lepton 1-2 sector that must be sufficiently small to satisfy other constraints from $\mu \to 3e$ and $\mu \to e\gamma$. As a result, this contribution to δm_W turns out to be safely negligible.

As expected, the shift in the W mass vanishes in the limit $\theta \to 0$ (due to the custodial protection). Hence the effect is small in the limit where the exotic gauge bosons couple dominantly to the third family – the same limit that is also strongly favoured by collider searches (see below), *as well as* by our independent consideration of tuning in the Higgs potential. Importantly, the shift in the W mass is necessarily positive (with respect to the SM prediction), going in the same direction as a current small deviation in combined measurements of the W mass [64]. The size of this deviation (roughly the same size as the current uncertainty on the measurement) is a few tens of MeV. From (4.45) we see that, taking as a benchmark $v_\Sigma = 3$ TeV, one has $\delta m_W \approx 10$ MeV for $g_{Y,12} \approx g_{R,3}/3 \implies g_{R,3} \approx 1$, exactly in the region favoured by our considerations on the Higgs potential in §3.4.

We remind the reader that we have in mind a region of parameter space in which the effects of the composite sector on EWPOs are at the per-mille level (see §4.1.2), giving just small corrections to the leading-order effects from the (lighter) Z_R and W_R gauge bosons that we consider here.

Z-pole observables

Other EWPOs involve modifications to the electroweak gauge boson couplings to SM fermions, induced from integrating out the Z_R gauge boson (that mixes with the Z). Here the leading effects are due to the mixed fermion-Higgs operators $\mathcal{O}_{Hf}$.

Because of the right-handed chiral structure of the coupling to third generation fermions, we have $C_{Hl}^{33} = C_{Hq}^{(1)\,33} = 0$. Wilson coefficients involving light-generation fermion currents (which are proportional to the hypercharge of the fermion field involved) are suppressed by the small mixing parameter $\sin\theta$:[14]

$$\{C_{Hl}^{11,22}, C_{He}^{11,22}, C_{Hq}^{(1)\,11,22}, C_{Hu}^{11,22}, C_{Hd}^{11,22}\} \approx \frac{s_\theta^2 s_\phi^2}{6v_\Sigma^2}\{-3,-6,+1,+4,-2\} \tag{4.46}$$

The largest effects, however, involve the right-handed third family fermion fields. These come from the unsuppressed Higgs-bifermion operators:

$$C_{He}^{33} \approx -C_{Hu}^{33} \approx C_{Hd}^{33} \approx \frac{1}{2v_\Sigma^2}\,. \tag{4.47}$$

We can estimate the strength of the bound from EWPOs by taking the one-at-a-time constraints on these three coefficients. The analysis of Ref. [3] suggests the strongest of such bound comes from the modified Z coupling to right-handed taus, thanks to the LEP measurement of $R_\tau = \Gamma(Z \to \tau^+\tau^-)/\Gamma(Z \to q\bar{q})$. From the anlsysis of Ref. [3] we deduce

$$v_\Sigma \gtrsim 2.7 \text{ TeV}\,, \tag{4.48}$$

which is comparable to the $B \to X_s\gamma$ bound discussed in §4.2.4. We therefore identify these kind of precision flavour and electroweak observables involving third generation fermions (taus and bottoms) as particularly promising probes of a flavour deconstructed composite Higgs scenario.

[14] Note that we can further reduce EWPO effects involving light leptons and quarks by considering the $\phi \ll 1$ (i.e $g_{R,3} \ll g_{B-L,3}$) limit.

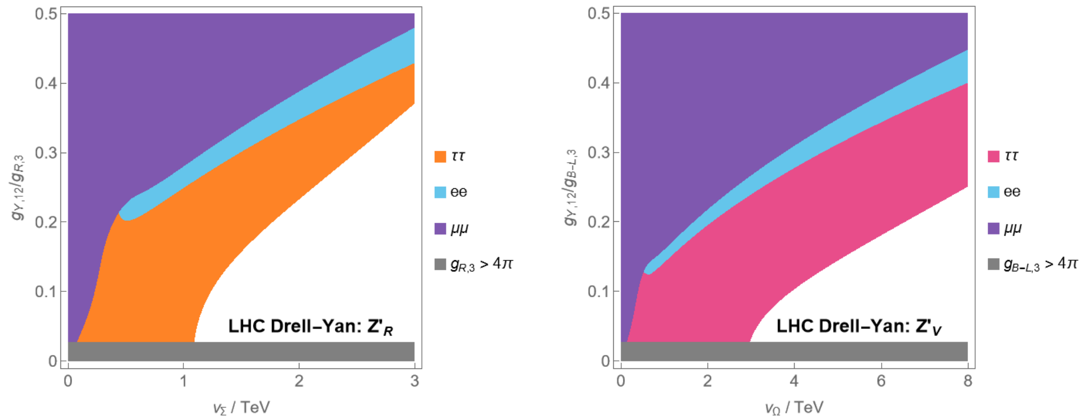

Figure 4: Constraints from high-p_T Drell–Yan data at the LHC on the two Z' bosons arising from the flavour deconstruction in our model. In the region of parameter space plotted, for which $g_{Y,12} \ll g_{R,3}$, $g_{B-L,3}$, the constraint from $pp \to \tau\tau$ is the strongest.

4.2.6 LHC bounds from Drell-Yan data

Lastly, bounds on the Z' bosons masses can be derived from pp collisions at the LHC. We present here an EFT estimate of such bounds, as derived from high-p_T Drell–Yan production of di-lepton pairs, measured by ATLAS [65] and CMS [66]. These analyses use a total integrated luminosity of 139 fb^{-1} (140 fb^{-1}) respectively.

While a light enough Z' in the s-channel would generate a peak in the $m_{\ell\ell}$ distribution, we here approximate the impact of heavy Z' bosons by the four-fermion contact interaction the induce. As pointed out in various studies (see *e.g.* [67]), the inclusion of four-fermion operators lifts the tail of the $m_{\ell\ell}$ distribution allowing to derive stringent bounds also for Z' masses well above the kinematical threshold. We use the `HighPT` package [68, 69] in 'EFT mode' to approximate a χ^2 likelihood function for our model, turning on all dimension-6 semi-leptonic operators obtained by tree-level matching of the two heavy Z' bosons to the SMEFT. We include both light lepton and tau channels.

The results are shown in Fig. 4. We present the constraints in terms of the relevant VEV v_Σ (v_Ω), vs. the ratio of the $g_{Y,12}$ to $g_{R,3}$ ($g_{B-L,3}$) gauge coupling, which is assumed to be small. As can be seen, the high-p_T constraints on the Z_R boson even permits $v_\Sigma \gtrsim 1$ TeV, in the limit $g_{Y,12} \ll g_{R,3}$ ($\theta \ll 1$), when $g_{R,3}$ approaches 4π. For more realistic $g_{R,3}$ values, namely for $g_{R,3} \lesssim 1.5$, we get

$$v_\Sigma \gtrsim 2.0 \text{ TeV}\,. \tag{4.49}$$

It is the $pp \to \tau\tau$ search that set the most stringent limits the small θ region. Looking at Fig. 4, one can see that the régime of light v_Σ (compared to the mass M_ρ of the lightest spin-1 composite resonance) favoured by our considerations of the Higgs potential is, for now, completely consistent with high-p_T LHC data. Once more, this is because of the flavour non-universal structure of the Z_R couplings, with suppressed interactions with the light quarks. From our previous considerations, we expect the indirect tests (from both flavour, in particular $B \to X_s\gamma$, and electroweak, in particular the R_τ ratio) to be slightly more relevant than high-p_T bounds at present; which may of course change as the LHC program advances.

The second Z_V boson, that couples mostly to the $J_{B-L}^{[3]}$ fermion current, has a seemingly stronger bound on its VEV, $v_\Omega \gtrsim 3 \div 6$ TeV or so depending on the desired upper limit for the coupling. This factor of roughly 3 difference in the bounds just comes from the fact that the gauge boson mass is related to the VEV as $M_{Z_V} \approx \frac{1}{6} g_{B-L,3} v_\Omega$, vs. $M_{Z_R} \approx \frac{1}{2} g_{R,3} v_\Sigma$, in units where the fermion charges are comparable. Actually, the relation between M_{Z_V} and v_Ω changes to $M_{Z_V} \approx \frac{1}{2} g_{B-L,3} v_\Omega$ if the extra link field Ω_ℓ (see Table 1) is taken into consideration. In that case the bounds on v_Ω and v_Σ turn out to be very comparable.

We further recall that the scale v_Ω plays no role in the Higgs potential because the Higgs does not couple to this (dominantly vector-like) gauge boson; thus, the constraint on v_Ω does not have implications for naturalness, in contrast to v_Σ which is crucially tied to the generation of the electroweak scale within our model.

5 Conclusions and Outlook

In this paper we have explored a model for addressing the origin of flavour and the stability of the electroweak scale, by combining the idea of flavour non-universality with Higgs compositeness. In the UV, flavour non-universal gauge interactions encode the $2+1$ family structure, motivated by the observed pattern in the Yukawa couplings and the smallness of flavor-violating effects involving the light families. On the other hand, the Higgs mass is protected from quantum corrections induced by heavier degrees of freedom thanks to its compositeness: the Higgs emerges as a light pNGB of a spontaneously broken symmetry and is unaffected by dynamics above the compositeness scale – in particular, it is insensitive to the mass scale of the heavy fermion which is responsible for generating the hierarchical flavour structure needed to solve the flavour puzzle.

The Higgs potential is generated at one-loop by explicit symmetry-breaking terms in the fermion sector and by the gauging of a subgroup of the strong symmetry. We have quantified the fermionic, scalar, and gauge contributions to the Higgs mass. We found that the flavour non-universal gauge structure of our model combined with a close-by symmetry breaking scale allows for a cancellation between fermionic and gauge contributions in the Higgs potential, thereby justifying *a posteriori* the observed little hierarchy between v and $f_{\rm HC}$. This important feature is connected to the possibility of having a large enough gauge coupling $g_{R,3} = O(1) \gg g_{R,12} \approx g_Y^{\rm SM}$, together with relatively light massive flavoured gauge bosons, $M_{W_R} \sim$ few TeV. It is important to stress that this finding is reasonably general and would hold also in variations of the model, especially as far as the UV gauge group is concerned. The key observation is that combining partial compositeness with flavour deconstruction results in a more predictive framework than when each hypothesis is considered independently.

The interesting interplay between the scale of flavour deconstruction and the tuning in the Higgs potential motivated us to perform a study of the phenomenological implications of the model. The masses and couplings of the flavoured gauge bosons turn out to be mostly constrained by $B \to X_s \gamma$ and Z-pole observables and, to a lesser extent (at least at present), by LHC measurements of $\sigma(pp \to \tau^+\tau^-)$. Present data allow these bosons to have masses in the vicinity of the TeV scale, as required by the consistency of the model. Interestingly enough, signals of their presence could appear via a modified high-p_T behaviour of $pp \to \tau^+\tau^-$ in the high-luminosity phase of the LHC.

On the theory side, one direction forward is to further speculate regarding the next layer of UV physics underlying the framework we have presented. Quite conspicuously, the model we sketched features fundamental scalar link fields Σ_R and Ω which would themselves introduce (large) hierarchy problems. One approach is to realise the scalar link fields also as composite particles of an enlarged strong sector. For instance, one can embed Σ_R alongside the Higgs as composite pNGBs arising from a strong sector with global symmetry breaking

$$Sp(6)_{\text{global}} \longrightarrow SU(2)_L \times SU(2)_R^{[3]} \times SU(2)_R^{[12]} \,. \tag{5.1}$$

This delivers pNGBs transforming in bi-fundamentals of each pair of $SU(2)$s, corresponding to the Higgs pNGB, the Σ_R, plus an extra bidoublet charged under $SU(2)_L \times SU(2)_R^{[12]}$ (which would need to have a vanishing VEV). The natural mass scale for these pNGBs is around $f_{\text{HC}} \sim \text{TeV} \sim v_\Sigma$, so the Σ_R state is completely naturally described as a composite pNGB; it is only the pNGB corresponding to the SM Higgs that must end up anomalously light due to fine-tuning in the loop-generated potential.

Acknowledgments

We are grateful to Lukas Allwicher for helpful discussions. This work is funded by the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme under grant agreement 833280 (FLAY), and by the Swiss National Science Foundation (SNF) under contract 200020-204428.

A Further details on the CCWZ formalism

The decomposition of $U^\dagger \partial_\mu U$ in (3.12) can be ordered in terms of the number of ϕ-insertions. The LO and NLO terms in Γ_μ^a and u_μ^a are given by:

$$\Gamma_\mu^a = -\frac{1}{2}\phi_b\partial_\mu\phi_c \cdot f_U^{cba} + \frac{1}{6}\phi_b\phi_c\partial_\mu\phi_d \cdot f_B^{cde} f_U^{bea} + \dots \tag{A.1}$$

$$u_\mu^a = \partial_\mu\phi_a - \frac{1}{2}\phi_b\partial_\mu\phi_c \cdot f_B^{cba} + \frac{1}{6}\phi_b\phi_c\partial_\mu\phi_d \cdot \left(f_B^{cde} f_B^{bea} - f_U^{cde} f_{UB}^{eba}\right) + \dots \tag{A.2}$$

where the commutation relations among unbroken generators T_U^a and broken generators T_B^a are given by:

$$\left[T_B^a, T_B^b\right] = i f_B^{abc} \cdot T_B^c + i f_U^{abc} \cdot T_U^c \tag{A.3}$$

$$\left[T_U^a, T_B^b\right] = i f_{UB}^{abc} \cdot T_B^c \,. \tag{A.4}$$

Notice that the LO contribution to Γ_μ^a starts at second order in the ϕ_a, and hence plays no role in the expansion of $\mathcal{L}_U^{(2)}$ up to quadratic terms in H.

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