PROBING TRANSVERSE IMPEDANCES IN THE HIGH FREQUENCY RANGE AT THE CERN-SPS

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Abstract

The SPS transverse impedance model, which includes the major impedance contributions in the machine, can be benchmarked through measurements of the Head-Tail mode zero instability. Since the SPS works above transition energy, the Head-Tail mode zero is unstable for negative values of chromaticity. The measured instability growth rate is proportional to the real part of the transverse impedance. Studies performed after the LHC Injectors Upgrade (LIU) showed a relevant impedance around 2 GHz with high-gamma transition optics (Q26). This paper presents a comprehensive follow-up to probe the behavior of this beam coupling impedance contribution. Our studies include measurements of instability growth rates in both vertical and horizontal planes, spanning a broad spectrum of negative chromaticity values. To address the uncertainties in the high chromatic frequency range, the SPS Head-Tail monitor data is used to calculate the intrabunch chromatic content through a bidimensional frequency domain analysis.

INTRODUCTION

In accelerators operating above transition energy, the presence of negative chromaticity Q' leads to the occurrence of Head-Tail instability [1]. This instability is primarily driven by the non-zero machine impedance inherent to the accelerator system and is further magnified by the longitudinal oscillations performed by individual particles within the bunch [2]. The short-range wakefield produced by the leading part of a bunch (head) excites an oscillation of the trailing part (tail) of the same bunch, causing, after several turns, the characteristic exponential growth of the bunch's transverse centroid position [3].

The instability mode zero growth rate τ^{-1} is proportional to the real part of the effective driving impedance $Z_{\perp,\mathrm{dip}}^{\mathrm{eff}}$ as a function of relative chromaticity $\xi = Q'/Q$ [4]:

$$\tau^{-1}(\xi) = \pi^{-\frac{3}{2}} \frac{Re\left[Z_{\perp,\mathrm{dip}}^{\mathrm{eff}}(\xi)\right] N r_0 c^2}{8\pi^2 \gamma Q \sigma_z}, \tag{1}$$

where N is the number of protons per bunch, r_0 is the electron radius, c is the speed of light, γ is the relativistic factor, Q is the betatron tune and σ_z is the RMS bunch length.

Chromaticity can be expressed in terms of chromatic frequency $f_\xi = \xi Q f_{\mathrm{rev}}/\eta$. Therefore, studying the instability growth rate for a wide range of negative chromaticities provides information on the frequency dependence of the SPS transverse impedance. The smaller slip factor $\eta = 1/\gamma_t^2 - 1/\gamma^2$ of the high-gamma transition Q26 optics allows us to explore a wider spectrum of chromatic frequencies.

Pre-LS2 measurements of Head-Tail mode zero growth rates are reported in Ref. [5]. In the framework of the LHC Injectors Upgrade (LIU), a further benchmark of the present impedance model is required. Post-LS2 measurements reported in Ref. [6] showed a relevant impedance around 2 GHz. The trustworthiness of this impedance contribution was hampered by scarce data and the absence of consistent chromaticity measurements at highly negative values. In these studies, the measurements were replicated in the vertical plane and extended to the horizontal plane with a finer negative chromaticity scan. Finally, SPS Head-Tail monitor data is used to estimate the negative chromaticity values, in order to mitigate the uncertainty in the high-frequency regime.

GROWTH RATE MEASUREMENTS

The measurements were performed with a single-bunch beam with low intensity $2.8 \cdot 10^{10}$ p/b. Prior to performing the chromaticity scans, the tunes were set to the nominal operation values, measured, and adjusted with the SPS Laslett correction tool [7], the octupole strengths (LOF and LOD) were set to zero, and the transverse feedback was disabled. Chromaticity measurements were performed with the MultiQ procedure [8] to find the small offset between the set value Q'_{knob} and the real machine chromaticity, finding $\Delta Q'_V = 0.07$ and $\Delta Q'_H = 0.14$. The chromaticity function is defined following the methodology established in Ref. [6], applying a delay of 1000 ms to avoid sextupole current hysteresis and injection oscillations.

Figure 1 shows the main beam properties observed during the measurements. When a negative chromaticity value is applied at 1000 ms, the beam becomes unstable, showing sharp intensity losses (Fig. 1, upper-left). If the scan of negative chromaticity is performed in the vertical plane, the bunch vertical centroid position exhibits the expected exponential growth (upper-right). However, if coupling is present between transverse planes, some exponential growth can be observed also in the horizontal plane. To obtain the growth rate from the centroid turn-by-turn positions, the Moving Window Fourier Transform (MWFT) was used, as described in Refs. [1] and [6]. The final value of the growth rate τ^{-1}

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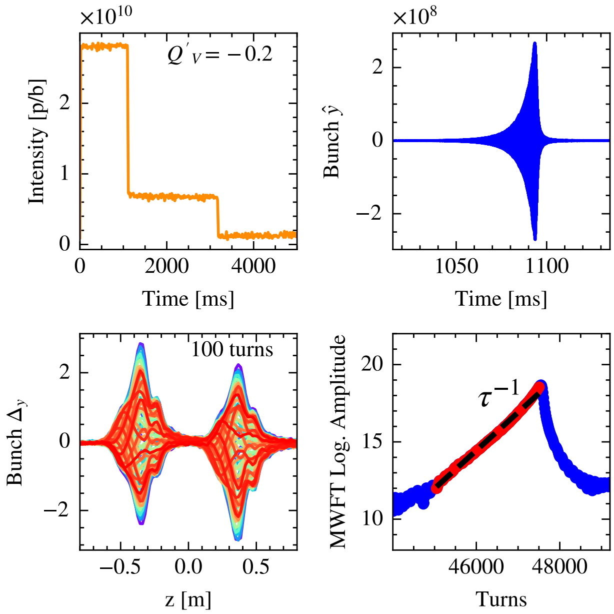

Figure 1: Head-Tail mode-zero instability observables. (upper-left) Bunch intensity over time. (upper-right) bunch centroid position over time. (lower-left) Intrabunch motion. (lower-right) linear fit slope of MWFT analysis gives the instability growth rate τ^{-1}.

is given by a linear fit of the MWFT logarithmic amplitudes of each transformed window (lower-right). Throughout the negative chromaticity scan, the SPS Head-Tail monitor and Head-Tail viewer [9] were used to visualize the mode zero pattern (Fig. 1, lower-left) described by the intrabunch particle trajectories.

CHROMATICITY ESTIMATION FROM HEAD-TAIL MOTION

Chromaticity is defined as the difference in a particle's tune from the nominal tune due to a difference in its energy from the nominal energy $\Delta Q = Q'\Delta p/p$. Thus, the conventional method to measure positive chromaticities is based on RF modulation of the beam energy [8]. For chromaticity below zero, the beam is unstable, and the conventional measurement gives non-reproducible results for $Q' < -0.3$ [10]. As an alternative for unstable beams, chromaticity can be deduced from the instability's intrabunch motion using the Head-Tail phase shift technique [9, 11].

The longitudinal motion (change in $\Delta p/p$) is coupled via chromaticity Q' to the transverse motion (change in tune Q). When Q' is non-zero, the particle oscillations show a betatron phase modulation that can be described as [12]:

$$y(n) = A\cos\left[2\pi n Q + \overbrace{Q'\frac{\omega_0\hat{\tau}}{\eta}}^{\Delta\psi_{HT}} \cdot\{\cos(2\pi Q_s n) - 1\}\right], \quad (2)$$

where $\omega_0 = 2\pi f_{rev}$, Q_s is the synchrotron tune, and $\hat{\tau}$ the position of a given slice within the bunch. This phase modulation is translated to a bunch longitudinal crabbing, whose angle depends on the chromaticity value.

Figure 2 shows Head-Tail monitor measurements for different negative chromaticity values. The color code corresponds to the bunch horizontal offset Δ_H amplitude, where the increasing crabbing angle due to chromaticity is clearly observed. In order to retrieve the chromaticity-dependent phase modulation of the grating, a 2D Fourier analysis can be performed [13], as shown in Fig. 3. The offset of the fundamental peak located at the betatron frequency determines the Head-Tail phase shift $\Delta\psi_{HT} = Q'\frac{\omega_0}{\eta}$, which is directly proportional to the chromaticity.

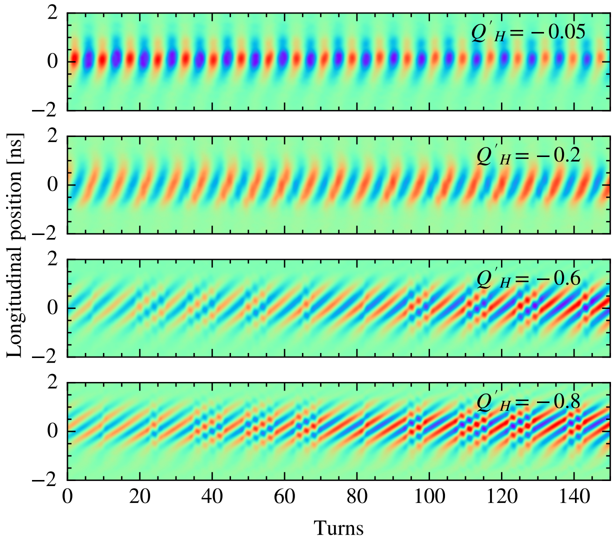

Figure 2: Head-Tail monitor measurements for different negative chromaticity values over 150 turns.

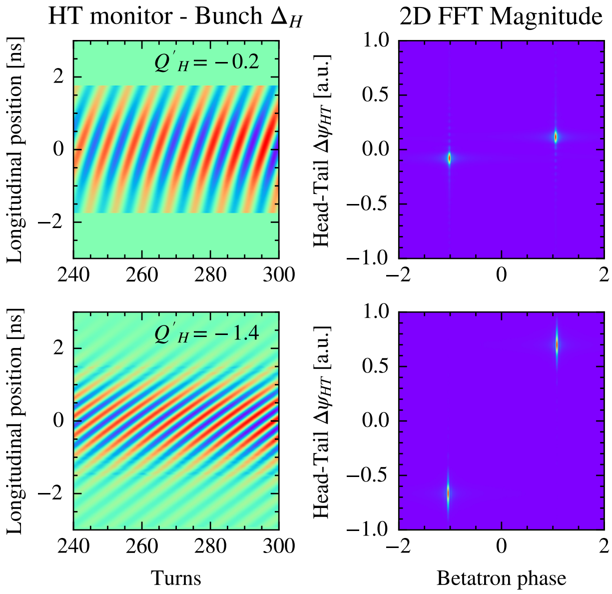

Figure 3: Chromaticity estimation from intrabunch motion using a 2D Fourier analysis.

COMPARISON WITH SIMULATIONS

Simulations of the instability growth rates in the presence of the latest Q26 SPS transverse impedance model's wake [14, 15], for a wide range of negative chromaticities, have been matched to beam observations using the PyHEADTAIL macroparticle tracking code [16]. RF modulation technique based measurements of non-linear chromaticity coefficients Q'', Q''' were conducted shortly before our studies, finding an average $Q''_H = (61 \pm 0.5) \cdot 10^2$ and $Q'''_H = (-5.5 \pm 1.0) \cdot 10^4$ for the horizontal plane, and $Q''_V = (-54 \pm 1.6) \cdot 10^2$ and $Q'''_V = (33 \pm 2.7) \cdot 10^4$ for the vertical plane.

Measurements and simulations for the horizontal plane are shown in Fig. 4. The first set of measurements was conducted with $2.8 \cdot 10^{10}$ p/b. However, for $Q'_H < -0.6$, the instability exponential growth rate was heavily degraded and the Head-Tail monitor could not capture the mode-zero motion. To enhance the amplitude of the bunch's centroid motion, a second set of measurements were conducted at higher intensity $8.5 \cdot 10^{10}$, where the instability could be observed up to $Q'_H = -1.4$. Simulations using the linear (Q'', $Q''' = 0$) and non-linear chromaticity models are shown in dotted and solid lines, respectively, the latter having a stronger impact in the lower intensity set of results. The shaded area on the simulation curves shows the uncertainty of the MWFT's linear fit that yields the growth rate.

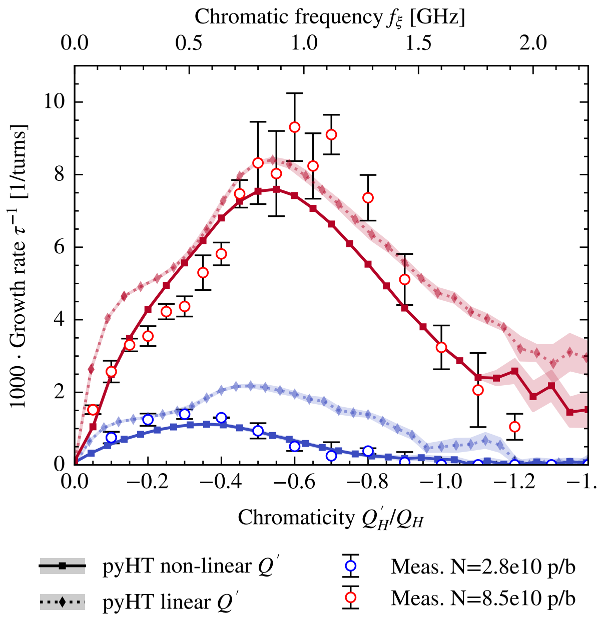

Figure 4: Comparison of PyHEADTAIL simulations with measurements for the horizontal plane.

For the vertical plane, the new set of measurements taken in 2023 and previous 2022 measurements are compared with simulations in Fig. 5. Two sets of finer scans were performed over different days, to confirm the results on the high chromatic frequency range. The discrepancy around 2.0 GHz is confirmed and hints to a possible impedance contribution that is not present in the current impedance model. Such an impedance could be modeled by a broadband resonator with $R_s/Q = 2 \cdot 10^5\ \Omega$ located at a frequency around $f_R = 2.3$ GHz, as introduced in Ref. [6].

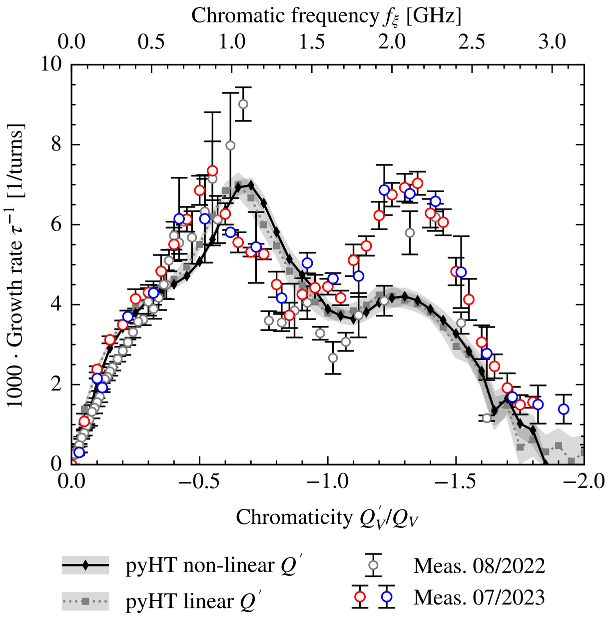

Figure 5: Comparison of PyHEADTAIL simulations with measurements for the vertical plane.

CONCLUSIONS

As part of the LHC-LIU initiative, a further benchmarking of the current transverse SPS impedance model was conducted through reference impedance measurements of the mode-zero Head-Tail instability in both transverse planes, extending to high chromatic frequencies with the Q26 optics. To address the uncertainty of negative chromaticity values, we employed a 2D Fourier analysis to measure the Head-Tail phase shift. Comparing our measurements with PyHEADTAIL simulations, incorporating the latest Q26 transverse wake and non-linear chromaticity model, we found good agreement in the horizontal plane. However, in the vertical plane, measurements confirmed a discrepancy around 2.0 GHz, consistent with previous findings [6]. Future work will involve dedicated studies aimed at refining the existing impedance model in the high-frequency regime.

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