Constraining the time evolution of the propagation speed of gravitational waves with multimessenger astronomy

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Several modified gravity theories predict a possible time variation of the propagation speed of gravitational waves (GW) which could be tested with multimessenger astronomy. For this purpose we derive the relation between the redshift dependence of the propagation speed of GWs and the time delay between the detection of GWs and electromagnetic waves (EMWs) emitted by the same source. For theories with Einstein frame minimal matter-gravity coupling (EMC) the propagation speed of GWs can be jointly constrained by the time delay between GWs and EWs and the GW-EMW luminosity distance ratio, allowing to derive a consistency relation between these two observables. The event GW 170817 and its EM counterpart satisfy the consistency condition, confirming the EMC, and allow to set strong constraints on the time variation of the GWs speed.

I. INTRODUCTION

The detection of gravitational waves (GWs) [1] by the Laser Interferometer Gravitational Wave Observatory (LIGO) and Virgo, has started the era of gravitational multi-messenger astronomy. GW events with an electromagnetic counterpart [2], also knwon as bright sirens, are used to test modified gravity theories (MGT) [3–7] which predict a difference between the propagation speed of the GW and electromagnetic waves (EMW). The event GW170817 set tight constraints on the GW-EMW speed difference, but these constraints were derived assuming a constant speed difference, while the effective field theory approach [8, 9] predicts a possible time-dependence. Moreover, a frequency-dependence of the speed of GWs can arise in bigravity models, in Lorentz violating gravity models, as well as in theories of quantum gravity where the spectral dimension of spacetime changes with the probed scale. A general effective approach to study GWs was developed in [10, 11], showing that the speed of GWs could also depend on polarization.

In what follows, we go beyond the constant GW-EMW difference approximation, and derive new relations for the time delay between the detection of GWs and EMWs emitted by the same source, as well as the redshift dependence of the GW-EMW speed difference.

For theories with Einstein frame minimal matter-gravity coupling (EMC) the GW speed can be jointly constrained by the GW-EMW time delay and the GW-EMW luminosity distance ratio, allowing to derive consistency relations between these two observables. A violation of this consistency relation could support a non-minimal coupling of matter to the Einstein frame metric, as for instance in Chameleon models, allowing to test such theories with multimessenger astronomy.

II. TIME DELAY CONSTRAINTS ON GW-EMW PROPAGATION SPEED DIFFERENCE

The relation between the time delay and the GW-EMW speed difference can be derived by considering that the GW and EM waves travel the same comoving distance between the source and the observer [12–15],

$$
\begin{aligned}
r_{\rm GW} &= \int_{z_{\rm o,GW}}^{z_{\rm s,GW}} \frac{v_{\rm GW}}{H(z')}{\rm d}z' \\
&= \int_{z_{\rm o,EM}}^{z_{\rm s,EM}} \frac{v_{\rm EM}}{H(z')}{\rm d}z' = r_{\rm EM}\,. \qquad (1)
\end{aligned}
$$

In the above equation we have distinguished between the GW and EMW redshift because if the corresponding speeds are different the consequent detection time delay would induce a difference in the redshift, and could also account for a possible emission time delay.

In what follows we adopt the notation $\Delta z_{\rm s} \equiv z_{\rm s,GW} - z_{\rm s,EM}$, $\Delta z_{\rm o} \equiv z_{\rm o,GW} - z_{\rm o,EM}$, and set $z_{\rm GW} \equiv z_{\rm s,GW}$, $z_{\rm EM} \equiv z_{\rm s,EM}$, $z_{\rm o,EM} = 0$. Assuming photons propagate along null geodesics, we can express time intervals in terms of redshift intervals using the equation

$$
\begin{aligned}
{\rm d}t &= -\frac{{\rm d}z}{(1+z)H(z)} \\
&\approx -\frac{{\rm d}z}{(1+z_{\rm GW})H_0(1+\frac{3}{2}\Omega_m z)}\,, \qquad (2)
\end{aligned}
$$

where in the second equality we have assumed a flat ΛCDM model, and made the low-redshift approximation $H(z) \approx H_0(1+\frac{3}{2}\Omega_m z)$, giving

$$
\begin{aligned}
\Delta t_{\rm o} &= \frac{\Delta z_{\rm o}}{H_0}, \\
\Delta t_{\rm s} &= \frac{\Delta z_{\rm s}}{(1+z_{\rm GW})H_0(1+\frac{3}{2}\Omega_m z_{\rm GW})}\,, \qquad (3)
\end{aligned}
$$

with the definitions $\Delta t_{\rm s} \equiv t_{\rm s,EM} - t_{\rm s,GW}$ and $\Delta t_{\rm o} \equiv$

$t_{o,EM} - t_{o,GW}$, where Δt_s is the emission time delay at the source, while Δt_o is the observed time delay.

A. Constant case

In the low-redshift approximation $H(z) \approx H_0$, using eq.(1) we get

$$
\begin{aligned}
r_{GW} &= \frac{(z_{GW} - \Delta z_o)\,(\Delta v + v_{EM})}{H_0}\,, \\
r_{EM} &= \frac{v_{EM}(z_{GW} - \Delta z_s)}{H_0}\,,
\end{aligned} \tag{4}
$$

where we have assumed $\Delta v \equiv v_{GW} - v_{EM}$ to be constant. Since $r_{GW} = r_{EM}$, solving for Δv yields

$$\Delta v = \frac{(\Delta z_o - \Delta z_s) v_{EM}}{z_{GW}}\,. \tag{5}$$

At low-redshift, the EM luminosity distance is

$$D_L^{EM}(z_{EM}) = (1+z_{EM})\, r_{EM}(z_{EM}) \approx (1+z_{GW}) \frac{v_{EM}}{H_0} z_{GW}\,, \tag{6}$$

allowing to rewrite eq.(5) as

$$\Delta v \approx \frac{(1+z_{GW})(\Delta z_o - \Delta z_s) v_{EM}^2}{H_0 D_L^{EM}}\,. \tag{7}$$

From eqs.(3) and (7) we get [15] the relation between the speed difference and the time delay

$$\frac{\Delta v}{v_{EM}^2} = \frac{1+z_{GW}}{D_L^{EM}}\left[\Delta t_o - (1+z_{GW})\Delta t_s \left(1 + \frac{3}{2}\Omega_m z_{GW}\right)\right]. \tag{8}$$

B. Time-dependent case

The derivation given in the previous sub-section can be used to study the case of a redshift dependent GW propagation speed. This can be accomplished by substituting in eq.(1) a linear function

$$v_{GW} = v_{EM} + \Delta v(z) = v_{EM} + z\Delta v_1\,, \tag{9}$$

where Δv_1 is the GW speed redshift variation, giving the comoving distance traveled by a gravitational wave, as

$$r_{GW} = \frac{(z_{GW} - \Delta z_o)\,[2 v_{EM} + \Delta v_1(\Delta z_o + z_{GW})]}{2H_0}\,. \tag{10}$$

Since $r_{EM} = r_{GW}$, eqs.(4b) and (10) imply

$$\Delta v_1 \approx \frac{2(\Delta z_o - \Delta z_s) v_{EM}}{z_{GW}^2}\,. \tag{11}$$

which using eq.(6) yields

$$\Delta v_1 = \frac{2(z_{GW}+1)^2(\Delta z_o - \Delta z_s) v_{EM}^3}{(D_L^{EM} H_0)^2}\,. \tag{12}$$

Using eq.(3) we obtain the following expression for Δv_1 in terms of the time delay

$$
\begin{aligned}
\Delta v_1 = &\frac{2(z_{GW}+1)^2\; v_{EM}^3}{(D_L^{EM})^2 H_0} \\
&\times \left[\Delta t_o - (1+z_{GW})\Delta t_s (1 + \frac{3}{2}\Omega_m z_{GW})\right].
\end{aligned} \tag{13}
$$

Note that eq.(5) and eq.(11) imply $\Delta v_1 = 2\Delta v / z_{GW}$, a relation useful to check the constraints obtained from observations.

III. EINSTEIN VS. JORDAN FRAME FOR THE MATTER-GRAVITY COUPLING

A modified gravity theory with Jordan frame matter-coupling (JMC) can be formulated in the Jordan frame using a Lagrangian of the form

$$\mathcal{L}_{JMC} = \sqrt{g_J}\left[\Omega^2 R_J + L_J^{MG} + L_J^{matter}(g_J)\right]\,, \tag{14}$$

where L^{MG} and L^{matter} are respectively the modified gravity and matter Lagrangians. After a conformal transformation $g_E = \Omega^2 g_J$ the same theory can be formulated in the Einstein frame with Lagrangian

$$\mathcal{L}_{JMC} = \sqrt{g_E}\left[R_E + L_E^{MG} + L_E^{matter}(\Omega^{-2} g_E)\right]\,, \tag{15}$$

showing that JMC theories are non-minimally coupled to the Einstein frame metric. Examples of theories belonging to the JMC class are for instance the Chameleon field model [16], and Horndeski theories with Jordan frame coupling.

In EMC theories, matter is minimally coupled to the Einstein frame metric Lagrangian

$$\mathcal{L}_{EMC} = \sqrt{g_E}\left[R_E + L_E^{MG} + L_E^{matter}(g_E)\right]\,, \tag{16}$$

which in the Jordan frame takes the form

$$\mathcal{L}_{EMC} = \sqrt{g_J}\left[\Omega^2 R_J + L_J^{MG} + L_J^{matter}(\Omega^2 g_J)\right]\,, \tag{17}$$

showing that EMC theories are non-minimally coupled to the Jordan frame metric.

Note that EMC and JMC theories can both be studied in the either the Einstein or Jordan frame as shown above, and this has no effect on the prediction of physically observable quantities, while the type of matter coupling is observationally relevant, and leads to different predictions for the GW-EMW distance ratio.

For JMC theories the GW-EMW distance ratio $r(z)$ is [17]

$$r(z) = \frac{D_{\rm L}^{\rm GW}}{D_{\rm L}^{\rm EM}} = \frac{M_*(0)}{M_*(z)}\,, \quad (18)$$

where M_* is the effective Planck mass, while for EMC theories

$$r(z) = \frac{D_{\rm L}^{\rm GW}}{D_{\rm L}^{\rm EM}} = \frac{v_{\rm GW}(z)}{v_{\rm GW}(0)}\,. \quad (19)$$

For EMC theories eq.(19) allows to set constraints on the redshift evolution of the GW propagation speed using distance observations, which can be combined with the constraints from the time delay. These joint constrains allow to derive a consistency relation to test the minimal coupling of matter to the Einstein frame metric.

IV. CONSISTENCY RELATION FOR EINSTEIN FRAME MINIMAL COUPLING

Equation (19) implies that for EMC theories the GWs speed can be constrained by distance ratio observations

$$r(z) = \frac{D_{\rm L}^{\rm GW}}{D_{\rm L}^{\rm EM}} = \frac{v_{\rm GW}(z)}{v_{\rm GW}(0)} \approx 1 + \frac{z\Delta v_1}{v_{\rm EM}}\,, \quad (20)$$

where in the last equality we have assumed $\Delta v(z) \ll v_{\rm EM}$. Using eq.(6) we get

$$\Delta v_1 = \frac{(r-1)(z_{\rm GW}+1)v_{\rm EM}^2}{D_{\rm L}^{\rm EM} H_0}\,. \quad (21)$$

Combining the distance ratio (eq.(20) and time delay (eq.(11)) relations, we obtain the EMC consistency relation (CR)

$$\mathcal{E}(z_{\rm GW}) \equiv r(z_{\rm GW}) - \frac{2(\Delta z_o - \Delta z_{\rm s})}{z_{\rm GW}} = 1\,, \quad (22)$$

which using eq.(3) can be expressed in terms of time intervals as

$$\mathcal{E}(z_{\rm GW}) \equiv r(z_{\rm GW}) - \frac{2(1+z_{\rm GW})v_{\rm EM}}{D_{\rm L}^{\rm EM}} \times \left[\Delta t_{\rm o} - (1+z_{\rm GW})\Delta t_{\rm s}(1+\frac{3}{2}\Omega_m z_{\rm GW})\right] = 1. \quad (23)$$

As expected, for General Relativity the CR is satisfied, since $r(z)=1$ and $\Delta t_{\rm o} = (1+z)\Delta t_{\rm s}$. For any MGT in which matter is minimally coupled to the Einstein frame metric the CR should be satisfied. On the contrary, a violation of the CR would imply that that matter is not minimally coupled to the Einstein metric, such as for example in Chameleon fields theories [16]. For theories in which matter is minimally coupled to the Jordan frame metric, the distance ratio [17] depends on the effective Planck mass ratio, and the CR derived assuming Einstein frame metric minimal coupling does not hold.

V. OBSERVATIONAL CONSTRAINTS FROM GW170817

We will apply the theoretical results obtained above to constrain Δv_1 and Δv for different types of theories. Equation (13) and eq.(8) are valid for both EMC and JMC theories, while the consistency condition is only valid for EMC theories. Adopting an approach similar to [18], we assume that the short-duration gamma-ray burst (GRB) signal was emitted 10 s after the GW signal, and use the observed time delay $(+1.74 \pm 0.05)$ s between the GRB 170817A and the GW170817 event [19]. Following the conservative approach adopted in [18], we consider the lower bound of the 90% confidence interval of the gravitational wave luminosity distance $D_{\rm L}^{\rm GW} = 26$ Mpc, assume $D_{\rm L}^{\rm EM} = D_{\rm L}^{\rm GW}$, and hence determine the redshift $z_{\rm GW} = 0.006$ using the ΛCDM cosmological parameters given by the Planck mission [20]and consequently obtain the constraints shown in the first column of Table I. Note the constraint on Δv from $D_{\rm L}^{\rm GW}$ and $\Delta t_{\rm o}$ is the same as the one obtained in [18] without including the redshift effects, since at low redshift they are negligible.

Since optical follow-up observations have allowed to identify with high confidence the host galaxy as NGC 4993 [21], the redshift can be obtained directly, and from it one gets the EM luminosity distance. Using $z_{\rm EM} = 0.01$ and the same cosmological parameters we obtain the constraints listed in the second column of Table I. Note that the constraints using $z_{\rm EM}$ are better, since EM observations allow a high precision redshift measurement, and consequently a precise estimation of the EM luminosity distance $D_{\rm EM} \approx 44$ Mpc, while the gravitational luminosity distance has a larger uncertainty, giving the smaller 90% lower confidence value 26 Mpc.

For EMC theories there are two constraints on Δv_1, which can be combined together in the consistency condition for $\mathcal{E}(z)$ given in eq.(23). Since for these theories the GW-EMW distance ratio can constrain independently the GW speed, we cannot assume $D_{\rm L}^{\rm EM} = D_{\rm L}^{\rm GW}$, or this would introduce a bias. Assuming the GW event to have been hosted by NGC 4993 we can obtain $D_{\rm L}^{\rm EM}$ from it, while for $D_{\rm L}^{\rm GW}$ we take the 90% confidence interval, obtaining

$$0.56 < \mathcal{E}(z) < 1.05\,. \quad (24)$$

The constraint is consistent with matter minimally coupled to the Einstein frame metric, i.e. there is no evidence of a violation of the strong equivalence principle.

	$\Delta t_o + D_{GW}$	$\Delta t_o + z_{EM}$
Δv	$-3\times10^{-15} < \frac{\Delta v}{v_{EM}} < 7\times10^{-16}$	$-2\times10^{-15} < \frac{\Delta v}{v_{EM}} < 4\times10^{-16}$
Δv_1	$-1\times10^{-12} < \frac{\Delta v_1}{v_{EM}} < 2\times10^{-13}$	$-4\times10^{-13} < \frac{\Delta v_1}{v_{EM}} < 8\times10^{-14}$

TABLE I. Constraints on the GW speed from GW 170817 and its electromagnetic counterpart observations. The first column corresponds to the constraints from the observed time delay Δt_o and the gravitational luminosity distance, assuming $D_L^{EM} = D_L^{GW}$, as in [18] . The second column corresponds to the constraints from Δt_o and the electromagnetic counterpart redshift z_{EM}, computing D_{EM} using the ΛCDM model with the best fit parameters of [20].

VI. CONCLUSIONS AND DISCUSSION

The expected improved sensitivity of future GW detectors will allow the observation of an increasing number of bright sirens, leading to model independent constraints on the time variation of the speed of GWs. Using the observations of the event GW 170817 and its EM counterpart we have obtained strong constraints on the variation of the speed of GWs from the speed of light for frequencies at the kHz range.

An independent way to test modified gravity models is provided through the EMC consistency condition, eq.(23), and we found that the event GW 170817 does not violate the consistency condition, i.e. there is no evidence of non minimal coupling to the Einstein frame metric, or equivalently, of coupling to the Jordan frame metric.

We have provided a way to test the type of matter-gravity coupling using the CR. Violation of the consistency condition would support a non-minimal coupling of matter to the Einstein frame metric, such as in Chameleon field theories or other theories with matter coupled to the Jordan frame metric, and in this case the GW-EMW luminosity distance ratio could be used to probe the time variation of the effective Planck constant [17].

Finally, let us note that in deriving of eq.(13), eq.(8) and eq.(23) we have made some low-redshift approximations, which may not be justified for higher redshift observations. In this case higher order redshift expansions can be used, or eq.(1) can be integrated numerically.

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