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V.M. Korablev, L.M. Tkachenko

CALCULATION OF MAGNETIC FIELDS USING SURFACE CURRENTS AND LAW OF FULL CURRENT CONSERVATION

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Abstract

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A method of magnetic fields calculation using the boundary integral equations has been realized. The total field of magnetic induction $\vec{B}$ in an arbitrary point is presented as the sum of two components. The current field $\vec{B}_c$ is defined by Biot-Savart law. The contribution $\vec{B}_s$ due to the iron surface currents is calculated by the boundary integral equations method involving the law of full current conservation to take into account the volume magnetic flux distribution.

Аннотация

Кораблев В. М., Ткаченко Л. М. Вычисление магнитных полей с помощью поверхностных токов и закона сохранения полного тока.: Препринт ИФВЭ 95-44. – Протвино, 1995. – 8 с., 4 рис., библиогр.: 5.

Реализован метод вычисления магнитных полей с использованием граничных интегральных уравнений. Полное поле магнитной индукции $\vec{B}$ в произвольной точке представляется в виде суммы двух слагаемых. Токовая составляющая поля $\vec{B}_c$ определяется по закону Био-Савара. Вклад $\vec{B}_s$, обусловленный поверхностными токами магнитопровода, вычисляется посредством метода граничных интегральных уравнений с привлечением закона сохранения полного тока для учета распределения магнитного потока по объему магнетика.

INTRODUCTION

The volume integrals method (VIM) for the iron magnetization realized in the codes GFUN [1] and MULTIC [2] is more widely spread today in the three–dimensional magnetic field calculation tasks.

But there exist some magnet configurations, i.e., iron core with small air gaps, where the standard VIM method encounters difficulties. These difficulties have stimulated the authors to realize the method of magnetic field calculation with surface currents and the law of full current conservation (SCM), that allows one to calculate the magnetic field closer to the real one.

1. GENERAL CONSIDERATIONS

When an iron sample is placed in a magnetic field there arise molecular surface currents due to iron magnetization [3]. If we divide the iron surface into the definite number of elements, the tangential component of the total magnetic field in the iron for every element can be presented as

$$\vec{B}_{iron} = \vec{B}_c + \vec{B}_{ex} + \vec{B}_{int} \tag{1}$$

and in the air as

$$\vec{H}_{air} = \vec{H}_c + \vec{H}_{ex} - \vec{H}_{int}. \tag{2}$$

Here $\vec{B}_c$, $\vec{H}_c$ are magnetic fields from conductor, $\vec{B}_{ex}$, $\vec{H}_{ex}$ are magnetic fields from surface currents of other elements, $\vec{B}_{int}$ and $\vec{H}_{int}$ are magnetic fields from surface current of the given element.

From the tangential magnetic field conservation, we have

$$\vec{B}_{iron} = \vec{B}_c + \vec{B}_{ex} + \vec{B}_{int} = \mu\vec{H}_{air} = \mu(\vec{H}_c + \vec{H}_{ex} - \vec{H}_{int}). \tag{3}$$

Under the vacuum consideration of Eqs. (1)—(3) $\vec{B}_c = \vec{H}_c$, $\vec{B}_{ex} = \vec{H}_{ex}$, $\vec{B}_{int} = \vec{H}_{int}$ and Eq. (3) can be transformed to

$$\vec{B}_{int} = \frac{\mu - 1}{\mu + 1}(\vec{B}_c + \vec{B}_{ex}), \tag{4}$$

which connects internal and external tangentional field components.

To define the iron magnetic flux distribution, we apply the law of full current conservation:

$$\oint \frac{1}{\mu(l')} \left(\vec{B}(l') \cdot \vec{dl'}\right) = I, \tag{5}$$

that looks like the normalization factor and keeps right the B solution.

Let the two-component vector $\vec{\eta}_j = (\eta_{xj}, \eta_{yj})$ be the surface current density of j element in the intrinsic frame of reference, A_j is the transformation matrix from j element to space and C_j is the back transformation, then the three-dimensional space magnetic field $\vec{B}_j$ can be expressed through two-component current $\vec{\eta}_j$ as

$$\vec{B}_j = A_j \int\limits_{S_j} \frac{[\vec{\eta}_j \times (\vec{R} - \vec{R}')]}{|\vec{R} - \vec{R}'|^3}\, dS'_j. \tag{6}$$

Magnetic field from j element to i element looks like

$$\vec{B}_{ij} = C_i A_j \int\limits_{S_j} \frac{[\vec{\eta}_j \times (\vec{R} - \vec{R}')]}{|\vec{R} - \vec{R}'|^3}\, dS'_j. \tag{7}$$

If we return to Eq. (4) the connection between intrinsic surface current and all others in i element can be written as

$$\int\limits_{S_i} \frac{[\vec{\eta}_i \times (\vec{R} - \vec{R}')]}{|\vec{R} - \vec{R}'|^3}\, dS'_i = C_i \frac{\mu_i - 1}{\mu_i + 1} \left[\vec{B}_{ci} + \sum_{j \neq i} A_j \int\limits_{S_j} \frac{[\vec{\eta}_j \times (\vec{R} - \vec{R}')]}{|\vec{R} - \vec{R}'|^3}\, dS'_j \right]. \tag{8}$$

Using for the fit procedure only the tangential two–components part of Eq. (8) we get the system of $2N$ equations and $2N$ unknowns (η_{xi}, η_{yi}).

Eq. (5) allows one to include any number of additional connections for the law of full current conservation along arbitrary contour divided into k segments $\Delta\vec{l}_k$

$$\sum_k \frac{1}{\mu_k} \Delta\vec{l}_k \left(\vec{B}_{ck} + \sum_j A_j \int\limits_{S_j} \frac{[\vec{\eta}_j \times (\vec{R} - \vec{R}')]}{|\vec{R} - \vec{R}'|^3}\, dS'_j \right) = I. \tag{9}$$

If one assumes the surface current in the region of j element to be the constant vector and use m contours for the law of full current conservation, it is possible to solve the system of $2N + m$ equations relative to $2N$ unknowns (η_{xi}, η_{yi}) by the least squares method. New values (η_{xi}, η_{yi}) allow one to redefine the magnetic permeabilities μ_i and etc., up to the fit procedure convergence (fit details are in Appendix).

When magnetic permeability has a large gradient one can include additional internal iron surface elements in the large gradient region. Then instead of the coefficient $(\mu - 1)/(\mu + 1)$ in Eq. (8) we apply $(\mu_1 - \mu_2)/(\mu_1 + \mu_2)$, where μ_1 and μ_2 are the magnetic permeabilities on opposite sides of the internal surface element.

It is necessary to note that the choice of contours for the law of full current conservation, the amount of surface elements and their shape depend on the concrete magnet geometry. These aspects are the object for further methodical investigation.

2. COMPARISON OF THE SCM WITH VOLUME INTEGRALS METHOD

The basic idea of the VIM is the presentation of the magnetic field $\vec{H}$ as the sum of the field $\vec{H}_c$ due to a conductor and the field $\vec{H}_m$ due to a magnetization of the iron[3]

$$\begin{aligned} \vec{H}_i &= \vec{H}_{ci} + \sum_j \vec{H}_{mj}, \qquad (10) \\ \vec{H}_{mj} &= \frac{1}{4\pi} \int_V (\mu_j - 1) \frac{3[\vec{H}_j \cdot (\vec{R} - \vec{R}')](\vec{R} - \vec{R}') - (\vec{R} - \vec{R}')^2 \vec{H}_j}{|\vec{R} - \vec{R}'|^5} \, dV'. \end{aligned}$$

Sometimes for the tasks with full closed iron core and a weak external $\vec{H}_c$ field the expression

$$\mu_i \vec{H}_i = \mu_i \vec{H}_{ci} \qquad (11)$$

is in a closer egreement to the real one, than (10). Therefore, when comparing the SCM with the standard VIM we present both (10) and (11) fit results.

As the examples for the comparison we have taken the magnetic field calculations by both methods for the magnets with two arm spectrometer FODS [4] and with [5], where the VIM meets with difficulties.

The first magnet has the total current 324 kA, external iron core parameters $300 \times 252 \times 220$ cm^3. The variable air gap along a beam in both arms has the weighted average value 31 cm which is 7.4% from the minimal contour around the conductor.

The average value of the measured magnetic field in the air gap is 12.81 kGs. The VIM with 137 volume elements in the one quadrant gives 9.44 kGs, from the solution of Eq. (11) we have 16.71 kGs and the SCM with 105 surface elements and three contours (each divided on 33 segments) in the one quadrant gives 13.01 kGs.

The second magnet has the current 6 kA, external iron core parameters $200 \times 120 \times 50$ cm^3 (Fig.1). We have artificially inserted different air gaps (Fig.2) to demonstrate the magnetic field dependence on the value of air gap for both VIM and SCM. The VIM used for these calculations has 144 volume elements and the SCM has 72 surface elements and three contours (each divided on 48 segments) in the one octant.

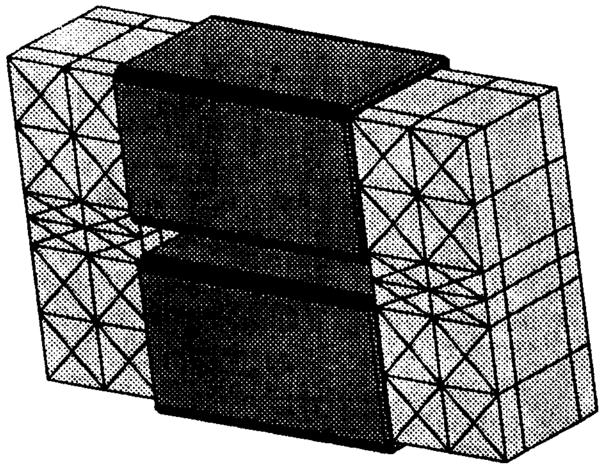

Fig. 1. The geometry of the magnet.

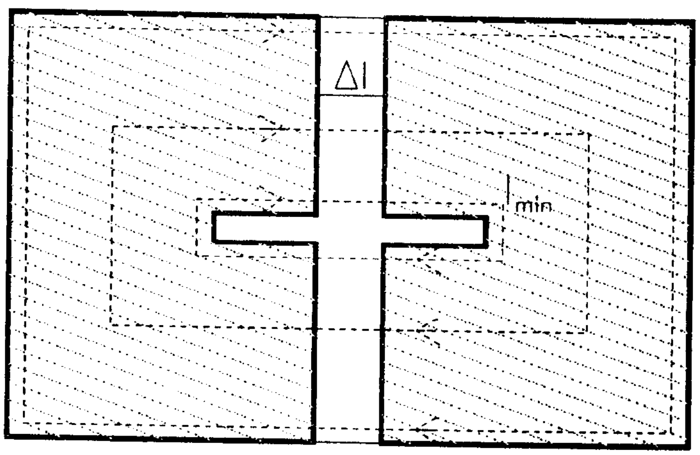

Fig. 2. The scheme of the air gap and contours for the law of full current conservation.

For this magnet, the magnetic field dependence on the air gap value for the central gap point is presented in Fig. 3, where the solid line stands for the SCM, the dotted line is for the VIM and the triangle markers are for Eq. (11). Normalization of air gap values was made on the minimal contour around the conductor ($Gap = 2\Delta l / l_{min} \times 100\%$).

Fig.4 shows the average iron core magnetization versus the air gap value.

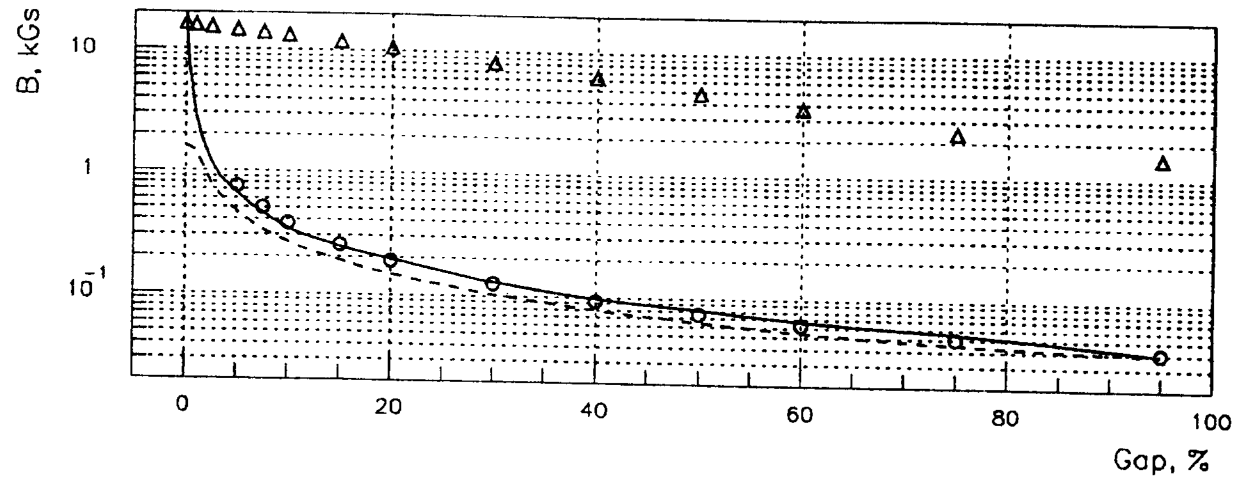

Fig. 3. The dependence of the magnetic field in the air gap on the air gap value.

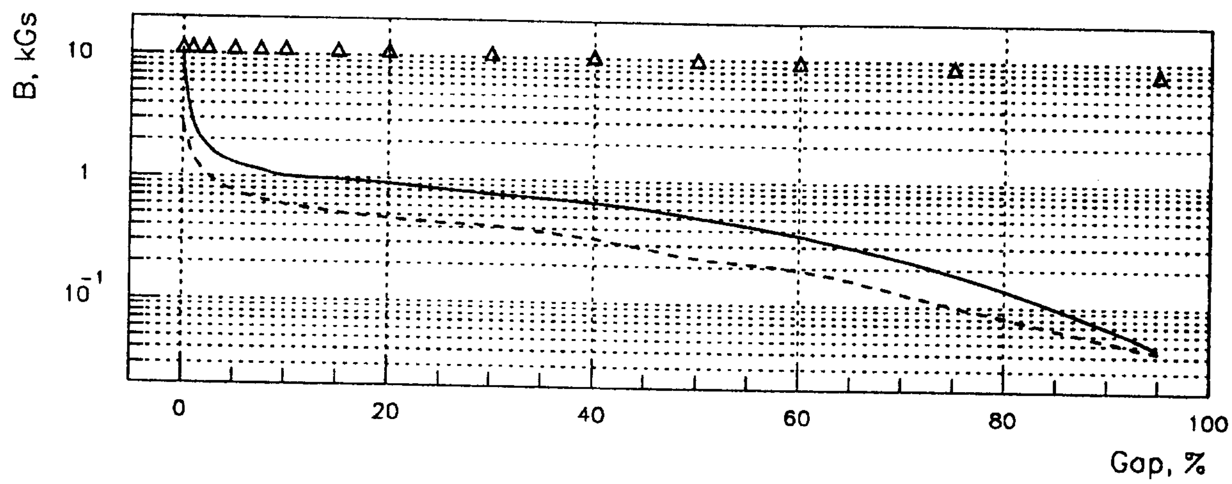

Fig. 4. The dependence of the average iron magnetization on the air gap value.

From comparison the SCM with the VIM we conclude that in the considered magnets standard the VIM gives the magnetic field that is systematically less, than the right value in the region of a small air gap (L_{gap}). At the same time the SCM gives a satisfactory result for any air gap and the magnetic field in the air (Fig.3) agrees, as expected, with the $1/L_{gap}$ — dependence (marked by circles on Fig.3) for $L_{gap} \ll L_{iron}/\langle\mu\rangle$. If $L = L_{gap} + L_{iron}$ is the total contour around a conductor, then

$$B \sim I/(L_{gap} + L_{iron}/\langle\mu\rangle) \sim I/L_{gap}, \qquad (12)$$

where $\langle\mu\rangle)$ is the average magnetic permeability along L_{iron}.

At zero air gap $B \sim \mu(I/L_{iron}) \sim \mu H_c$, that also agrees with the SCM calculation.

CONCLUSION

The results given in Section 2 demonstrate that the proposed method is capable of predicting the magnetic field, especially for the magnet configurations, where the standard VIM encounters some difficulties.

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Appendix 1

Here we present the integrals for the field calculation from the surface current density $\vec{\eta} = (\eta_x, \eta_y)$ in the intrinsic frame of reference, where Z is the normal to η direction.

If we direct radius vector $\vec{R}$ from the given space point to the surface element, the magnetic field in this point will be

$$\vec{B} = -\int_S \frac{[\vec{\eta} \times (\vec{R} - \vec{R}')]}{|R - R'|^3} dX'dY' \tag{13}$$

or

$$\begin{aligned} B_x &= -\eta_y Z \int_S \frac{dX'dY'}{|R - R'|^3}, \\ B_y &= \eta_x Z \int_S \frac{dX'dY'}{|R - R'|^3}, \\ B_z &= \eta_y \int_S \frac{(X - X')dX'dY'}{|R - R'|^3} - \eta_x \int_S \frac{(Y - Y')dX'dY'}{|R - R'|^3}. \end{aligned} \tag{14}$$

We replace the square integral on the contour integral using Green's formula and after integration we have

$$\begin{aligned} B_x &= \eta_y \sum_C \frac{Z}{|Z|}\frac{Y}{|Y|} \arctan \frac{|Y|X}{|Z|R}, \\ B_y &= -\eta_x \sum_C \frac{Z}{|Z|}\frac{Y}{|Y|} \arctan \frac{|Y|X}{|Z|R}, \\ B_z &= \eta_x \sum_C \alpha \log(X+R) - \eta_y \sum_C \beta \log(X+R). \end{aligned} \tag{15}$$

Here $\alpha = \Delta X/\Delta l$ and $\beta = \Delta Y/\Delta l$ are the direction cosines along the contour.

Let d_1, d_2, d_3 and d_4 denote the numerical values of integrals in expressions (15), then the magnetic field looks like

$$B_x = d_1\eta_y; \quad B_y = d_2\eta_x; \quad B_z = d_3\eta_x + d_4\eta_y. \tag{16}$$

For the matrix transformation from the surface element to space

$$A = \begin{pmatrix} X_x & X_y & X_z \\ Y_x & Y_y & Y_z \\ Z_x & Z_y & Z_z \end{pmatrix} \tag{17}$$

magnetic field in space looks like

$$\begin{aligned} B_x &= (d_2X_y + d_3X_z)\eta_x &+& (d_1X_x + d_4X_z)\eta_y &=& D_1\eta_x &+& D_4\eta_y, \\ B_y &= (d_2Y_y + d_3Y_z)\eta_x &+& (d_1Y_x + d_4Y_z)\eta_y &=& D_2\eta_x &+& D_5\eta_y, \\ B_z &= (d_2Z_y + d_3Z_z)\eta_x &+& (d_1Z_x + d_4Z_z)\eta_y &=& D_3\eta_x &+& D_6\eta_y. \end{aligned} \tag{18}$$

Thus, using six–component matrix D

$$D = \begin{pmatrix} D_1 & D_4 \\ D_2 & D_5 \\ D_3 & D_6 \end{pmatrix} \tag{19}$$

we can express the magnetic field in the space point from j two–component surface currents η_j and conductor current as

$$\vec{B} = \vec{B}_c + \sum_j D_j\vec{\eta}_j. \tag{20}$$

Now set the back transformation matrix from space to surface as

$$C = \begin{pmatrix} C_{xx} & C_{xy} & C_{xz} \\ C_{yx} & C_{yy} & C_{yz} \\ C_{zx} & C_{zy} & C_{zz}. \end{pmatrix} \tag{21}$$

For the tangential field connection it is enough to use only the first six–components

$$C = \begin{pmatrix} C_{xx} & C_{xy} & C_{xz} \\ C_{yx} & C_{yy} & C_{yz} \end{pmatrix} \tag{22}$$

and two–dimensional Eq. (8) for i element can be written as

$$C_i D_i \vec{\eta}_i = \frac{\mu_i - 1}{\mu_i + 1} C_i \left(\vec{B}_{ci} + \sum_{j \neq i} D_j \vec{\eta}_j \right). \tag{23}$$

One–dimensional Eq. (9) for the law of full current conservation looks like

$$\sum_k \frac{1}{\mu_k} \left[\Delta \vec{l}_k \cdot \left(\vec{B}_{ck} + \sum_i D_{ik} \vec{\eta}_i \right) \right] = I, \tag{24}$$

here index i means, that integral D_{ik} is calculated from the surface element i relative to the point in the center of Δl_k segment.

For Eq. (24) to be of the same scale as Eq. (23), it is reasonable to use the weight $w = \langle \mu / \Delta l \rangle$.

Now it is not difficult to transform $2N$ Eq. (23) and m Eq. (24) to the expression

$$\vec{A} = B \vec{\eta}, \tag{25}$$

where $\vec{A}$ is the $2N + m$ vector, B is the $(2N + m) \times 2N$ matrix and $\vec{\eta}$ is the vector of $2N$ unknowns.

Solving the system of equations by the least squares method we first find the approximation for η currents. Afterwards, we recalculate by formula (20) the magnetic field in every point involved in the fit procedure and redefine the magnetic permeability μ after i-th iteration as

$$\mu = k\mu_i + (1 - k)\mu_{i-1} \tag{26}$$

and repeat fit operation up to convergence:

$$\left| \frac{B_i - B_{i-1}}{B_i} \right|_{max} < \varepsilon. \tag{27}$$

Values k in (26) and ε in (27) dependent on the convergence behaviour in every concrete task.

В.М.Кораблев, Л.М.Ткаченко
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ГНЦ РФ Институт физики высоких энергий
142284, Протвино Московской обл.

Индекс 3649

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