Intermittent behaviour of bremsstrahlung photons produced in hadronic collisions at very high energies

Ján Pišút[a)b)], Neva Pišútová[b)] and Boris Tomášik[b)]

a) Laboratoire de Physique Corpusculaire, Université Blaise Pascal, Clermont-Ferrand, F-63177 Aubière, Cedex, France
b) Department of Physics, Comenius University, SK-842 15 Bratislava, Slovakia

Abstract

Hanbury-Brown and Twiss correlations of bremsstrahlung photons produced in hadronic collisions at very high energies show intermittent features. The phenomenon is due to large longitudinal dimensions of the space-time evolution of hadronic collisions and to the fact that rapidity of a bremsstrahlung photon need not be close to the rapidity of the charged particle radiating the photon.

Studies of the intermittent behaviour of hadrons produced in heavy ion collisions have been initiated by papers by Bialas and Peschanski [1], the present situation is reviewed e.g. in Refs.[2]. Detailed experimental studies of correlations of like- and un-like pions indicate [3] that Hanbury-Brown and Twiss (HBT) correlations [4] of identical pions are responsible for the observed intermittent behaviour.

The increased accuracy of experimental analyses is partly due to the use of technique of correlation integrals and star integrals by the Tucson-Vienna collaboration [5]. The state of art of HBT correlation can be found in Refs.[2, 6, 7]. The explanation of the intermittent behaviour has been sought either in cascade mechanisms [8] or in HBT correlations of particles produced by a source with fluctuating space-dimensions [9, 10]. To reech the agreement with experimental results one needs occasional fluctuations of the order of magnitude of 10 fm. Differences between correlations for like- and un-like pions make the latter mechanism more probable.

The purpose of the present note is to point out that the HBT correlation of bremsstrahlung photons emitted by final state charged particles in heavy ion collisions show the intermittent behaviour in the sence of Refs.[2, 9]. The main point is rather simple: the bremsstrahlung photons originated by charged hadrons during an inside-outside cascade proces can interfere even in the case when rapidities of their "parent" hadrons and consequently also the space-time distances of points of the "production" of hadrons, differ considerably.

We shall start with describing the HBT interference patterns of bremsstrahlung photons produced by final state hadrons during an inside-outside cascade proces and before concluding we shall make a few speculative comments on a possible relevance of this mechanism also for HBT correlations for pions.

The formalism for HBT correlations of bremsstrahlung photons has been described in our papers [11, 12], where standard amplitudes [13] for the radiation of bremsstrahlung photons by charged final state particles have been used. As explained in detail in [12] we are interested in bremsstrahlung radiation of charged hadrons emitted from the system which is complex enough to make the radiation of a photon by two different charged particles incoherent. In such a situation each charged hadron emitted from the system has a random phase factor $\exp(i\phi)$ and the phases connected with emissions of different hadrons are assumed to be random and uncorrelated.

The amplitude for the emission of single bremsstrahlung photon is then written

Using

$$\left\langle e^{i(\phi_j-\phi_l)}\right\rangle = \delta_{jl} \tag{3}$$

we obtain

$$P(k,\varepsilon) = \frac{1}{N}\sum_{j=1}^{N} Q_j^2\left(\frac{\varepsilon.p_j}{k.p_j}\right)^2 \tag{4}$$

The amplitude for emission of two photons consists of two terms

$$A(k_1,\varepsilon_1;k_2,\varepsilon_2) = \frac{1}{N}\left[\sum_l\sum_{j\neq l} e^{i\phi_j}e^{i\phi_l}e^{ik_1.x_j}e^{ik_2.x_l}Q_j\frac{\varepsilon_1.p_j}{k_1.p_j}Q_l\frac{\varepsilon_2.p_l}{k_2.p_l}\right.$$
$$\left. + \quad \sum_j e^{i\phi_j}e^{i(k_1+k_2).x_j}Q_j\frac{\varepsilon_1.p_j}{k_1.p_j}Q_j\frac{\varepsilon_2.p_j}{k_2.p_j}\right] \tag{5}$$

the former term corresponds to the radiation of photons by two different charged hadrons, the latter describes two photons radiated by the same charged hadron. The probability $P(k_1,\varepsilon_1;k_2,\varepsilon_2)$ is obtained after averaging over phases

$$P(k_1,\varepsilon_1;k_2,\varepsilon_2) = \left\langle |A(k_1,\varepsilon_1;k_2,\varepsilon_2)|^2\right\rangle \tag{6}$$

by using following rules

$$\left\langle e^{i(\phi_j-\phi_{j'})}\right\rangle = \delta_{jj'} \tag{7}$$

$$\left\langle e^{i(\phi_j+\phi_l-\phi_{j'})}\right\rangle = 0 \tag{8}$$

$$\left\langle e^{i(\phi_j+\phi_l-\phi_{j'}-\phi_{l'})}\right\rangle = \delta_{jj'}\delta_{ll'} + \delta_{jl'}\delta_{lj'} \tag{9}$$

The physics behind these rules is discussed in detail in Ref.[12].

The probability for radiation of two photons, obtained by using Eqs.(7-9) reads as follows

$$P(k_1,\varepsilon_1;k_2,\varepsilon_2)$$
$$= \frac{1}{N^2}\left[\sum_l\sum_{j>l}\left|e^{ik_1.x_j}e^{ik_2.x_l}Q_j\frac{\varepsilon_1.p_j}{k_1.p_j}\,Q_l\frac{\varepsilon_2.p_l}{k_2.p_l} + e^{ik_2.x_j}e^{ik_1.x_l}Q_j\frac{\varepsilon_2.p_j}{k_2.p_j}Q_l\frac{\varepsilon_1.p_l}{k_1.p_l}\right|^2\right.$$
$$\left. + \quad \sum_j\left|Q_j\frac{\varepsilon_1.p_j}{k_1.p_j}\,Q_j\frac{\varepsilon_2.p_j}{k_2.p_j}\right|^2\right] \tag{10}$$

The first term in the r.h.s. corresponds to a coherent sum of amplitudes for radiation of two photons by two charged particles. Apart of the "bremsstrahlung" factors $\varepsilon.p/k.p$ this is the expression well known in the HBT analyses. The four-vectors x_j and x_l denote the space-time points in which the final state hadrons 'j' and 'l' have been created. The second term in the r.h.s. of Eq.(10) corresponds to the probability of two photons radiated by the same charged particle. For large N this term will be suppressed with respect to the first one.

We shall now use expressions for $P(k_1, \varepsilon_1)$ and $P(k_1, \varepsilon_1; k_2, \varepsilon_2)$ as given by Eqs.(4) and (10) to calculate the correlation function

$$C(k_1, k_2) \equiv \frac{P(k_1, \varepsilon_1; k_2, \varepsilon_2)}{P(k_1, \varepsilon_1)P(k_2, \varepsilon_2)} \tag{11}$$

for a very simple model of the inside-outside cascade production of charged hadrons. To simplify the situation as much as possible, we shall work in the c.m.s. of the collision, denote the axis of the collision as the z-axis and we shall assume that

i) all charged particles in the final state are pions;

ii) transverse momenta of final state pions vanish;

iii) there exists a rigid correlation $y = \eta$ between the "momentum rapidity" y and "space-time rapidity" η

$$y = \frac{1}{2} \ln \frac{E+p}{E-p} \qquad \eta = \frac{1}{2} \ln \frac{t+z}{t-z} \tag{12}$$

iv) the radiation off a charged particle is neglected up to the moment when it appears in the final state;

v) the inside-outside cascade is characterized by a single parameter τ_0 and a pion appears as a final state particle when

$$t^2 - z^2 = \tau_0^2 \tag{13}$$

under these assumptions a pion with rapidity y_i appears in the final state in the space-time point

$$t_i = \tau_0 \cosh y_i \qquad z_i = \tau_0 \sinh y_i \tag{14}$$

its velocity being

$$v_i = \tanh y_i \tag{15}$$

We are interested in the probabilities for single and double photon production in the simple situation when

$$\vec{k}_1 = \vec{K} + \vec{q} \qquad \vec{k}_2 = \vec{K} - \vec{q} \qquad \omega_1 = \omega_2 \tag{16}$$

with $\vec{K}$ perpendicular and $\vec{q}$ parallel to the z-axis. The polarization vector $\vec{\varepsilon}$ is the same for both photons and it is taken as parallel to the z-axis.

The probability of the production of a photon with momentum k_1, weighted by probabilities $P(N)$ giving the distribution of the number of charged final state particles and $P(y_1, y_2, \ldots, y_N; N)$ giving the probability distribution of the charged particles in rapidity for a given N becomes

$$P(k_1) = \sum_N P(N) \int dy_1 \ldots dy_N P(y_1, y_2, \ldots, y_N) P(k_1; y_1, y_2, \ldots, y_N) \tag{17}$$

where

$$P(k_1; y_1, y_2, \ldots, y_N) = \frac{1}{N} \frac{Q^2}{\omega^2} \sum_{i=1}^{N} \frac{v_i^2}{(1 - \frac{q \cdot v_i}{\omega})^2} \tag{18}$$

The distribution $P(k_2; y_1, y_2, \dots, y_N)$ is obtained from Eq.(18) by changing the sign of q.

The probability $P(k_1, k_2)$ is given by an expression like Eq.(17) with $P(k_1; y_1, y_2, \dots, y_N)$ replaced by $P(k_1, k_2; y_1, y_2, \dots, y_N)$ where

$$P(k_1, k_2; y_1, y_2, \dots, y_N) = \frac{1}{N^2}\frac{Q^4}{\omega^4}\left\{\sum_{i,j}\frac{v_i^2 v_j^2}{(1-\frac{qv_i}{\omega})^2(1+\frac{qv_j}{\omega})^2} + \sum_{i\neq j}\frac{v_i^2 v_j^2 \cos[2q(z_i - z_j)]}{\left[1-(\frac{qv_i}{\omega})^2\right]\left[1-(\frac{qv_j}{\omega})^2\right]}\right\} \quad (19)$$

Note that the interference term $\cos(\Delta k \Delta z_{ij})$ contains here the term

$$\Delta z_{ij} \equiv z_i - z_j = \tau_0[\sinh y_i - \sinh y_j] \quad (20)$$

which takes on quite large values for two charged particles separated by a large rapidity interval. The parameter τ_0 is expected to be of the order of 1 fm/c but otherwise it is a free parameter.

A detailed study of the correlation function $C(k_1, k_2)$ given by Eq.(11) would require specification of probability distributions $P(N)$ and $P(y_1, y_2, \dots, y_N)$. The largest space separations Δz_{ij} are expected to come from small values of N and configurations in rapidity which contain large separations Δz_{ij}.

Neglecting these fluctuations we shall give here for illustrative purposes the correlation function corresponding to a single value of N and a single distribution of particle rapidities. We take two charged pions per unit of rapidity and an equidistant distribution of pions within the rapidity interval $(-Y_0, Y_0)$. Rapidities of pions are then given as

$$y_i = -Y_0 + \frac{1}{2}(n-1) \qquad n = 1,\ 2, \dots,\ N \quad (21)$$

with $N = 4Y_0 + 1$.

The correlation function in this situation becomes

$$C(k_1, k_2) = 1 + \frac{\sum_{i\neq j} v_i^2 v_j^2\left[1-(\frac{qv_i}{\omega})^2\right]^{-1}\left[1-(\frac{qv_j}{\omega})^2\right]^{-1}}{\left\{\sum_i v_i^2(1-\frac{qv_i}{\omega})^{-2}\right\}\left\{\sum_j v_j^2(1+\frac{qv_j}{\omega})^{-2}\right\}}\cos[2q(z_i - z_j)] \quad (22)$$

where v_i, v_j, $z_i - z_j$ are given by Eqs.(14, 15, 20).

In Fig.1 we present the dependence of $\ln C(q) \equiv \ln C(k_1, k_2)$ on $\ln q$ for a set of values of τ, ω and Y_0. The shape of curves consists of a slowly decreasing part followed by an almost linear decrease. This linear dependence is typical for the intermittent behaviour of the correlation function [2, 9, 10] the coefficient f_q characterizing the intermittency

$$\ln C(q) \approx \mathrm{const} - f_q \ln q \qquad q \geq q_0 \quad (23)$$

The fast linear decrease starts at the momentum q_0 which will be seen to depend strongly on the space-time evolution of the process. It is natural to ask what features of the Eq.(22) are responsible for the intermittent behaviour, since as pointed out in [11, 12] the effect connected with the photon formation length contribute substantially to the effectice dimensions of the region emitting photons.

In Fig.2 we plot $\ln C(q)$ versus $\ln q$ for $\tau = 0$. In this situation there is no space-time evolution of the collision, the model corresponds now to the free streaming and due to Eq.(14), $z_i - z_j = 0$. The term $\cos[2q(z_i - z_j)]$ in Eq.(22) is equal to one. The curves presented in Fig.2 show that the effects connected with the formation length of photons contribute to the intermittent behaviour, althought it sets up at higher values of q.

In Fig.3 we plot the correlation function in a situation when all the effects connected with the formation length of photons are switched off. The Eq.(22) then reads

$$C(k_1, k_2) = 1 + \frac{\sum_{i \neq j} \cos[2q(z_i - z_j)]}{\{\sum_i 1\}\{\sum_j 1\}} \tag{24}$$

The simplification corresponds to static spatially uniform sources placed in positions z_i.

A qualitative interpretation of the curves in Figs.1 – 3 is rather simple. In Fig.1 both the space-time evolution of the hadronization and the formation length of photons contribute to the distance which separates the production of the two photons. The simplest expectation for the relationship between the basic parameters is

$$\frac{\hbar c}{q_0} \approx \tau_0 2 \sinh Y_0 + 2\frac{v}{c}.\frac{\hbar c}{\omega} \tag{25}$$

The first term in the r.h.s. gives the distance $z_i - z_j$ for the pions with maximal positive and negative rapidities and the second term corresponds to the photon formation length. A simple calculation as well as results presented in Fig.2 indicate that the first term is dominant in cases we consider in Figs.1 – 3. For instance in the case of the curve c) in Fig.1 we have $\tau_0 2 \sinh Y_0 = 20$ fm whereas $2(v/c)(\hbar c/\omega) \approx 4$ fm. The corresponding value of q_0 calculated from Eq.(25) is almost 8 MeV and $\ln(q_0/\mathrm{MeV}) = 2$ what roughly corresponds to the value of $\ln(q_0/\mathrm{MeV})$ at which the first linear decrease sets in.

The uniform distribution of charged particles is not a realistic assumption. In Fig.4 we present the correlation function corresponding to the expression

$$C(k_1, k_2) = 1 + \frac{\sum_{i \neq j} P_i P_j v_i^2 v_j^2 \left[1 - (\frac{qv_i}{\omega})^2\right]^{-1} \left[1 - (\frac{qv_j}{\omega})^2\right]^{-1}}{\left\{\sum_i P_i v_i^2 (1 - \frac{qv_i}{\omega})^{-2}\right\} \left\{\sum_j P_j v_j^2 (1 + \frac{qv_j}{\omega})^{-2}\right\}} \cos[2q(z_i - z_j)] \tag{26}$$

where

$$P_i = A \exp\left(-\frac{y_i^2}{2\sigma^2}\right) \tag{27}$$

The results show again the same features as ones shown in Figs.1, 2 and 3 what indicates that more realistic rapidity distribution of charged hadrons does not destroy the intermittent features of the HBT correlation.

This indicates that the linearly expanding system of a type of an inside-outside cascade products bremsstrahlung photons which show an intermittent behaviour. The resulting shape of the correlation function is due both to the effects of increasing linear dimensions of the system and the effects connected with the formation length of photons.

We shall now add a few speculative comments concerning a possible application of the present mechanism to the HBT interferometry of identical hadrons.

Large longitudinal dimensions entered the correlation function $C(k_1, k_2)$ of bremsstrahlung photons because the rapidities of photons [illegible] differ substantially from

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Figure captions

Fig.1
The correlation function as given by Eq.(22) $C(k_1, k_2)$ plotted as $\ln C(q)$ versus $\ln q$ for

a) $\tau_0 = 1$, $\omega = 100$ MeV, $Y_0 = 1$;
b) $\tau_0 = 1$, $\omega = 100$ MeV, $Y_0 = 2$;
c) $\tau_0 = 1$, $\omega = 100$ MeV, $Y_0 = 3$;
d) $\tau_0 = 2$, $\omega = 100$ MeV, $Y_0 = 3$.

Fig.2
$\ln C(q)$ versus $\ln q$ for $\tau_0 = 0$:

a) $Y_0 = 1$, $\omega = 100$ MeV;
b) $Y_0 = 2$, $\omega = 100$ MeV;
c) $Y_0 = 3$, $\omega = 100$ MeV.

Fig.3
$\ln C(q)$ as given by Eq.(24). Notation:

a) $\omega = 100$ MeV, $\tau_0 = 1$, $Y_0 = 1$;
b) $\omega = 100$ MeV, $\tau_0 = 1$, $Y_0 = 2$;
c) $\omega = 100$ MeV, $\tau_0 = 1$, $Y_0 = 3$;
d) $\omega = 100$ MeV, $\tau_0 = 2$, $Y_0 = 3$.

Fig.4
$\ln C(q)$ given by Eqs.(26) and (27) vs $\ln q$. Notation of curves and values of parameters:

a) $Y_0 = 3$, $\tau_0 = 1$, $\sigma = 1$;
b) $Y_0 = 3$, $\tau_0 = 1$, $\sigma = 2$;
c) $Y_0 = 3$, $\tau_0 = 1$, $\sigma = 3$;
d) $Y_0 = 3$, $\tau_0 = 2$, $\sigma = 3$.

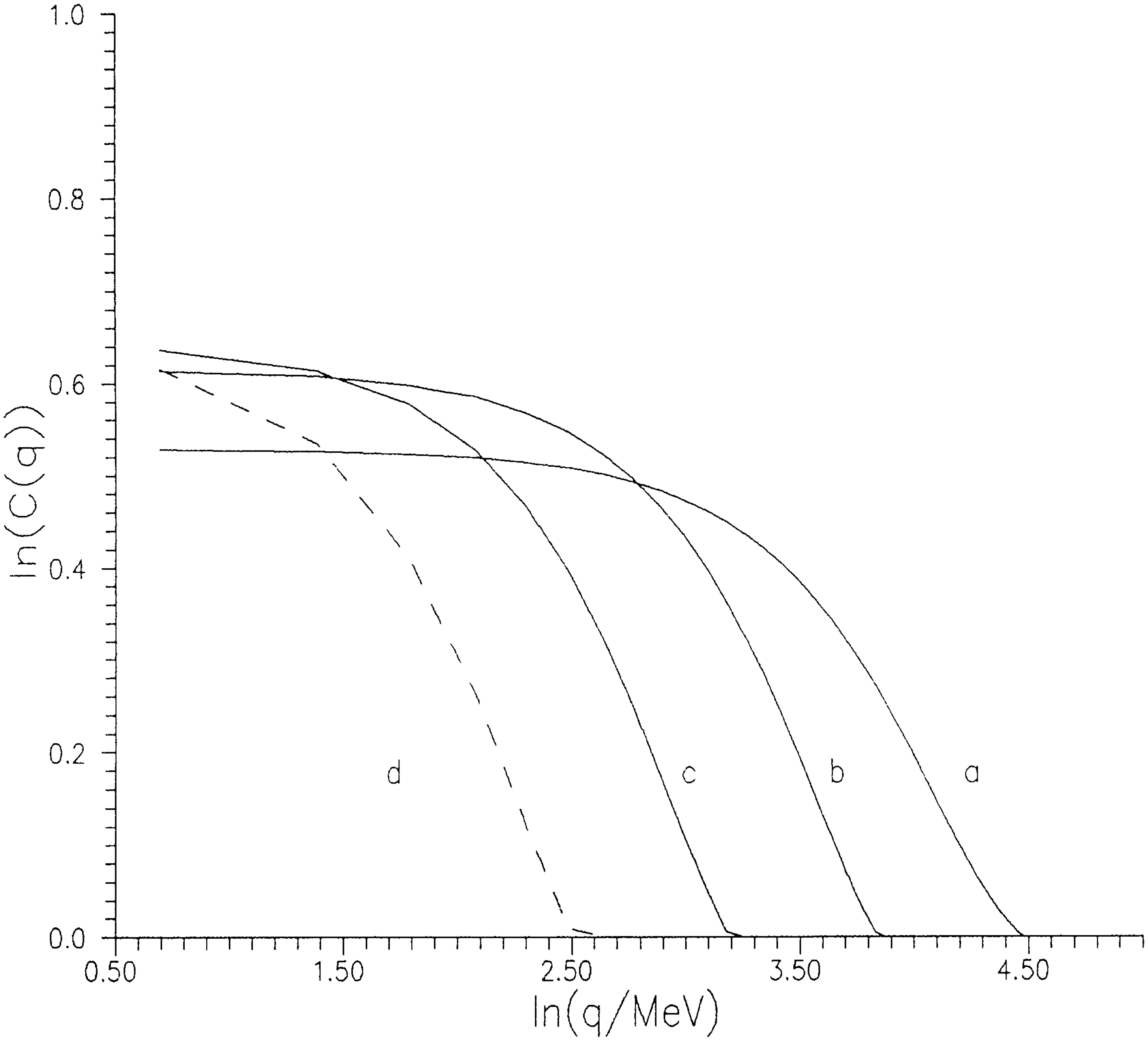

Fig.1

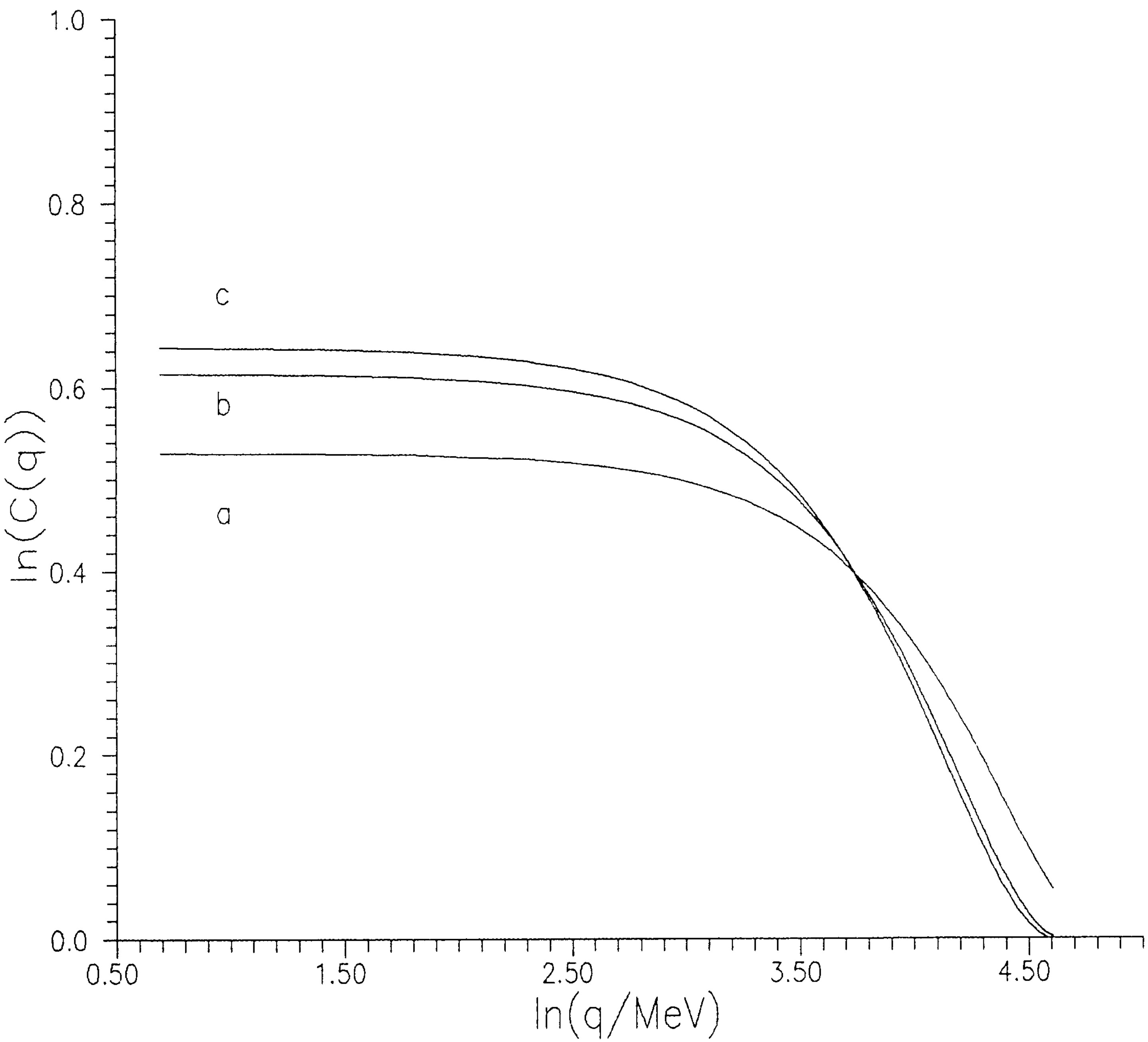

Fig.2

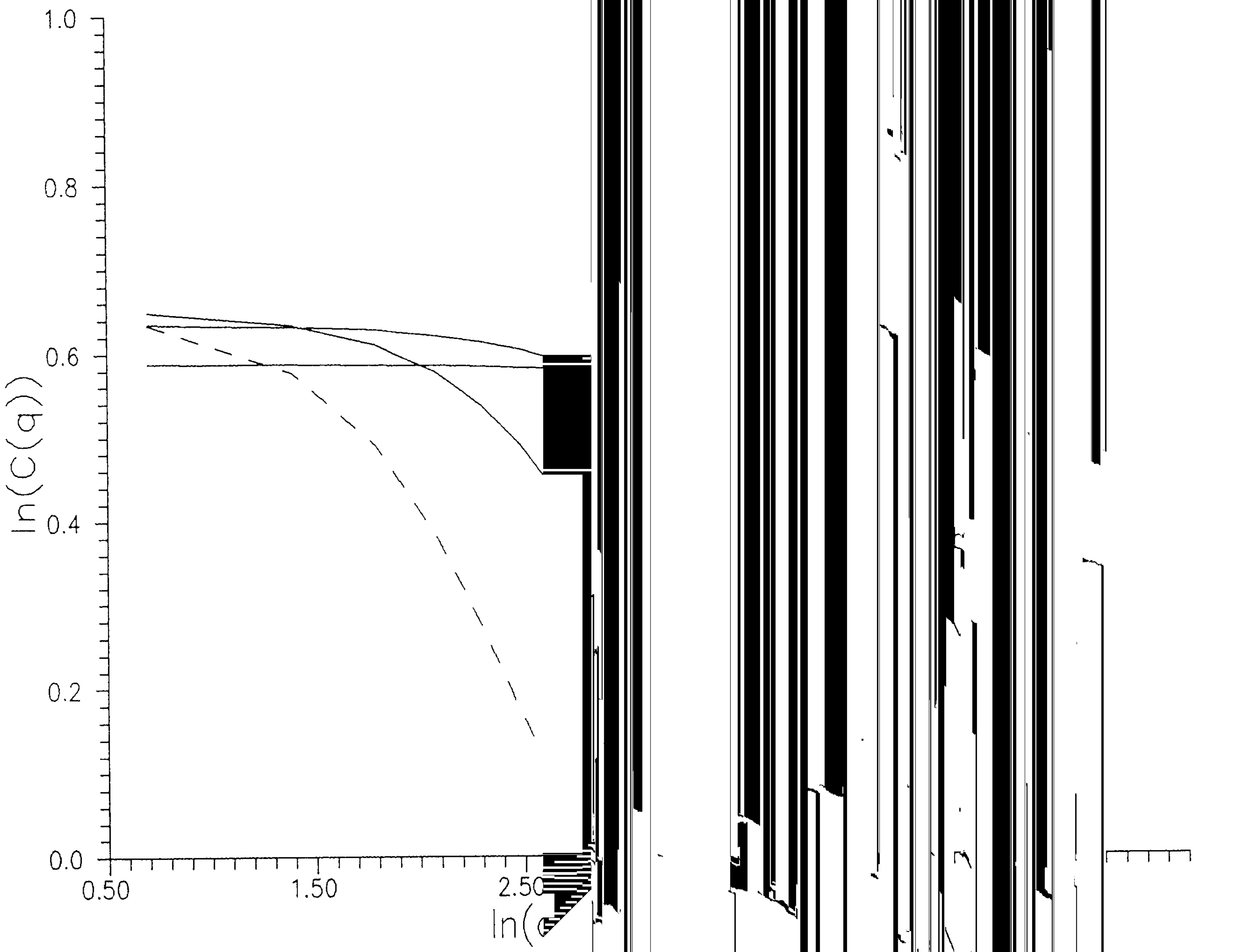

Fig.3

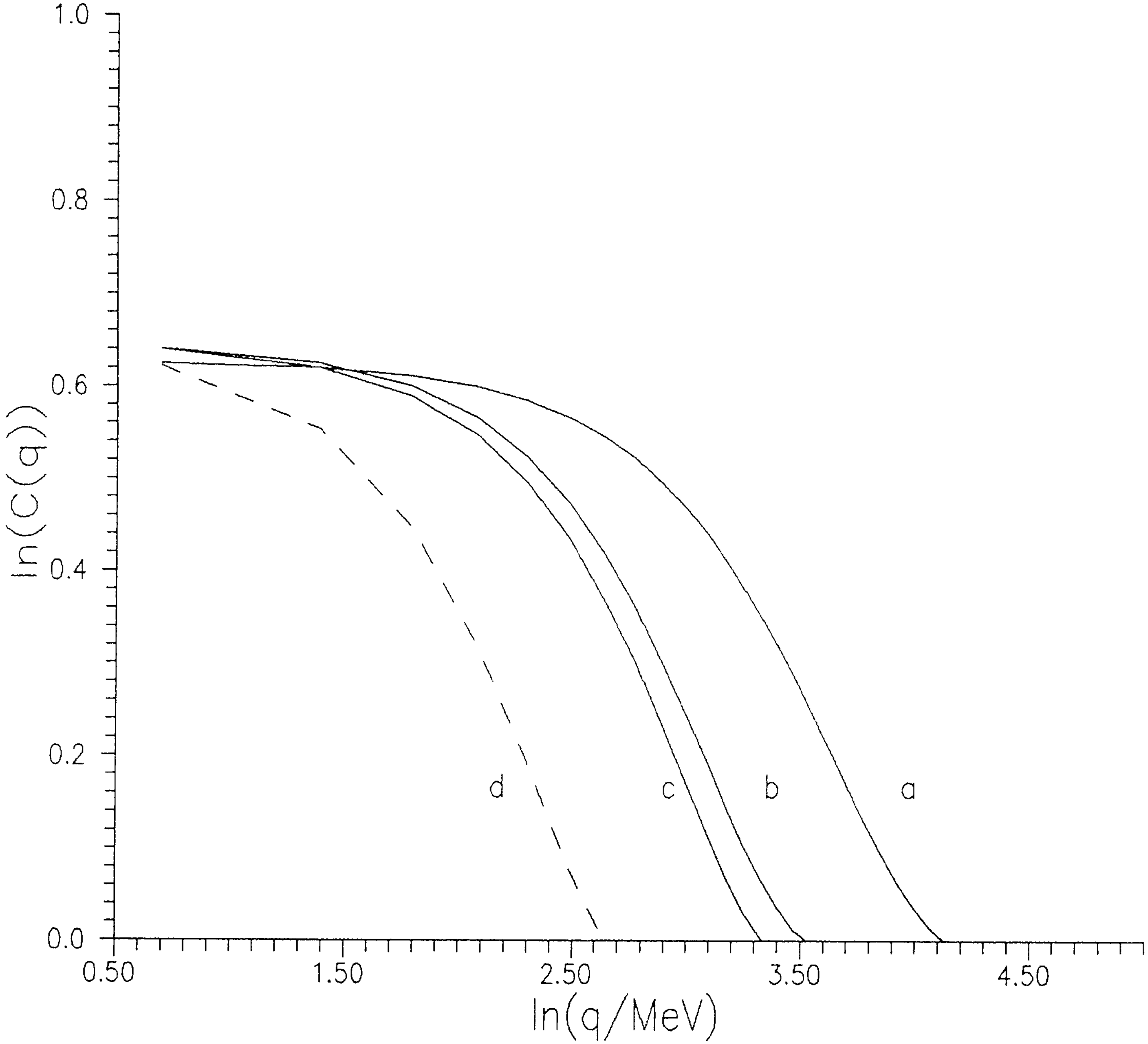

Fig.4