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Coherent Radiation

in the CLIC Isochronous Ring

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Abstract

Conditions for emission of coherent synchrotron radiation and its power are evaluated for the Isochronous Ring which is being designed for the CLIC Drive Beam. There will be no coherent radiation for the present design with 6 mm bunch length, but excessive radiated power would occur for submillimeter bunches.

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1 Introduction

The present scheme for the CLIC drive beam generation includes an isochronous ring[1] in each leg of the machine. In this ring, trains of bunches, while have been accelerated to 2.6 GeV in the injector linac, are stored and wait to be transferred into the drive linac. Each bunch circulating in the isochronous ring is designed to have a rather large charge of 17 nC/bunch. The radius of curvature of the beam trajectory is 100 m, so the energy loss due to incoherent synchrotron radiation is $\approx$ 46 KeV/turn. That corresponds to a modest synchrotron radiation power of $\approx$ 62.8 kW for 340 bunches of $\approx$ 4.64 mA each.

On the other hand, the large number of electrons per bunch in rather short bunches leads one to think of a possibility of strong coherent synchrotron radiation. For example, by applying the formulas derived by Schiff [2] or by Nodvick and Saxon [3] to the isochronous ring, one finds for the power of shielded coherent radiation the estimates of $\approx$ 0.93 and $\approx$ 1.4 MW, respectively.

An alternative estimate of the power of shielded coherent synchrotron radiation with an assumption of the Gaussian longitudinal charge distribution in a bunch was obtained in the Report [4]. The present note evaluates the power of coherent synchrotron radiation which can be expected in the the isochronous ring. As it is shown here, this radiation is negligible for a Gaussian bunch if the rms length of a bunch is $\approx$ 6 mm. The situation would be completely different if the rms bunch length were 0.6 mm.

Table 1 contains parameters of the isochronous ring relevant for the evaluation of coherent synchrotron radiation.

2 Main Formulas and Numerical Estimates for Isochronous Ring

For convenience of the reader I reproduce here main formulas for incoherent and coherent synchrotron radiation together with an estimates for an Isochronous Ring with parameters listed in Table 1.

2.1 Incoherent Synchrotron Radiation

The angular and frequency spectra of incoherent synchrotron radiation (emitted by a single particle), i.e. the intensity of radiation per unit solid angle $d\Omega$ and for a given radiation harmonic number $n \equiv \omega/\omega_0$ ($\omega_0 = c/R$ is the revolution frequency) is conveniently given by the following formula:

$$dP(n,\alpha) = \frac{e^2\omega_0}{\rho}\frac{n^2}{6\pi^3}\Big(\epsilon^2 K_{2/3}^2(\frac{n}{3}\epsilon^{3/2}) + \epsilon\alpha^2 K_{1/3}^2(\frac{n}{3}\epsilon^{3/2})\Big)d\Omega \qquad (1)$$

where the parameter

$$\epsilon \equiv \frac{1}{\gamma^2} + \alpha^2 \tag{2}$$

depends both on the angle α between the direction of radiation and the instantaneous velocity of a particle (radiation angle), and the Lorentz factor γ.

Due to property of the modified Bessel functions $K_{1/3}$ and $K_{2/3}$ which decrease exponentially when their argument is larger than 1, the intensity of radiation grows with n like $\propto n^{1/3}$ until it reaches a maximum at $n \approx n_{max}$, where

$$n_{max} \equiv \frac{3}{2}(\frac{1}{\gamma^2} + \alpha^2)^{-3/2} \ . \tag{3}$$

For $\alpha = 0$, $n_{max} \equiv n_c \approx (3/2)\gamma^3$ is called the critical radiation harmonic number. Beyond the maximum, i.e. for $n > n_c$ the spectrum falls down exponentially as $\sqrt{n}\exp(-n/n_c)$.

On the other hand, when $1/\gamma \ll \alpha$, $n_{max} \approx (3/2)\alpha^{-3}$. For the main part of the spectrum $n < n_{max}$, and for those harmonic numbers $\alpha < n^{-1/3}$. That fact is important in respect to coherent synchrotron radiation. As we will see shortly, it limits from below values of harmonic numbers on which coherent radiation can be emitted.

The total power of incoherent synchrotron radiation emitted by a train of bunches circulating on an orbit is given by

$$P_{tot} = \frac{2}{3}\frac{r_0\omega_0}{\rho} k_B N_B \frac{\rho}{R}\gamma^4 mc^2 \ , \tag{4}$$

where r_0 is the electron classical radius, ω_0 is circular revolution frequency, ρ and R are radii of the orbit curvature and the average orbit, k_B and N_B are the numbers of bunches and electrons per bunch. This formula can also be interpreted as the product of the beam current $I = k_B N_B ec/2\pi\rho$ times the energy loss per revolution in units of volts:

$$V = \frac{4\pi}{3}\frac{e}{\rho}\gamma^4 \ . \tag{5}$$

The last formula can be rewritten in a convenient way for the amount of energy loss per one revolution:

$$\Delta E_{KeV} = 88.5\frac{E^4_{GeV}}{\rho_m} \ . \tag{6}$$

2.2 Necessary Condition for Coherent Synchrotron Radiation

The natural coherence condition in free space

$$\sigma_s < \lambda \equiv \frac{2\pi c}{\omega} = \frac{2\pi\rho}{n} \tag{7}$$

is modified in the presence of a good conductor due to *shielding*. The image charges and currents in a screen tend to compensate the electromagnetic field of the circulating charge decreasing the amount of radiation. This compensation is the more effective the closer are the radiating and image charges to each other, i.e. the smaller is the distance h from the radiating charge to the screen.

In the presence of a screen the wave vector of radiation field acquires transverse component: $\vec{k} = \vec{k}_{||} + \vec{k}_{\perp}$. Its absolute value $k_{\perp} \approx k\alpha = n\alpha/\rho$. Since the tangential component of the electric field of the radiation should be zero on the surface of a perfect conductor (or to be very small on the surface of a good conductor), the transverse component of the wave vector can not be smaller then π/h. Here h is a characteristic distance to a screen. In a model with two screens positioned equidistantly form the plane of the orbit, h is the distance between shielding screens. Hence, $k\alpha > \pi/h$, or $\alpha > \pi\rho/nh$. At the same time, as was shown before, $\alpha < n^{-1/3}$. From here follows the condition that the radiation harmonic number on which coherent radiation is possible is limited from below:

$$n > n_{th} \equiv \sqrt{\frac{2}{3}} (\frac{\pi\rho}{h})^{3/2}. \tag{8}$$

A clear physical picture which explains the appearance of this condition is presented in Ref.[5].

From Eq. 7 follows the limitation of the harmonic numbers from above:

$$n < n_{coh} \equiv \frac{2\pi\rho}{\sigma_s} . \tag{9}$$

Combining the last two inequalities together we find the following necessary condition for coherent radiation in the presence of shielding:

$$n_{th} \equiv \sqrt{\frac{2}{3}} (\frac{\pi\rho}{h})^{3/2} < n < n_{coh} \equiv \frac{2\pi\rho}{\sigma_s} . \tag{10}$$

Since the ratio $\pi\rho/h$ is usually large, the lower limit for n is higher than the one which comes from the cut-off frequency of the beam pipe $n > n_{c.o.} \equiv \pi\rho/h$.

The same condition can be formulated in terms of the wave length:

$$\sigma_s < \lambda < \lambda_{th} \equiv \frac{2\pi\rho}{n_{th}} = h\sqrt{\frac{6h}{\pi\rho}} . \tag{11}$$

Coherent radiation in a storage ring is possible only when there is a range of harmonic numbers or wave lengths satisfying conditions Eqs. 10 and 11. As one can see, they are always fulfilled if the bunch length is short enough[1]:

$$\sigma_s < 2\pi\sqrt{\frac{3}{2}}\rho(\frac{h}{\pi\rho})^{3/2} . \tag{12}$$

If $n_{th} > n_{coh}$ all harmonics are emitted incoherently.

For the Isochronous Ring with $h = 100$ mm, $n_{th} = 1.76 \cdot 10^5$. For $\sigma_s = 6$ mm, $n_{coh} = 1.05 \cdot 10^5$, and no radiation can be emitted coherently. If the rms length of a bunch would be ten times smaller: $\sigma_s = 0.6$ mm, then $n_{coh} = 1.05 \cdot 10^6$ and one should expect such radiation.

2.3 Unshielded Coherent Synchrotron Radiation

The total power of coherent synchrotron radiation in the absence of any shielding was estimated by Schiff[2]:

$$P^{coh}_{unsh} \approx CN^2 \frac{e^2 \omega_o}{\rho} (\frac{\rho}{\sigma_s})^{4/3} , \tag{13}$$

which is valid for the range $1 \gg \sigma_s/2\pi\rho \gg \gamma^{-3}$, $\gamma \gg 1$. The value of the constant is $C = 2^{7/3}3^{1/6}\Gamma^2(2/3)\pi^{1/3} \approx 16.24$.

For the Isochronous Ring the total power of unshielded coherent radiation would be $P^{coh}_{unsh} \approx 2.15$ MW. The local density of radiation $P^{coh}_{unsh}/2\pi\rho \approx 3.42$ KW/m.

2.4 Shielded Coherent Synchrotron Radiation

Two estimates for the reduction of the power of coherent synchrotron radiation in the presence of shielding exist in the literature, due to to Schiff[2] and due to Nodvick and Saxon[3].

An upper limit of the power has been evaluated by Schiff[2]. For the reason of simplicity he assumed that only one shielding screen is present:

$$P^{coh}_{sh} \approx CN^2 \frac{e^2 \omega_o}{\rho} (\frac{h}{\rho})^2 (\frac{\rho}{\sigma_s})^{8/3} , \tag{14}$$

which is valid for the same range of parameters as Eq.13. The value of constant in this equation is $C = 2(2\pi)^{8/3}\Gamma^2(1/3)/3^{7/6}\pi \approx 1.7 \cdot 10^2$.

For the Isochronous Ring the total power of shielded coherent radiation according to Schiff's estimate $P^{coh}_{sh} \approx 0.93$ MW. The local density of radiation $P^{coh}_{sh}/2\pi\rho \approx 1.47$ KW/m. The shielding reduction factor is $R_S \equiv (h/\rho)^2(\rho/\sigma_s)^{4/3} = 0.43$.

Another estimate is due to to Nodvick and Saxon[3]. They evaluated the effect of two screens positioned at distances $\pm h/2$ from the radiation plane. According to this estimate, the total power of radiation is:

$$P^{coh}_{sh} \approx CN^2 \frac{e^2 \omega_o}{\rho} (\frac{h}{\rho}) (\frac{\rho}{\sigma_s})^2 . \tag{15}$$

The constant here is $C = \sqrt{3}/2 \approx 0.87$. The shielding Reduction Factor (Ratio $P^{coh}_{sh}/P^{coh}_{unsh}$) is: $R_{NS} \approx (h/\rho)(\rho/\sigma_s)^{2/3}$.

The Nodvick and Saxon estimate of the power of shielded coherent radiation for the Isochronous Ring is $P^{coh}_{sh} \approx 1.4$ MW. The local density of radiation $P^{coh}_{sh}/2\pi\rho \approx 2.23$ KW/m. The shielding reduction factor $R_{NS} \equiv (h/\rho)(\rho/\sigma_s)^{2/3} = 0.65$.

2.5 Shielded Coherent Synchrotron Radiation for a Gaussian Bunch

The total power of coherent synchrotron radiation emitted by a Gaussian bunch[4] can be derived by summing all contributions of the radiated power on the n-th harmonic, weighted by the longitudinal density distribution of a bunch. In Ref. [4] a rigorous calculation of this sum is presented. Here we present a simplified version of this derivation.

The first factor in each term of the sum is given by:

$$P_n^{coh} = Z_n I_n^2 \ , \tag{16}$$

where the real part of the longitudinal impedance due to radiation is:

$$Z_n = Z_0 \frac{\Gamma(2/3)}{3^{1/3}} \frac{\sqrt{3}}{2} n^{1/3} \ . \tag{17}$$

$Z_0 = 4\pi/c \approx 377$ Ohm is the free space impedance.

Hence, we need to evaluate the sum in the expression for P_{sh}^{coh}:

$$P_{sh}^{coh} \propto \Sigma_{n_{th}}^{n_{coh}} n^{1/3} \exp(-(\frac{n\sigma_s}{2\pi\rho})^2) \ . \tag{18}$$

Here we assume that $n_{th} < n_{coh}$, i.e. that the condition for coherent radiation is fulfilled, cf. Eq. 10.

Since the number of essential harmonics $n \gg 1$, the summation can be replaced by integration. Introducing the variable $x \equiv (n\sigma_s/2\pi\rho)^2$ and performing the integration from

$$x_{th} \equiv (\frac{n_{th}\sigma_s}{2\pi\rho})^2 = \frac{2}{3}(\frac{\pi\rho}{h})^3(\frac{\sigma_s}{2\pi\rho})^2 \tag{19}$$

to

$$x_{coh} = (\frac{n_{coh}\sigma_s}{2\pi\rho})^2 = 1 \tag{20}$$

we get:

$$P_{sh}^{coh} \approx P_{unsh}^{coh} F(x_{th}) \ , \quad x_{th} \leq 1 \ , \tag{21}$$

where the power of unshielded coherent radiation is given in Eq. 13 and the reduction factor

$$F(x_{th}) = \int_{x_{th}}^{1} dx x^{\frac{2}{3}-1} e^{-x} = \Gamma(\frac{2}{3}, x_{th}) - \Gamma(\frac{2}{3}, 1) \tag{22}$$

is expressed as difference of two incomplete Γ-functions[6] with the parameter 2/3 shown in Fig.1.

The reduction factor $F(x_{th})$ simplifies in two limiting cases:

1. If $x_{th} \ll 1$, $F(x_{th}) \approx 1$ and we recover the result Eq. 13 for unshielded coherent radiation derived by Schiff[2].

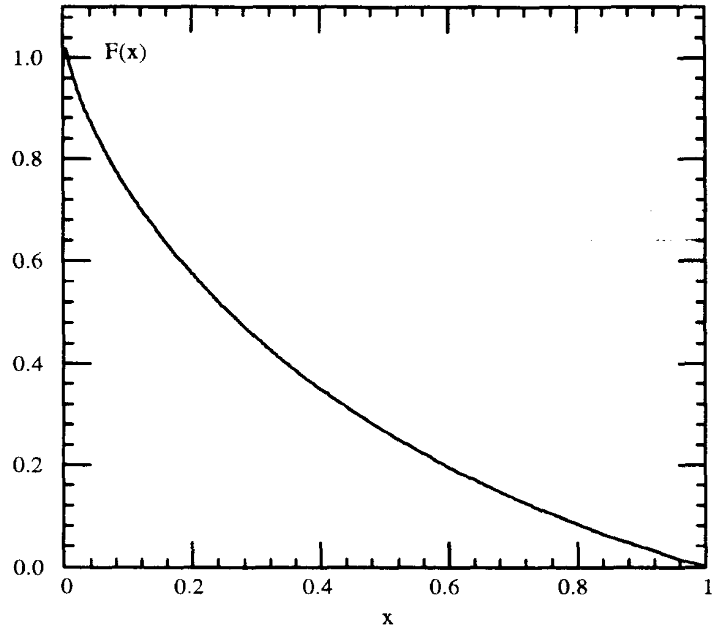

Figure 1: Reduction factor $F(x)$ of the power of coherent synchrotron radiation for a Gaussian bunch.

2. When $x_{th} \rightarrow 1, F \rightarrow 0$. In this case coherent synchrotron radiation is suppressed exponentially. For $x_{th} > 1, F = 0$ and there is no coherent radiation.

Note, that if the upper limit of integration is extended to infinity,

$$F(x_{th})|_{x_{th}\gg 1} \approx x_{th}^{-1/3} e^{-x_{th}} . \tag{23}$$

If one further neglects here the exponential factor $\exp(-x_{th})$ the result by Nodvick and Saxon[3] Eq. 15 follows.

The condition $x_{th} < 1$ reproduces Eq. 12 (apart from the factor 2π) which was used in Ref.[1]. For the present design (Table 1) $x_{th} \gg 1$ and there is no coherent radiation. For smaller bunch lengths, the value of the parameter x_{th} could be calculated from Eq.19 and the value of the corresponding reduction factor $F(x_{th})$ from Fig.1. For example, for $\sigma_s = 0.6$ mm, the power of coherent synchrotron radiation would reach level of 270 KW.

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Table 1. **Input Parameters**

Energy	E	2.6	GeV
Lorentz Factor	γ	$5.1 \cdot 10^3$	
Radius of Curvature	ρ	100	m
Average Radius	R	175	m
Number of Electrons/bunch	N_B	$1.0 \cdot 10^{11}$	
Number of Bunches	k_B	340	
RMS Bunch Length	σ_s	$6.0 \cdot 10^{-3}$	m
Diameter of Vacuum Chamber	h	$100 \cdot 10^{-3}$	m

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