VERTICAL SHAPE DETERMINATION OF A STRETCHED WIRE FROM OSCILLATION MEASUREMENTS

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Abstract

The Geodetic Metrology group at CERN uses stretched wires as a reference for the position monitoring and alignment of accelerator components. Until now, stretched wires find in particular use as horizontal offset measurement references, since their vertical projection is a line. However, the wire positioning system is able to measure not only the horizontal but also the vertical wire position. In order to use this data as vertical reference of the alignment system, a framework to describe the vertical wire shape is required. This work re-conceptualises a previously proposed optimization based algorithm, that calculates the vertical wire shape via its fundamental frequency from oscillation measurements. As a result, the determination of the vertical shape with respect to a static parabola fitting model was improved one order of magnitude compared to the previously available oscillation-based algorithm. Now, it is possible to determine the wire position with respect to static wire measurements with a precision of the same order of magnitude as the static parabolic fitting model for wires of up to $140\,\mathrm{m}$ length. Furthermore, the study of wire oscillations revealed methods to localize restrictions of the wire. With these means, an alternative evaluation method to the static parabolic fitting model is provided that adds information to already existing alignment systems and offers new sensor configuration possibilities for future alignment systems.

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INTRODUCTION

In order to collide particle beams, machine components of particle accelerators need to be positioned with high accuracy. In the Large Hadron Collider (LHC), stretched wires are used for the position monitoring and alignment of the Inner Triplet cryo-assemblies. Currently each of these cryo-assemblies is equipped with minimum two wire positioning sensors (WPS) and a minimum of three hydrostatic levelling sensors (HLS), to follow horizontal, vertical and roll movements over time [1, 2]. The HLS are using the principle of communicating vessels to determine relative vertical positions of the cryostats. The WPS are capturing the horizontal and the vertical position of a wire that is stretched along the cryostats and use this information as a position reference. Therefore, models of horizontal and vertical wire shape are required. Horizontally, it is assumed that the stretched wire is a straight line. Vertically, its sag has to be taken into account.

In the last decades, several approaches have been developed to determine a vertical wire model, that serves as a vertical reference in the context of accelerator alignment. Following Timoshenko and Young, the shape of a stretched wire can be described in dependence of the gravity g, its tension T and its linear mass q by the catenary model [3]. Mainaud showed that the catenary model can be approximated by a second order polynomial with sufficient precision. This polynomial depends besides of the parameters of the catenary model also on the endpoints of the wire [4]. According to Touzé, the linear mass of the wire is affected by the absorption of humidity [5]. Therefore, models dependant on this parameter suffer from large uncertainties. To overcome this issue, Touzé proposes to substitute the quotient of tension over linear mass by a term that includes the fundamental frequency of the stretched wire. In order to determine the fundamental frequency, Schott Guilmault [6] derived a model to describe an oscillating wire. Furthermore, he implemented an optimization-based method to determine the fundamental frequency by fitting the Fourier transformed oscillation model on Fourier transformed vertical position measurements of the oscillating wire. To verify his model, he compared the resulting sag of the vertical wire shape to the sag of the best fitting second order polynomial obtained from static wire measurements.

Although the oscillation based approach of Touzé and Schott Guilmault [5, 6] avoids parameters with large uncertainties, the latter comparison revealed a non-negotiable difference of the resulting vertical wire sags. The objective of this work was the improvement of their approach with respect to the vertical wire shape that is determined from the best fitting second order polynomial on static wire measurements.

For this purpose, their approach is modified in three steps. First, the wire oscillations are conducted and recorded horizontally. Second, the oscillation model is directly fitted on the wire oscillation measurements. Third, the dimension of the search space of the optimized variables is decreased to one, by fitting only the fundamental frequency.

Having an algorithm to determine the vertical shape of a stretched wire as an alignment reference adds information to existing measurement installations and offers the possibility for new configurations with regard to the quantity and ratio of HLS and WPS for future alignment projects, e.g. High Luminosity LHC (HL-LHC) [7] or Compact Linear Collider (CLIC) [8].

METHODS

Vertical Wire Models

For the description of the vertical shape of a stretched wire, various models are available, that differ in terms of the required parameters and measurements. For the method presented in this work, the fundamental frequency dependent parabolic model derived by Mainaud [4] and Touzé [5] is used. The static parabolic fitting model is applied to validate the results of method presented in this paper.

Fundamental Frequency Dependent Parabolic Model. As illustrated in Fig. 1, Mainaud [4] proposes to describe the vertical position $y(x)$ of a stretched wire in the longitudinal position x with the second order polynomial

$$\begin{aligned} y(x) =& y_O + \frac{gq}{2T}(x - x_O)^2 \\ &+ \left(\frac{h}{l} - \frac{gql}{2T}\right)(x - x_O) \ , \end{aligned} \tag{1}$$

that approximates the catenary model [3]. The coefficients of the polynomial depend on the wire's extremities, the gravity g, the linear mass q and the tension T.

The sag f of the wire is defined to be the maximal difference of the straight line d between the wire extremities and the wire as shown in Fig. 1. Following Mainaud, the sag is given by

$$f = \frac{gql^2}{8T}. \tag{2}$$

Inserting Eq. 2 in Eq. 1 leads to the second order polynomial

$$y(x) = y_O + \frac{4f}{l^2}(x - x_O)^2 + \frac{h - 4f}{l}(x - x_O). \tag{3}$$

This model for the vertical wire position still depends via the sag on the the linear mass q. According to Touzé [5], the latter is affected by humidity, such that high uncertainties in the model are caused. Therefore, he proposes the insertion of the equation

$$\phi = \frac{1}{2l}\sqrt{\frac{T}{q}} = \sqrt{\frac{g}{32f}}, \tag{4}$$

that relates the fundamental frequency ϕ to the quotient of linear mass and tension and to the sag in Eq. 1. As a result, the fundamental frequency dependent parabolic model (FFDPM)

$$\begin{aligned} y(x) =& y_O + \frac{4g}{32l^2\phi^2}(x - x_O)^2 \\ &+ \frac{h - \frac{4g}{32\phi^2}}{l}(x - x_O) \end{aligned} \tag{5}$$

is obtained.

Static Parabolic Fitting Model. Instead of describing the vertical shape of a wire with respect to parameters, vertical position measurements of the wire itself can be used to determine a second order polynomial.
Since a second order polynomial of one variable is uniquely characterized by three coefficients a, b and c,

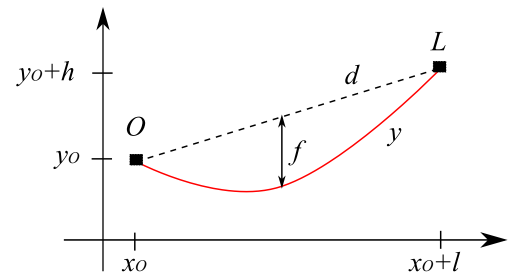

Figure 1: Sketch of the vertical shape of a stretched wire.

three vertical position measurements are sufficient for determining a parabolic vertical shape model of the wire. If more measurements are available, suitable values for the coefficients can be obtained by minimizing the least square error between the polynomial

$$y_{\text{model}}(x, a, b, c) = ax^2 + bx + c \tag{6}$$

and a set of vertical position measurements $\{y_{\text{meas}}(x_1), \ldots, y_{\text{meas}}(x_n)\}$ in the optimization problem

$$\min_{a,b,c\in\mathbb{R}} \sum_{i=1}^{n} \|y_{\text{model}}(x_i, a, b, c) - y_{\text{meas}}(x_i)\|^2. \tag{7}$$

The resulting best fitting parabolic model is referred to as static parabolic fitting model (SPFM).

Wire Oscillation Model

Schott Guilmault [6], derived from the solution of the wave equation the oscillation model

$$w(t, x) = H(t)\sum_{n\in\mathbb{N}} A_n(x)e^{-\alpha t}\cos(2\pi\phi n t) \tag{8}$$

that describes the position of an oscillation wire at time t in the longitudinal position x. Supplementary, the Heavyside function H is used for modelling the beginning of the oscillation at $t = 0$ and the amortisation of the oscillation over time is taken into account by including the factor $e^{-\alpha t}$. The time-independent factor

$$A_n(x) := \sin\left(\frac{n\pi x}{l}\right)\frac{2}{l}\int_0^l w(0, x)\sin\left(\frac{n\pi x}{l}\right)\,\mathrm{d}x \tag{9}$$

is derived from the Fourier series and depends on the initial position $w(0, x)$ and the longitudinal distance l of the extremities of the wire (see Fig. 1). The fundamental frequency ϕ is a parameter of the oscillation model, thus the notation $w_\phi(t, x)$ is used.

Experimental Setup

Oscillation Wire Measurement Setup. The measurement setup consists of a carbon-PEEK wire, that is at one extremity fixed and at the other extremity deflected 90 degrees by a ball bearing bedded wheel and stretched with a weight of $15\,\text{kg}$ that is attached to the wire.

In order to generate the wire oscillation, the wire is deflected in one point by a fibre glass finger and released after the wire reached a stable position. The wire position is measured after the relaxation by a WPS, that is mounted in the position interval $\left[\frac{1}{4}l, \frac{1}{2}l\right]$ along the wire. This interval is based on the available access points to the wire and chosen such that fast amortisations of the wire close to its extremities and cancellations of waves in the middle of the wire are avoided. The WPS data is read by an oscilloscope with a $50\,\mathrm{kHz}$ sampling rate and saved with 12 bit resolution.
The position of the fibreglass finger along the wire is chosen symmetrically to the position of the evaluated WPS with respect to the middle of the wire. In contrast to Schott Guilmault, who deflected the wire vertically and estimated the initial position of the wire from the relative vertical position of the wire after the deflection, the wire is deflected horizontally, allowing to assume a piece-wise affine initial position of the wire.

Metrological Measurement Setup. The metrological measurement setup is shown schematically in Fig. 2. It contains three carbon-PEEK wires of different length. The distances between their extremities are $141.465\,\mathrm{m}$, $92.736\,\mathrm{m}$ and $49.456\,\mathrm{m}$. They are numbered with decreasing length. The shorter wires are stretched by weights, while the longest wire is stretched by the wire stretching device described by Herty et al. [9].

The longest wire is observed by 7 WPS, the intermediate by 5 WPS and the shortest by 3 WPS. For all wires, its corresponding WPS are mounted on metrological plates, that are equally distributed along the wire. The metrological plates are additionally equipped with HLS, which measure their vertical position continuously. As a consequence, the static vertical positions of the wires at each WPS are also determinable.

The position of the metrological plates in the setup is known from laser tracker measurements. Thereof, the longitudinal distance l of the wire extremities and the position of the fibreglass finger were sufficiently precisely obtained by tape measurements to the closest measuring plate. The vertical distance h of the wire extremities was not measurable with the tape method. Hence, h was determined by inserting the measured calm wire position at the WPS close to the wire extremities in the fundamental frequency dependent parabolic model Eq. 5 and resolving the equation for h.

For the shortest wire, the fibreglass finger is installed near WPS 3-2 that is also used for the recording of the oscillations. For the intermediate wire, the fibreglass finger is installed near WPS 2-2 respectively 2-4 and the recordings of WPS 2-4 respectively 2-2 are evaluated. Equivalently, the measurement setup is chosen with WPS 1-3 and 1-5 for the longest wire.

Fundamental Frequency and Sag Determination Method

Given oscillation measurements $\hat{w}(t, x_{\mathrm{WPS}})$ at the longitudinal position x_{WPS} of a stretched wire, the fundamental frequency ϕ is determined by solving the following optimization problem

$$\min_{\phi \in \mathbb{R}} \left\| w_\phi(t, x_{\mathrm{WPS}}) - \hat{w}(t, x_{\mathrm{WPS}}) \right\|^2 . \tag{10}$$

Inserting the resulting fundamental frequency in Eq. 5 and 4 yields the description of the vertical shape of the wire via the fundamental frequency dependent parabolic model and the sag of this model respectively.

The optimization problem in Eq. 10 is solved in MATLAB using the the Levenberg-Marquardt algorithm implementation called `LMFnlsq2` as programmed by Balda [10]. The procedure of determining the fundamental frequency by solving the optimization problem from Eq. 10 is referred to as fundamental frequency determination method. Obtaining the sag by subsequently inserting in Eq. 4 is referred to as sag determination method.

Before solving the optimization problem, the amortization parameter α and a bound $N \in \mathbb{N}$ to cut of the infinite sum in Eq. 8 have to be set to enable the evaluation of the oscillation model $w_\phi(t, x_{\mathrm{WPS}})$. A suitable amortization parameter α is found by fitting the function

$$f(t) := \hat{w}(t_1, x_{\mathrm{WPS}}) e^{-\alpha t} \tag{11}$$

to the set of peaks $\{\hat{w}(t_1, x_{\mathrm{WPS}}), \ldots, \hat{w}(t_n, x_{\mathrm{WPS}})\}$ of the oscillation measurement. The cut off bound N is chosen in advance by setting it to the number of observable harmonics in the Fourier transform of the oscillation measurements.

Fundamental Frequency Dependant Parabolic Model Determination Method

Besides the fundamental frequency, the parabolic model of Eq. 5 requires the gravity g and the parameters x_O, y_O, l and h that describe the position of the wire extremities. In the metrological measurement setup, only g, x_O and l were available by measurements. Therefore, the measured static wire positions $\hat{y}(x_{\mathrm{WPS1-1}})$ and $\hat{y}(x_{\mathrm{WPS1-7}})$ of the first and the last WPS were considered to be fixed such that the equation system

$$\begin{cases} \hat{y}(x_{\mathrm{WPS1-1}}) = y(x_{\mathrm{WPS1-1}}) \\ \hat{y}(x_{\mathrm{WPS1-7}}) = y(x_{\mathrm{WPS1-7}}) \end{cases}$$

can be resolved for h and y_O. Subsequently, the fundamental frequency dependant parabolic model can be evaluated.

Algorithmic Error and Uncertainties

The fundamental frequency and the sag determination method were tested on simulated wire oscillations in order to estimate the algorithmic error. The latter increases with

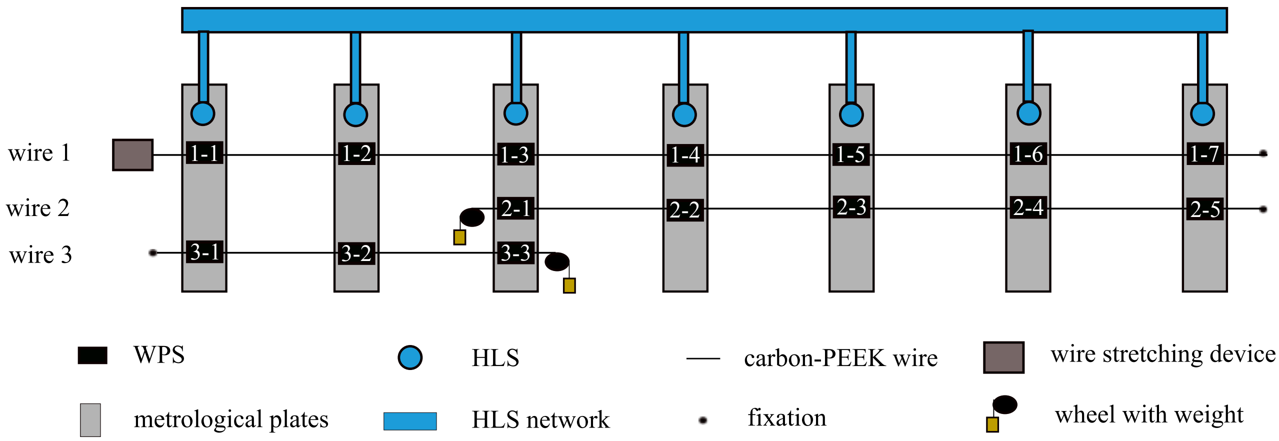

Figure 2: Schematic illustration of the metrological measurement setup.

increasing wire length. For wires with a length of $140\,\mathrm{m}$ the algorithmic error was maximal of magnitude $1 \times 10^{-7}\,\mathrm{m}$.

For validating the results, not only the algorithmic error but also the propagation of uncertainties of the parameters to the sag has to be taken into account. The uncertainty of the WPS reading is assumed to be of magnitude $1 \times 10^{-6}\,\mathrm{m}$. The uncertainty of the wire extremity distance l and the WPS position x_{WPS} along are assumed to be of magnitude $1 \times 10^{-3}\,\mathrm{m}$. The uncertainty of the initial deflection of the wire is assumed to be of magnitude $1 \times 10^{-5}\,\mathrm{m}$. Also the magnitude of the uncertainty of the gravity g is assumed to be $1 \times 10^{-5}\,\mathrm{m\,s^{-2}}$. The uncertainty propagation was calculated using the METAS UNC-library in MATLAB [11]. The values were expected to be normal distributed and the propagation was assumed linear. The resulting uncertainty of the sag f and the vertical positions of the fundamental frequency dependant parabolic model is thus of magnitude $1 \times 10^{-6}\,\mathrm{m}$ for wires of $140\,\mathrm{m}$ length and accordingly better for shorter wires.

RESULTS

Resulting Fundamental Frequencies and Sags

Table 1 shows the results of the sag determination method applied to different measurement configurations and repeated oscillation measurements of the wires in the metrological measurement setup. For the shortest wire, the obtained sags vary $6\,\mathrm{\mu m}$ at most, for the intermediate wire they differ up to $29\,\mathrm{\mu m}$ and for the longest wire up to $32\,\mathrm{\mu m}$. The algorithmic error and the uncertainty of the sag have been estimated to not exceed the micrometer scale. Consequently, a repeatability of the sag determination within micrometer range was assumed but could not be verified in the experiments. Thus, either uncertainties were underestimated or the wire oscillation model ignores further effects.

Comparison between FFDPM and SDFM

The wire positions derived with the fundamental frequency dependant parabolic model (FFDPM) determination method are compared to static wire position measurements (SDFM) and the static parabolic fitting model on the latter.

The average sags of the resulting parabolic models are compared in Table 2. For the shortest wire they differ the most. Nevertheless, the difference stays less than $29\,\mathrm{\mu m}$. Schott Guilmault [6] analysed also the difference between the sag derived from oscillation measurements and the sag of the static parabolic fitting model. He obtained a difference of $277\,\mathrm{\mu m}$ between the two sags. With our adjustments of his approach this difference is decreased by one order of magnitude.

The static vertical wire position measurements can also directly be used as a validation reference for the vertical wire shape that is determined by the two vertical wire models. Table 3 shows the differences between the predicted vertical wire position by the parabolic models and the measured static vertical wire positions. Note, that for the determination of the fundamental frequency dependent parabolic model for the metrological measurement setup the vertical position of the wire at the outer WPS were considered to be given by the static measurements. Therefore, the difference in these positions equals zero. Over the range of all WPS positions, both parabolic models differ in the order of $1 \times 10^{-5}\,\mathrm{m}$ from the static vertical measurements. Being the best fitting parabolic model by definition, the differences of the static parabolic fitting model are smaller than the differences of the fundamental frequency dependent parabolic model.

DETECTION OF WIRE RESTRICTIONS

Besides the determination of the vertical shape of a stretched wire, the fundamental frequency determination method is found to be useful for the detection of wire restrictions. The stretched wires that are installed for the position monitoring and alignment of the LHC Inner Triplet cryo-assemblies are covered by a wire protection system. However, they might accidentally touch an object in their proximity or even get blocked at one point.

Table 1: Results of the fundamental frequency and the sag determination method applied to repeated oscillation measurements and different fibreglass finger positions (x_{finger}) and WPS positions (x_{WPS}) for the stretched wires in the metrological measurement setup.

wire	x_{WPS}	x_{finger}	ϕ [Hz]	f [m]
3	3-2	3-2	7.88472	0.004929
			7.88633	0.004927
			7.88713	0.004926
			7.88633	0.004927
			7.88944	0.004923
			7.88883	0.004924
2	2-2	2-4	4.16624	0.017654
			4.16683	0.017649
			4.16660	0.017651
	2-4	2-2	4.16365	0.017676
			4.16342	0.017678
			4.16318	0.017680
1	1-3	1-5	2.77252	0.039864
			2.77231	0.039870
			2.77239	0.039868
			2.77263	0.039861
			2.77260	0.039862
			2.77266	0.039860
			2.77246	0.039866
			2.77242	0.039867
	1-5	1-3	2.77284	0.039855
			2.77343	0.039838
			2.77246	0.039866
			2.77298	0.039851
			2.77267	0.039860
			2.77274	0.039858

Both scenarios influence the wire shape and deteriorate the reference line. Consequently, the touching object needs to be localized and removed. Due to the wire protection system that hides the wire, the localization can be time consuming. As an alternative to the manual localization of wire restrictions, approaches that include the determination of the fundamental frequency were studied for two scenarios.

Localization of Blocking Points

Given that the wire is totally blocked in one point, the fundamental frequency determination method can still be carried out. By inserting the resulting fundamental frequency in

$$l = \frac{1}{2\phi}\sqrt{\frac{T}{q}} \qquad (12)$$

the length of the remaining oscillating section of the wire can be estimated and thus the blocking point be localized. In the metrological measurement setup, the intermediate wire was intentionally blocked such that the remaining oscillating part had a length of $68.696\,\mathrm{m}$. The average resulting fundamental frequency derived from seven oscillation measurements was inserted in Eq. 12, leading to the result $l = 68.602\,\mathrm{m}$. Thus, the blocked point is localized with the accuracy of centimetres.

Localization of Wire Touching Objects

Given that the wire touches an object at one point but the wire is not completely blocked at this point. In this case, the effect is used that the wire oscillates also when it is deflected by the fibreglass finger and not only when it is released. During the deflection, the fibreglass finger blocks the wire and consequently separates it in two independently oscillating sections. The local restriction will be only in one of these sections and affects the determination of the fundamental frequency in this section. Nonetheless, two fundamental frequencies ϕ_1 and ϕ_2 of the wire sections can be calculated with the fundamental frequency determination method from the wire oscillations after the deflection. Subsequently, the product $\phi_1 \cdot l_1$ and $\phi_2 \cdot l_2$ can be computed. If the wire is untouched, the following equation holds

$$\phi \cdot l = \frac{1}{2}\sqrt{\frac{T}{q}}. \qquad (13)$$

Therefore, it can be checked if the products $\phi_1 \cdot l_1$ and $\phi_2 \cdot l_2$ equal the constant $\frac{1}{2}\sqrt{\frac{T}{q}}$ to determine the wire section that contains the touching object.

Repeating this procedure systematically by putting the fibreglass finger in different positions along the wire decreases the length of the section that contains the touching object.

DISCUSSION

The algorithm of Schott Guilmault to determine the fundamental frequency and thereof the vertical shape of a stretched wire has been revised. Comparisons of the resulting wire shape to static wire measurements and the best fitting parabolic model on these measurements have shown that the approach of Schott Guilmault has been improved by one order of magnitude. However, it remains impossible, due to the lack of repeatability of the determination of the fundamental frequency, to obtain the vertical wire shape with micrometric precision.

Nevertheless, the resulting vertical wire shape did not exceed a difference of $24\,\mu\mathrm{m}$ compared to static wire measurements for wires of up to $140\,\mathrm{m}$ length. Thus, the fundamental frequency determination method provides an alternative evaluation method to the static parabolic fitting method to obtain a vertical reference. The latter adds information to existing alignment setups and offers new configuration possibilities, especially regarding the number of required HLS in future projects. Furthermore,

Table 2: Comparison of the average wire sag of the fundamental frequency dependent parabolic model and the wire sag of the static parabola fitting model for the wires in the metrological measurement setup.

wire number	**1**	**2**	**3**
average sag according to fundamental frequency	$39.860\,\mathrm{mm}$	$17.665\,\mathrm{mm}$	$4.926\,\mathrm{mm}$
sag of static parabolic fitting model	$39.842\,\mathrm{mm}$	$17.667\,\mathrm{mm}$	$4.955\,\mathrm{mm}$
difference	$18\,\mu\mathrm{m}$	$2\,\mu\mathrm{m}$	$29\,\mu\mathrm{m}$

Table 3: Difference between the vertical wire position determined by the fundamental frequency dependent parabolic model (FFDPM) and the static parabolic fitting model (SPFM) and the measured static wire positions in the position of the WPS for the longest wire in the metrological measurement setup.

model	**WPS 1-1**	**WPS 1-2**	**WPS 1-3**	**WPS 1-4**	**WPS 1-6**	**WPS 1-7**
FFDPM	$0\,\mu\mathrm{m}$	$3\,\mu\mathrm{m}$	$20\,\mu\mathrm{m}$	$24\,\mu\mathrm{m}$	$5\,\mu\mathrm{m}$	$0\,\mu\mathrm{m}$
SPFM	$1\,\mu\mathrm{m}$	$6\,\mu\mathrm{m}$	$4\,\mu\mathrm{m}$	$7\,\mu\mathrm{m}$	$12\,\mu\mathrm{m}$	$6\,\mu\mathrm{m}$

the fundamental frequency is a helpful indicator for the localization of wire restrictions.

Future research could focus on the optimization of the oscillation measurement setup in order to improve the repeatability. Besides, further investigations in the vertical wire model, that include non-constant linear masses and different directions of gravity along the wire, could be conducted to decrease the model error.

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