

Goldstone boson decays and chiral anomalies

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Abstract

Martinus Veltman was the first to point out the inconsistency of the experimental value for the decay rate of $\pi^0 \rightarrow \gamma\gamma$ and its calculation by J. Steinberger with the very successful concept of the pion as the (pseudo)Nambu-Goldstone boson of the spontaneously broken global axial symmetry of strong interactions. That inconsistency has been resolved by J. Bell and R. Jackiw in their famous paper on the chiral anomalies. We review the connection between the decay amplitudes of an axion into two gauge bosons in Abelian vector-like and chiral gauge theories. The axion is the Nambu-Goldstone boson of a spontaneously broken axial global symmetry of the theory. Similarly as for the vector-like gauge theory, also in the chiral one the axion decay amplitude is determined by the anomaly of the current of the axial symmetry in its non-linear realization. Certain subtlety in the calculation of the anomaly in chiral gauge theories is emphasised.

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1 Introduction

In 1999, Martinus Veltman shared with Gerard t’Hooft the Nobel Prize in physics for their contribution to the proof of renormalisability of non-Abelian gauge theories. It is less remembered that he also was the first, together with D. Sutherland [1,2], to point out the inconsistency of the experimental value for the decay rate of $\pi^0 \rightarrow \gamma\gamma$ and its direct calculation by J. Steinberger [3] with the very successful concept of the pion as the (pseudo)Nambu-Goldstone boson (PNGB) of the spontaneously broken global axial symmetry of strong interactions. That inconsistency has been resolved by J. Bell and R. Jackiw in their famous paper on the chiral anomalies [4]. In beyond the Standard Model theories there may be new PNGBs that play important roles in particle physics and cosmology. The most famous example is the QCD axion that can solve the strong CP problem [5–7] and/or explain the origin of dark matter [8–10] (for a review, see [11]). Axion-like particles (ALPs) may also drive inflation [12,13] or make dark matter dynamical [14–16]. The important aspect of the ALPs physics is the link of their properties to the chiral anomalies. The PNGB playing the role of the QCD axion must have anomalous couplings to gluons, similarly as the pion to photons to explain the $\pi^0 \rightarrow \gamma\gamma$ decay. Such couplings are not needed for the ALPs that play the other roles mentioned above but their experimental signatures depend on whether the anomalous couplings are present or not.

Extensions of the Standard Model with ALPs in the particle spectrum have been under continuous research for various reasons. Some of them are: a global symmetry as a remnant of gauge symmetries to protect the axion potential against gravitational corrections [17–20], the potential link of ALPs to the fermion mass theories [21], ALPs in chiral gauge theories [22,23] and the experimental signatures of ALPs.

In this brief review we recall some selected topics and subtleties related to the link between the properties of ALPs and the global chiral anomalies. For simplicity (and capturing the main points) we work with global $U(1)$ and Abelian gauge symmetries.

2 Axion decay in gauge theories

2.1 Vector-like gauge theories

The model we consider first is defined by the Lagrangian with a local $U(1)$ symmetry:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + \overline{\psi}_L i \not{D} \psi_L + \overline{\psi}_R i \not{D} \psi_R + |\partial_\mu \phi|^2 - V(|\phi|^2) - (y\phi \overline{\psi}_L \psi_R + h.c.), \quad (2.1)$$

where $D_\mu = \partial_\mu - iqqA_\mu$ and the gauge symmetry is vector-like, that is the gauge charges of the left and right-handed Weyl fermions are: $q_L = q_R \equiv q$. Without loss of generality one can normalise the gauge charge as $q = 1$. The scalar field ϕ is a singlet of the gauge symmetry. The Lagrangian is classically invariant under two orthogonal vector and axial global symmetries, $U(1)_V$ and $U(1)_A$, respectively, defined by the transformations

$$\psi_{L,R} \rightarrow e^{iQ_{L,R}^{V,A}\theta} \psi_{L,R}, \quad \phi \rightarrow e^{iQ_\phi^{V,A}\theta} \phi, \quad (2.2)$$

with the charges

$$\begin{aligned} U(1)_V : \quad & Q_R^V = Q_L^V \equiv Q^V, \quad Q_\phi^V = 0, \\ U(1)_A : \quad & Q_R^A = -Q_L^A \equiv Q^A/2, \quad Q_\phi^A = -Q^A. \end{aligned} \quad (2.3)$$

Without loss of generality, the global charges are normalised as $Q^V = Q^A = 1$.¹ In the Dirac fermion notation, $\psi = (\psi_R, \psi_L)^T$, and $U(1)_V$ and $U(1)_A$ transformations are written as $\psi \rightarrow e^{i\theta}\psi$ and $\psi \rightarrow e^{i\gamma_5\theta}\psi$, respectively, where $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3 = \text{diag}(1, -1)$.

Associated with those global symmetries, one can find the Noether currents

$$\begin{aligned} J_V^\mu &= -(\bar{\psi}_L\gamma^\mu\psi_L + \bar{\psi}_R\gamma^\mu\psi_R) = -\bar{\psi}\gamma^\mu\psi, \\ J_A^\mu &= -\frac{1}{2}(\bar{\psi}_R\gamma^\mu\psi_R - \bar{\psi}_L\gamma^\mu\psi_L) + i(\phi^*\partial^\mu\phi - \phi\partial^\mu\phi^*) \\ &= -\frac{1}{2}\bar{\psi}\gamma^\mu\gamma_5\psi + (i\phi^*\partial^\mu\phi + \text{h.c.}). \end{aligned} \quad (2.4)$$

Classically, these currents are conserved; $\partial_\mu J_V^\mu = \partial_\mu J_A^\mu = 0$ (classically).

Note that since there are two orthogonal $U(1)$ symmetries, any linear combinations of them are also classical symmetries of the Lagrangian. For example, one can define the two symmetry axes as $Q_i^1 = \cos\varphi Q_i^V - \sin\varphi Q_i^A$ and $Q_i^2 = \sin\varphi Q_i^V + \cos\varphi Q_i^A$ with $i = L, R, \phi$. The corresponding symmetry currents $J_\mu^1 = \cos\varphi J_\mu^V - \sin\varphi J_\mu^A$ and $J_\mu^2 = \sin\varphi J_\mu^V + \cos\varphi J_\mu^A$ are also conserved classically.

Among infinitely many choices of global symmetry axes, $U(1)_V$ and $U(1)_A$ directions are special since a non-zero vacuum expectation value of the field ϕ

$$\phi = \frac{1}{\sqrt{2}}(f + \sigma)e^{ia(x)/f} \quad (2.5)$$

breaks spontaneously the $U(1)_A$, while its orthogonal one, $U(1)_V$, remains unbroken.² The physical spectrum of the theory below the scale f contains then the Nambu-Goldstone boson $a(x)$ of the spontaneously broken $U(1)_A$ symmetry, which we also call the axion, the massive Dirac fermion and the massless gauge boson, γ . The Lagrangian for these fields takes the form

$$\mathcal{L} \supset -\frac{1}{4}F_{\mu\nu}^2 + \bar{\psi}i\not{D}\psi - M\bar{\psi}\psi + \frac{1}{2}(\partial_\mu a)^2 - i\lambda a\bar{\psi}\gamma_5\psi + \dots, \quad (2.6)$$

where $M = \frac{yf}{\sqrt{2}}$, $\lambda = \frac{y}{\sqrt{2}}$ and (\dots) corresponds to the higher order terms of the axion field. The axial symmetry is realized non-linearly, by a shift on the axion field

$$a(x) \rightarrow a(x) - f\theta, \quad (2.7)$$

with the fermion fields transforming as in Eq. (2.2). By removing the σ field from Eq. (2.4), the axial symmetry current becomes

$$\tilde{J}_A^\mu = -\frac{1}{2}\bar{\psi}\gamma^\mu\gamma_5\psi - f\partial_\mu a(x). \quad (2.8)$$

In order to discuss phenomenology of axions, in particular the decay of axions, we add the axion mass term

$$-\frac{1}{2}m_a^2 a^2 \quad (2.9)$$

to our non-linear Lagrangian (2.6). This term breaks the $U(1)_A$ symmetry explicitly. With this modification, the axial current is conserved classically up to the axion mass parameter

$$\partial_\mu \tilde{J}_A^\mu = fm_a^2 a \quad (\text{classically}). \quad (2.10)$$

¹In this example, $U(1)_V$ transformation is a special case of the gauge transformation with the constant gauge transformation parameter. Still it is useful to talk about the $U(1)_V$ global symmetry here for later discussions.

²The more general case is discussed in detail in [23].

The axion decay rate into two gauge bosons can be calculated in the standard way. From Lorentz and CP invariance we see that the amplitude must be proportional to $\epsilon_{\mu\nu\rho\sigma} k_1^\mu k_2^\nu \epsilon_1^\rho \epsilon_2^\sigma$ where $k_{1,2}$ are the photon momenta and $\epsilon_{1,2}$ are their polarisation vectors. Since there is no direct coupling between the axion and gauge bosons, the leading contribution to the amplitude is given by triangle diagrams with fermions with mass $M = yf/\sqrt{2}$ running in the loop. The coupling between the axion and fermions is given by $-i\lambda = -iy/\sqrt{2}$, as can be seen in Eq. (2.6), and the amplitude picks up this coupling. The result reads

$$i\mathcal{M}(a \rightarrow \gamma\gamma) = q^2 \frac{ig^2}{4\pi^2} \frac{\lambda}{M} \left[1 + \mathcal{O}\left(\frac{m_a^2}{M^2}\right) \right] \epsilon_{\mu\nu\rho\sigma} k_1^\mu k_2^\nu \epsilon_1^\rho \epsilon_2^\sigma. \quad (2.11)$$

Note that the leading order term is independent of the Yukawa coupling λ , since $\lambda/M = 1/f$.

The above amplitude can be obtained at tree level by the effective Lagrangian:

$$\mathcal{L}_{\text{eff}} \ni \frac{1}{2}(\partial_\mu a)^2 - \frac{1}{2}m_a^2 a^2 + q^2 \frac{g^2}{16\pi^2 f} a F_{\mu\nu} \tilde{F}^{\mu\nu}. \quad (2.12)$$

Under a shift, $a(x) \rightarrow a(x) - f\theta$, the last term in the lagrangian Eq. (2.12) of this effective theory breaks the axial symmetry explicitly. The divergence of the axial current can be computed classically as

$$\partial_\mu \tilde{J}_A^\mu = fm_a^2 a(x) - q^2 \frac{g^2}{16\pi^2} F_{\rho\sigma} \tilde{F}^{\rho\sigma}. \quad (2.13)$$

This shows that there is a quantum effect which breaks the axial symmetry explicitly. We see that the leading contribution to the $a \rightarrow \gamma\gamma$ amplitude is directly related to this anomaly and that link will be reviewed in more detail in the next section. This anomalous violation of the global axial symmetry reconciles an apparent inconsistency of the decay rate for $\pi^0 \rightarrow \gamma\gamma$ with the concept of the pion as a pseudo-Nambu-Goldstone boson associated with the spontaneous breaking of (approximate) axial symmetry of strong interactions of the light quarks [4].

One can highlight this point by considering a model with another fermion pair (ψ'_L, ψ'_R) with the same gauge charge, $q'_L = q'_R = q$, and the opposite $U(1)_A$ charge compared to those of the original pair, (ψ_L, ψ_R) . Classical symmetries allow the Lagrangian to have the Yukawa term

$$-y'\phi^* \bar{\psi}'_L \psi'_R + \text{h.c.} \quad (2.14)$$

After ϕ acquires the vev in Eq. (2.5), the new fermions obtain the mass $M' = y'f/\sqrt{2}$. Since they couple to ϕ^* rather than ϕ (due to the opposite $U(1)_A$ charge), the coupling to the axion has the opposite sign, $i(y'/\sqrt{2})a\bar{\psi}'\gamma_5\psi'$, compared to the previous case. The new fermions give the same contribution to $i\mathcal{M}(a \rightarrow \gamma\gamma)$ as Eq. (2.11) but with the opposite sign.

The leading contributions to the $i\mathcal{M}(a \rightarrow \gamma\gamma)$ from ψ and from ψ' cancel out. This is consistent with the fact that the theory with the new fermion pair is free from the axial anomaly (see Sec.3). The next to leading terms in this case do not cancel and give

$$i\mathcal{M}(a \rightarrow \gamma\gamma) = q^2 \frac{ig^2}{4\pi^2 f} \left[\frac{m_a^2}{24} \left(\frac{1}{M^2} - \frac{1}{M'^2} \right) + \mathcal{O}\left(\frac{1}{M^4}\right) \right] \epsilon_{\mu\nu\rho\sigma} k_1^\mu k_2^\nu \epsilon_1^\rho \epsilon_2^\sigma. \quad (2.15)$$

Before closing this subsection, we comment on the case where the vector-like $U(1)$ gauge symmetry is broken by the Brout-Englert-Higgs mechanism. This can easily be realised by adding to the above model (2.1) a new scalar, ϕ' , with a non-vanishing gauge charge $q' \neq 0$ and assume that ϕ' gets a vev. The Yukawa terms for ϕ' is forbidden due to the non-zero gauge charge and

the previous calculation of the axion decay is unchanged except that the gauge bosons (we call them Z in this case) are now massive. We have

$$i\mathcal{M}(a \rightarrow ZZ) = q^2 \frac{ig^2}{4\pi^2 f} (1 + \Delta) \epsilon_{\mu\nu\rho\sigma} k_1^\mu k_2^\sigma \epsilon_1^\rho \epsilon_2^\sigma \quad (2.16)$$

with

$$\Delta = \frac{m_a^2 + 2m_Z^2}{24M^2} + \mathcal{O}\left(\frac{1}{M^4}\right). \quad (2.17)$$

It is somewhat amusing that the expression of the leading term of the axion decay amplitude is unchanged from the previous case with the unbroken $U(1)$ despite the fact that gauge bosons in this case have a longitudinal component. The latter effect is encapsulated in the polarization vectors $\epsilon_1(k_1)$ and $\epsilon_2(k_2)$, which are different from the ones for massless gauge bosons in Eq. (2.11).

2.2 Chiral gauge theories

When the gauge theory is chiral, the model of (2.1) needs extensions. First of all, when the gauge charges of left- and right-handed Weyl fermions that couple to a scalar, ϕ , are chiral ($q_L \neq q_R$), the gauge invariance of the Yukawa term requires that the scalar necessarily carries a non-zero gauge charge, $q_\phi = q_L - q_R \neq 0$. Therefore, in this case the vev of ϕ breaks a global $U(1)_A$ spontaneously and also breaks the local $U(1)$. Secondly, since the gauge boson acquires a mass, for the axion (the pseudo-Nambu-Goldstone boson of the $U(1)_A$ breaking) to remain in the physical spectrum one needs at least two scalars (or two phases) because one combination of them is eaten up by the Brout-Englert-Higgs mechanism.

We illustrate these points in an explicit model. Our model contains two scalars (ϕ_1, ϕ_2) and one pair of fermions (ψ_L, ψ_R).³ We assume ϕ_1 and ϕ_2 have non-zero but different gauge charges $q_{\phi_1} \neq q_{\phi_2}$ and $q_{\phi_1} = q_L - q_R \neq 0$. In this case, only ϕ_1 can have a gauge invariant Yukawa term with the fermions;

$$\mathcal{L} \ni -y\phi_1 \overline{\psi_L} \psi_R + \text{h.c.} \quad (2.18)$$

We assume both ϕ_1 and ϕ_2 develop non-zero vevs; $\langle \phi_i \rangle = f_i \neq 0$ ($i = 1, 2$). Writing $\phi_i = \frac{1}{\sqrt{2}}(f_i + \sigma_i(x))e^{ia_i(x)/f_i}$, the phase degrees of freedom transform as $a_i(x) \rightarrow a_i(x) + q_{\phi_i} f_i \alpha(x)$ under the gauge transformation. Therefore, defining

$$\begin{aligned} a(x) &= \cos \varphi a_1(x) - \sin \varphi a_2(x), \\ \tilde{a}(x) &= \sin \varphi a_1(x) + \cos \varphi a_2(x), \end{aligned} \quad (2.19)$$

with

$$\cos \varphi = \frac{q_{\phi_2} f_2}{\tilde{f}}, \quad \sin \varphi = \frac{q_{\phi_1} f_1}{\tilde{f}}, \quad \tilde{f} = \sqrt{(q_{\phi_1} f_1)^2 + (q_{\phi_2} f_2)^2}, \quad (2.20)$$

$\tilde{a}(x)$ transforms as $\tilde{a}(x) \rightarrow \tilde{a}(x) + \tilde{f} \alpha(x)$, while $a(x)$ is invariant under the gauge transformation. We can thus identify $\tilde{a}(x)$ as the would-be Nambu-Goldstone boson to be eaten by the gauge boson and $a(x)$ remains physical in the low energy spectrum.

Similarly as for the vector-like gauge theory, classically, the theory has two global symmetries: $U(1)_V$ and $U(1)_A$. The $U(1)_A$ symmetry ($Q^A(\psi_R) = -Q^A(\psi_L) = \frac{1}{2}$, $Q^A(\phi_1) = -1$) is spontaneously broken by $\langle \phi_1 \rangle = f_1$. The Nambu-Goldstone mode of this broken symmetry is $a_1(x)$,

³We assume the existence of additional fermions that cancel the $[U(1)]^3$ gauge anomaly. Such fermions can always be introduced so that they do not couple to the scalars and do not modify the axion decay.

which can be expressed in terms of the physical field $a(x)$ and the would-be Nambu-Goldstone boson $\tilde{a}(x)$ as $a_1(x) = \cos \varphi a(x) + \sin \varphi \tilde{a}(x)$. At the leading order, the interaction between the physical axion $a(x)$ and the fermions is given by

$$\mathcal{L} \ni i \frac{y \cos \varphi}{\sqrt{2}} a(x) \bar{\psi}_L \psi_R + \text{h.c.} = i \frac{y \cos \varphi}{\sqrt{2}} a(x) \bar{\psi} \gamma_5 \psi. \quad (2.21)$$

In the last expression, we combine the Weyl fermions into the four-component Dirac spinor field as $\psi = (\psi_R, \psi_L)^T$. In the Dirac spinor notation, the fermion kinetic term is organised as

$$i \bar{\psi} \gamma^\mu (\partial_\mu - ig [\alpha - \beta \gamma_5] A_\mu - M) \psi, \quad (2.22)$$

where $\alpha = (q_L + q_R)/2$, $\beta = (q_L - q_R)/2$ and $M = y f_1 / \sqrt{2}$. The lagrangian is invariant under the $U(1)_A$ symmetry realized non-linearly by a shift $\cos \phi a(x) \rightarrow \cos \phi a(x) + f_1 \theta$ and Eq. (2.8) holds with replacements $a(x) \rightarrow \cos \phi a(x)$ and $f \rightarrow f_1$.

Understanding the axion-fermion and gauge boson-fermion interactions in Eqs. (2.21) and (2.22), respectively, we are ready to compute the axion decay amplitude, $a \rightarrow ZZ$, in this scenario. A diagrammatic calculation of the amplitude is performed in Appendix A and the result reads [23]

$$i\mathcal{M}(a \rightarrow ZZ) = \frac{i \cos \varphi g^2}{4\pi^2 f_1} \left[\left(\alpha^2 + \frac{1}{3} \beta^2 \right) + \Delta \right] \epsilon^{\mu\nu\rho\sigma} \epsilon_\mu^1 \epsilon_\nu^2 q_1^\rho q_2^\sigma, \quad (2.23)$$

where Δ is higher order terms in m_a^2/M^2 and m_Z^2/M^2 given in Eq. (A.23). Eq. (2.12) and Eq. (2.13) remain true, with the replacements as above and $q^2 \rightarrow (\alpha^2 + \frac{1}{3} \beta^2)$.

The leading term with the factor $(\alpha^2 + \beta^2/3)$ is related to the axial anomaly, similarly as for the axion decay to two vector gauge currents. In the next section we provide an explicit calculation of the mixed anomaly in the three-current-correlator of the axial current and two gauge currents, and interpret the factor $(\alpha^2 + \beta^2/3)$ from the anomaly view point.

3 Axion decays and chiral anomaly

We consider the three current correlator $\langle \tilde{J}_A^\rho(x) J_G^\mu(y) J_G^\nu(z) \rangle$, where we use a short-handed notation, $\langle \dots \rangle \equiv \langle \Omega | T \{ \dots \} | \Omega \rangle$, for Green functions of a time-ordered product. The global axial current and gauge currents are defined as

$$\tilde{J}_A^\rho = J_5^\rho - f \partial^\rho a(x), \quad J_G^\mu = -\bar{\psi}(x) \gamma^\mu (\alpha - \beta \gamma_5) \psi(x), \quad (3.1)$$

where we introduced the fermionic part of the axial current

$$J_5^\rho \equiv -\frac{1}{2} \bar{\psi} \gamma^\rho \gamma_5 \psi. \quad (3.2)$$

A vector-like gauge theory implies $\beta = 0$, while a chiral gauge theory can be examined with $\beta \neq 0$.

Classically, the gauge current is conserved exactly, while the axial current is conserved up to the axion mass, since it breaks $U(1)_A$ explicitly. On the equation of motion, classically we have

$$\begin{aligned} \partial_\mu J_G^\mu &= 0, \\ \partial_\mu J_5^\mu(x) &= M \bar{\psi} F(a) \psi \\ \partial_\mu \tilde{J}_A^\mu(x) &= f m_a^2 a(x), \end{aligned} \quad (3.3)$$

with $F(a) \equiv \sin(a/f) - i\gamma_5 \cos(a/f)$. At the quantum level, these relations *without anomalies* would imply

$$\begin{aligned}
i(k_1)_\mu \Gamma_5^{\rho\mu\nu}(-p, k_1, k_2) &\stackrel{?}{=} i(k_2)_\nu \Gamma_5^{\rho\mu\nu}(-p, k_1, k_2) \stackrel{?}{=} 0, \\
-ip_\rho \Gamma_5^{\rho\mu\nu}(-p, k_1, k_2) &\stackrel{?}{=} f \Omega^{\mu\nu}(-p, k_1, k_2), \\
-ip_\rho \tilde{\Gamma}_A^{\rho\mu\nu}(-p, k_1, k_2) &\stackrel{?}{=} f m_a^2 \Delta^{\mu\nu}(-p, k_1, k_2),
\end{aligned} \tag{3.4}$$

in the momentum space, respectively, where

$$\begin{aligned}
\Gamma_5^{\rho\mu\nu}(-p, k_1, k_2) &\equiv \langle J_5^\rho(-p) J_G^\mu(k_1) J_G^\nu(k_2) \rangle, \\
\Delta^{\mu\nu}(-p, k_1, k_2) &\equiv \langle a(-p) J_G^\mu(k_1) J_G^\nu(k_2) \rangle, \\
\Omega^{\mu\nu}(-p, k_1, k_2) &\equiv \frac{M}{f} \langle \bar{\psi} F(a) \psi \cdot J_G^\mu(k_1) J_G^\nu(k_2) \rangle, \\
\tilde{\Gamma}_A^{\rho\mu\nu}(-p, k_1, k_2) &\equiv \langle \tilde{J}_A^\rho(-p) J_G^\mu(k_1) J_G^\nu(k_2) \rangle \\
&= \Gamma_5^{\rho\mu\nu}(-p, k_1, k_2) + i f p^\rho \Delta^{\mu\nu}(-p, k_1, k_2).
\end{aligned} \tag{3.5}$$

Here we notice that $\Delta^{\mu\nu}(-p, k_1, k_2)$ and $\Omega^{\mu\nu}(-p, k_1, k_2)$ are related to the axion decay amplitude at the leading order as

$$\begin{aligned}
\Delta^{\mu\nu}(-p, k_1, k_2) &= \frac{i}{p^2 - m_a^2} \mathcal{M}^{\mu\nu}(a \rightarrow ZZ) \\
\Omega^{\mu\nu}(-p, k_1, k_2) &= \mathcal{M}^{\mu\nu}(a \rightarrow ZZ),
\end{aligned} \tag{3.6}$$

with

$$i\mathcal{M}(a(p) \rightarrow Z(k_1)Z(k_2)) = (ig)^2 \epsilon_\mu^{*1}(k_1) \epsilon_\nu^{*2}(k_2) \mathcal{M}^{\mu\nu}(a \rightarrow ZZ), \tag{3.7}$$

and at the heavy fermion mass limit we have (see Appendix A)

$$\mathcal{M}^{\mu\nu}(a \rightarrow ZZ) = \frac{i}{4\pi^2 f} \left(\alpha^2 + \frac{1}{3} \beta^2 \right) \epsilon^{\mu\nu\rho\sigma} (k_1)_\rho (k_2)_\sigma. \tag{3.8}$$

Our goal here is to check whether Eqs. (3.4) are indeed hold at the quantum level.

In the following we sketch the calculation of the fermion triangle contribution to those Ward identities. Note that in our toy model, mimicing the linear pion-nucleon σ model, spontaneous breaking of the axial symmetry gives a Nambu-Goldstone boson in the spectrum and simultaneously is the origin of the Dirac fermion mass. Thus, the calculation of the triangle contribution to the three-current correlator with one of the currents being J_5^ρ has to be performed with the massive Dirac fermion in the loop.

The leading contribution to $\Gamma_5^{\rho\mu\nu}(-p, k_1, k_2)$ is obtained by two triangle diagrams. Their contribution is given by⁴

$$\begin{aligned}
\Gamma_5^{\rho\mu\nu}(-p, k_1, k_2) &= (-1) \cdot (i)^3 \cdot \left(-\frac{1}{2} \right) \cdot \int \frac{d^4 q}{(2\pi)^4} \left\{ \right. \\
&\quad \text{Tr} \left[\gamma^\rho \gamma_5 \frac{(\not{q} - \not{k}_1) + M}{(q - k_1)^2 - M^2} \gamma^\mu (\alpha - \beta \gamma_5) \frac{\not{q} + M}{k^2 - M^2} \gamma^\nu (\alpha - \beta \gamma_5) \frac{(\not{q} + \not{k}_2) + M}{(q + k_2)^2 - M^2} \right] \\
&\quad \left. + [(k_1, \mu) \leftrightarrow (k_2, \nu)] \right\}
\end{aligned} \tag{3.9}$$

⁴In Eq. (3.9), the (-1) factor is due to the fermion loop, $(i)^3$ is from the three propagators and the $(-1/2)$ factor comes from Eq. (3.2).

The last term $[(k_1, \mu) \leftrightarrow (k_2, \nu)]$ comes from the diagram with the external momenta interchanged with respect to the first one. We first use

$$(\alpha - \beta\gamma_5)(\not{q} + M)\gamma^\nu(\alpha - \beta\gamma_5) = [(\alpha^2 + \beta^2)(\not{q} + M) - 2\beta^2 M - 2\alpha\beta\gamma_5\not{q}]\gamma^\nu. \quad (3.10)$$

We see that the first term, $(\alpha^2 + \beta^2)(\not{q} + M)\gamma^\nu$, produces exactly the same expression as the vector-like gauge theory with $J_G^\mu = i\sqrt{\alpha^2 + \beta^2} \cdot \bar{\psi}\gamma^\mu\psi$. It is well known that the result of this part is subject to the ambiguity originating from shifts of loop momenta, since each diagram is separately divergent. We will come back to this point shortly. The contribution from the second term, $-2\beta^2 M\gamma^\nu$, can be computed straightforwardly since each diagram is separately finite. Finally, the third term, $-2\alpha\beta\gamma_5\not{q}\gamma^\nu$, does not contribute to the anomaly, since it will not produce ϵ -tensor due to the γ_5 .

We are interested in the limit of the fermion mass $M \rightarrow \infty$ to check the link between the leading, Yukawa coupling independent, term in the axion decay amplitude and the mixed anomaly of the three-current correlator. Taking the fermion mass to infinity, $M \rightarrow \infty$, we obtain the following result:

$$\begin{aligned} (k_1)_\mu \Gamma_5^{\rho\mu\nu}(-p, k_1, k_2) &= \frac{i}{4\pi^2} \epsilon^{\rho\nu\alpha\beta} (k_1)_\alpha (k_2)_\beta \left[+ \frac{1}{4}(2 + c_2)(\alpha^2 + \beta^2) - \frac{\beta^2}{3} \right], \\ (k_2)_\nu \Gamma_5^{\rho\mu\nu}(-p, k_1, k_2) &= \frac{i}{4\pi^2} \epsilon^{\rho\mu\alpha\beta} (k_1)_\alpha (k_2)_\beta \left[- \frac{1}{4}(2 - c_1)(\alpha^2 + \beta^2) + \frac{\beta^2}{3} \right], \\ p_\rho \Gamma_5^{\rho\mu\nu}(-p, k_1, k_2) &= \frac{i}{4\pi^2} \epsilon^{\mu\nu\alpha\beta} (k_1)_\alpha (k_2)_\beta \left[+ \frac{1}{4}(c_1 - c_2)(\alpha^2 + \beta^2) - \left(\alpha^2 + \frac{1}{3}\beta^2 \right) \right], \end{aligned} \quad (3.11)$$

where c_1 and c_2 are some real numbers parametrising the ambiguity originated from the shift of loop momenta; $q_\alpha \rightarrow (q + l)_\alpha$ in the first diagram and $q_\alpha \rightarrow (q + r)_\alpha$ in the second one with $(l - r)_\alpha = c_1(k_1)_\alpha + c_2(k_2)_\alpha$. We observe that choosing

$$c_1 = -c_2 = 2 \frac{\alpha^2 + \frac{1}{3}\beta^2}{\alpha^2 + \beta^2} \quad (3.12)$$

all vanish simultaneously

$$(k_1)_\mu \Gamma_5^{\rho\mu\nu}(-p, k_1, k_2) = (k_2)_\nu \Gamma_5^{\rho\mu\nu}(-p, k_1, k_2) = p_\rho \Gamma_5^{\rho\mu\nu}(-p, k_1, k_2) = 0. \quad (3.13)$$

Vanishing of $k_1 \cdot \Gamma_5$ and $k_2 \cdot \Gamma_5$ is consistent with the gauge current conservation, expected classically in Eqs. (3.4). On the other hand, the last equation differs from the second line of Eqs. (3.4). This implies that the axial anomaly cancels the classical non-conservation piece of the fermionic axial current. This can be also seen in the following way. By contracting p_ρ with $\Gamma_5^{\rho\mu\nu}$ and writing $\not{p} = \not{k}_1 + \not{k}_2 = [-(\not{q} - \not{k}_1) - M] + [(\not{q} - \not{k}_2) - M] + 2M$, the trace of Eq. (3.9) becomes

$$\begin{aligned} & \text{Tr} \left[\gamma_5 \gamma^\mu (\alpha - \beta\gamma_5) \frac{\not{q} + M}{q^2 - M^2} \gamma^\nu (\alpha - \beta\gamma_5) \frac{(\not{q} + \not{k}_2) + M}{(q + k_2)^2 - M^2} \right] \\ + & \text{Tr} \left[\gamma_5 \frac{(\not{q} - \not{k}_1) + M}{(q - k_1)^2 - M^2} \gamma^\mu (\alpha - \beta\gamma_5) \frac{\not{q} + M}{q^2 - M^2} \gamma^\nu (\alpha - \beta\gamma_5) \right] \\ + & 2M \text{Tr} \left[\gamma_5 \frac{(\not{q} - \not{k}_1) + M}{(q - k_1)^2 - M^2} \gamma^\mu (\alpha - \beta\gamma_5) \frac{\not{q} + M}{q^2 - M^2} \gamma^\nu (\alpha - \beta\gamma_5) \frac{(\not{q} + \not{k}_2) + M}{(q + k_2)^2 - M^2} \right]. \end{aligned} \quad (3.14)$$

In this expression, the contribution from the first two lines are quadratically divergent. Adding these terms together with the corresponding pieces from the second diagrams gives a finite and

M -independent result (the term proportional $(\alpha^2 + \beta^2)$ in the last line of Eq. (3.11)) since it comes from the UV part of the momentum integral. One can interpret this part of the contribution as the anomaly.

On the other hand, the last line of Eq. (3.14) has the exactly the same expression as the axion decay amplitude up to the $(-f)$ factor, which corresponds to the $(\alpha^2 + \beta^2/3)$ term in the last line of Eq. (3.11). As can be seen in Eqs. (3.4) and (3.6), this part can be interpreted as the classical non-conservation piece, $if\Omega^{\mu\nu} = if\mathcal{M}^{\mu\nu}(a \rightarrow ZZ)$, and the cancellation between the anomaly and the classical non-conservation piece can be understood.

Using this result and Eq. (3.5), we have

$$\begin{aligned} p_\rho \tilde{\Gamma}_A^{\rho\mu\nu} &= ifp^2 \Delta^{\mu\nu} \\ &= if \left(m_a^2 \frac{i}{p^2 - m_a^2} \mathcal{M}^{\mu\nu}(a \rightarrow ZZ) + i \mathcal{M}^{\mu\nu}(a \rightarrow ZZ) \right) \\ &= ifm_a^2 \Delta^{\mu\nu} - f \mathcal{M}^{\mu\nu}(a \rightarrow ZZ), \end{aligned} \quad (3.15)$$

where we have used $p^2 = m_a^2 + (p^2 - m_a^2)$ and Eq. (3.6). Compared this with the last line of Eq. (3.4), we see that $p_\rho \tilde{\Gamma}_A^{\rho\mu\nu}$ has a piece that is not present in the classical relation. We call this the *anomaly piece of the divergence of the axial current*.

The fact that the anomaly piece is proportional to the axion decay amplitude can also be understood by the following argument. The fact that the axion decay amplitude is given by Eq.(2.23) (in our present case, $f_1 = f$ and $\cos \varphi = 1$) implies the effective Lagrangian must have terms,

$$\mathcal{L}_{\text{eff}} \ni \frac{1}{2}(\partial_\mu a)^2 - \frac{1}{2}m_a^2 a^2 + \left(\alpha^2 + \frac{1}{3}\beta^2 \right) \frac{g^2}{16\pi^2 f} a F_{\mu\nu} \tilde{F}^{\mu\nu}. \quad (3.16)$$

Calculating the divergence of the axial current, $\tilde{J}_A^\mu = f\partial^\mu a$, in this effective theory, one finds

$$\partial_\mu \tilde{J}_A^\mu = fm_a^2 a - \left(\alpha^2 + \frac{1}{3}\beta^2 \right) \frac{g^2}{16\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu}. \quad (3.17)$$

This is consistent with the above result (3.15) and clarifies the relation between the anomaly and the axion decay amplitude.

4 Summary

We have reviewed the calculation of the axion decay amplitudes into two gauge bosons in vector-like and chiral $U(1)$ gauge theories and its connection to the chiral anomalies. The axion is a (pseudo)Nambu-Goldstone boson (or its component invariant under gauge transformations) of the axial $U(1)_A$ global symmetry of the lagrangian. The leading contribution to the decay amplitude depends on whether the gauge theory is vector-like or chiral. In both cases it is directly linked to the anomalous divergence of the current of the axial global symmetry. The calculation of the divergence of the current-current-current Green's function requires a special attention.

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A Appendix: Calculation of the axion decay

We compute the $a \rightarrow ZZ$ amplitude with general interactions and a mass. The Lagrangian is given by:

$$\mathcal{L} \ni \bar{\psi} \gamma^\mu [i\partial_\mu + g(\alpha - \beta\gamma_5) A_\mu] \psi - M\bar{\psi}\psi - i\lambda\alpha\bar{\psi}\gamma_5\psi \quad (\text{A.1})$$

The matrix element takes a form

$$i\mathcal{M}(a(p) \rightarrow Z(k_1)Z(k_2)) = (ig)^2 \epsilon_\mu^{*1}(k_1) \epsilon_\nu^{*2}(k_2) \mathcal{M}^{\mu\nu}(a \rightarrow ZZ), \quad (\text{A.2})$$

where

$$\begin{aligned} \mathcal{M}^{\mu\nu}(a \rightarrow ZZ) = & (-1)\lambda(i)^3 \int \frac{d^4q}{(2\pi)^4} \left\{ \right. \\ & \text{Tr} \left[\gamma_5 \frac{(\not{q} - \not{k}_1) + M}{(k - k_1)^2 - M^2} \gamma^\mu (\alpha - \beta\gamma_5) \frac{\not{q} + M}{k^2 - M^2} \gamma^\nu (\alpha - \beta\gamma_5) \frac{(\not{q} + \not{k}_2) + M}{(k + k_2)^2 - M^2} \right] \\ & \left. + [(k_1, \mu) \leftrightarrow (k_2, \nu)] \right\} \quad (\text{A.3}) \end{aligned}$$

Since the matrix element should be invariant under the simultaneous exchange $(k_1, \mu) \leftrightarrow (k_2, \nu)$, it has to be proportional to $k_1^\mu k_2^\nu$ or $\epsilon^{\mu\nu\rho\sigma} k_1^\rho k_2^\sigma$. For both cases, the integral is convergent since $\sim \int d^4q \frac{k^2 q}{(q^2)^3}$.

First note that the numerator of the first trace can be organised as

$$\begin{aligned} & \text{Tr} [\gamma_5 [(\not{q} - \not{k}_1) + M] \gamma^\mu (\omega_+ \not{q} + \omega_- M) \gamma^\nu [(\not{q} + \not{k}_2) + M]] \\ & - 2\alpha\beta \text{Tr} [[(\not{q} - \not{k}_1) - M] \gamma^\mu \not{q} \gamma^\nu [(\not{q} + \not{k}_2) + M]] \quad (\text{A.4}) \end{aligned}$$

where $\omega_\pm \equiv \alpha^2 \pm \beta^2$. One can calculate these traces using the formulae

$$\text{Tr} [\text{odd \# of } \gamma's] = 0, \quad (\text{A.5})$$

$$\text{Tr} [\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma] = 4(g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma} + g^{\mu\sigma} g^{\nu\rho}), \quad (\text{A.6})$$

$$\text{Tr} [\gamma_5 \gamma^\mu \gamma^\nu] = 0, \quad (\text{A.7})$$

$$\text{Tr} [\gamma_5 \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma] = -4i\epsilon^{\mu\nu\rho\sigma}. \quad (\text{A.8})$$

In the first trace of Eq. (A.4), the M^3 term vanishes due to Eq. (A.7). The M^2 term and the mass independent term also vanish since they have odd numbers of γ matrices. The only non-vanishing term in the first trace of Eq. (A.4) is linear in M and may be calculated in the form

$$4i\epsilon^{\mu\nu\rho\sigma} M [\omega_-(k_1)_\rho (k_2)_\sigma + 2\beta^2 k_\rho (k_1 + k_2)_\sigma]. \quad (\text{A.9})$$

The non-vanishing term in the second trace of Eq. (A.4) must have four γ matrices. This term can be calculated as

$$8\alpha\beta M [k^\mu (k_1 + k_2)^\nu + k^\nu (k_1 + k_2)^\mu - g^{\mu\nu} k \cdot (k_1 + k_2)]. \quad (\text{A.10})$$

We are left with the evaluation of the momentum integration with the denominator. One must calculate

$$\int \frac{d^4 q}{(2\pi)^4} (a + b^\alpha q_\alpha) \frac{1}{[(q - k_1)^2 - M^2]} \frac{1}{[q^2 - M^2]} \frac{1}{[(q + k_2)^2 - M^2]} \quad (\text{A.11})$$

Using the Feynman parameter formula

$$\frac{1}{A_1 A_2 \cdots A_n} = \int dx_1 \cdots dx_n \delta(\sum x_i - 1) \frac{(n-1)!}{[x_1 A_1 + x_2 A_2 + \cdots x_n A_n]^n} \quad (\text{A.12})$$

Eq. (A.11) becomes

$$\begin{aligned} & 2 \int \frac{d^4 q}{(2\pi)^4} \int_0^1 dx \int_0^{1-x} dy \frac{a + b^\alpha q_\alpha}{[x((q - k_1)^2 - M^2) + y((q + k_2)^2 - M^2) + (1-x-y)(q^2 - M^2)]^3} \\ &= 2 \int \frac{d^4 q}{(2\pi)^4} \int_0^1 dx \int_0^{1-x} dy \frac{a + b^\alpha q_\alpha}{[q^2 + 2q \cdot (yk_2 - xk_1) + (x+y)m_Z^2 - M^2]^3}, \end{aligned} \quad (\text{A.13})$$

where $k_1^2 = k_2^2 \equiv m_Z^2$ has been used. The $\int d^4 q$ integral can be performed by using the formula [25]

$$\int \frac{d^4 q}{(2\pi)^4} \frac{a + b^\alpha q_\alpha}{[k^2 + 2k \cdot p + M^2]^n} = \frac{i}{16\pi^2} \frac{\Gamma(n-2)}{\Gamma(n)} \frac{a - b^\alpha p_\alpha}{[M^2 - p^2]^{n-2}}. \quad (\text{A.14})$$

The result reads

$$-\frac{i}{16\pi^2} \int_0^1 dx \int_0^{1-x} dy \frac{a + b^\alpha (xk_1 - yk_2)_\alpha}{M^2 - (x+y-x^2-y^2)m_Z^2 - xy(m_a^2 - 2m_Z^2)}, \quad (\text{A.15})$$

where $2k_1 \cdot k_2 = m_a^2 - 2m_Z^2$ was used. Let's assume the fermion mass in the loop is much larger than the masses of the axion and the gauge boson, $M \gg m_a, m_Z$. To get the leading order expression, we take $m_a, m_Z \rightarrow 0$. Then we finally find Eq. (A.11) to be

$$-\frac{i}{32\pi^2} \frac{1}{M^2} \left[a + \frac{1}{3} b^\alpha (k_1 - k_2)_\alpha \right]. \quad (\text{A.16})$$

Now we combine this result with the numerators (A.9) and (A.10). First, we note that the fact that $\int d^4 q \cdot q_\alpha$ term is proportional to $(k_1 - k_2)_\alpha$ implies that the pieces in Eq. (A.10) do not contribute to the amplitude. This can be seen by replacing q with $(k_1 - k_2)$ in Eq. (A.10);

$$8\alpha\beta M [(k_1 - k_2)^\mu (k_1 + k_2)^\nu + (k_1 - k_2)^\nu (k_1 + k_2)^\mu - g^{\mu\nu} (k_1^2 - k_2^2)]. \quad (\text{A.17})$$

The last term vanishes since $k_1^2 = k_2^2 = m_Z^2$. The first two terms cancel when they are contracted with the polarization tensors $\epsilon_\mu^1 \epsilon_\nu^2$ and demand $\epsilon^1 \cdot k_1 = \epsilon^2 \cdot k_2 = 0$.

Now what is left is the pieces that come from Eq. (A.9). The result can be obtained by taking $a = 4i\epsilon^{\mu\nu\rho\sigma} M(\alpha^2 - \beta^2)k_1^\rho k_2^\sigma$ and $b^\alpha = 8i\epsilon^{\mu\nu\rho\sigma} M\beta^2 \delta_\rho^\alpha (k_1 + k_2)^\sigma$ in Eq. (A.16). This leads to

$$\frac{1}{8\pi^2} \frac{1}{M} \epsilon_{\mu\nu\rho\sigma} \left[(\alpha^2 - \beta^2) k_1^\rho k_2^\sigma + \frac{2}{3} \beta^2 (k_1 - k_2)^\rho (k_1 + k_2)^\sigma \right] = \frac{1}{8\pi^2} \frac{1}{M} \left(\alpha^2 + \frac{1}{3} \beta^2 \right) \epsilon_{\mu\nu\rho\sigma} k_1^\rho k_2^\sigma. \quad (\text{A.18})$$

The contribution from the second trace in Eq. (A.3) can be obtained by replacing $(k_1, \mu) \leftrightarrow (k_2, \nu)$, which is identical. Therefore, the final result is obtained as

$$\mathcal{M}^{\mu\nu}(a \rightarrow ZZ) = \frac{i\lambda}{4\pi^2 M} \left(\alpha^2 + \frac{1}{3} \beta^2 \right) \epsilon^{\mu\nu\rho\sigma} k_1^\rho k_2^\sigma + \mathcal{O}(m_a^2, m_Z^2). \quad (\text{A.19})$$

Let's find out the next-to-leading terms in Eq. (A.19) that are linear in m_a^2 and m_Z^2 . The next higher order terms in the expansion of Eq. (A.15) go as

$$-\frac{i}{32\pi^2} \frac{1}{M^2} \frac{1}{24M^2} \left[a(m_a^2 + 2m_Z^2) + \frac{1}{5}(2m_a^2 + 3m_Z^2)b^\alpha(k_1 - k_2)_\alpha \right]. \quad (\text{A.20})$$

Due to the $(k_1 - k_2)$ structure, there is no contribution from Eq. (A.10), and the contribution from Eq. (A.9) can be obtained by taking $a = 4i\epsilon^{\mu\nu\rho\sigma} M(\alpha^2 - \beta^2)k_1^\rho k_2^\sigma$ and $b^\alpha = 8i\epsilon^{\mu\nu\rho\sigma} M\beta^2\delta_\rho^\alpha(k_1 + k_2)^\sigma$. This leads to

$$\begin{aligned} & \frac{1}{8\pi^2} \frac{1}{M} \frac{1}{24M^2} \epsilon_{\mu\nu\rho\sigma} \left[(\alpha^2 - \beta^2)(m_a^2 + 2m_Z^2)k_1^\rho k_2^\sigma + \frac{2}{5}\beta^2(2m_a^2 + 3m_Z^2)(k_1 - k_2)^\rho(k_1 + k_2)^\sigma \right] \\ &= \frac{1}{8\pi^2} \frac{1}{M} \frac{1}{24M^2} \left[(m_a^2 + 2m_Z^2)\alpha^2 + \frac{1}{5}(3m_a^2 + 2m_Z^2)\beta^2 \right] \epsilon_{\mu\nu\rho\sigma} k_1^\rho k_2^\sigma. \end{aligned} \quad (\text{A.21})$$

So, the final result up to the next-to-leading order is

$$\mathcal{M}(a(p) \rightarrow Z(k_1)Z(k_2)) = -\frac{\lambda g^2}{4\pi^2 M} \left[\left(\alpha^2 + \frac{1}{3}\beta^2 \right) + \Delta \right] \epsilon^{\mu\nu\rho\sigma} \epsilon_\mu^{*1} \epsilon_\nu^{*2} k_1^\rho k_2^\sigma \quad (\text{A.22})$$

with

$$\Delta = \frac{1}{24M^2} \left[(m_a^2 + 2m_Z^2)\alpha^2 + \frac{1}{5}(3m_a^2 + 2m_Z^2)\beta^2 \right] + \left(\text{higher order in } \frac{m_a^2}{M^2}, \frac{m_Z^2}{M^2} \right). \quad (\text{A.23})$$

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