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SO(10) Model Today

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Abstract

The SO(10) model has got some determined results because it may interpret the unification of the three interaction. The limit of the Z' mass is the 495 GeV $< m_{Z'} < 10^9\ GeV$. The formulas of the width and asymmetry for Z' decay will only depend on the Z' masses. We apply the method of Boudjema et al for identifying a theoretical origin of Z' boson between SO(10) and other models.

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1. The establishment of the Standard Model[1] (SM or the Glashow-Weinberg-Salam theory) is one of the major accomplishments in particle physics. The SM is compatible with known experimental data, including a wide variety of low energy neutral current[2]. But there are many question that cannot be answered satisfactorily within the framework of the SM. For example, why are there so many undetermined free parameters (21 of them)? Why is Higgs put in by hand and not yet found on experiment? Why is the gauge structure a product of three gauge groups rather than just a single one? Why ... ...?

In order to try to answer before mentioned question in SM many authors sought a fundamental theory which will reduce to the SM at present energies. On the other hand many theorists attempt to seek a contradiction between the experimental data and SM. However they are often disappointed. If we think that the electromagnetism and the weak and strong interactions ought to unifies then last a few year high-precision experimental results[3] shall be the contradiction. Because they can not be interpreted by SM. In order to interpret the contradiction we must go to beyond SM. There are many beyond the SM. However at present only SO(10) and the supersymmetric[1] SU(5) model may interpret the unification. This will express that SO(10) and supersymmetric SU(5) model has been supported by the experiment. Both these models will merite more attention in comparison with other beyond SM. In other words whether we may consider that SO(10) and supersymmetric SU(5) models may be first candidacy of the models after SM. For this reason both these models will necessitate further research.

2. The SO(10) model[4] has already been explored by a number of authors. It included only one extra fermion, one extra neutral gauge boson (Z' boson) and 32 extra non-neutral gauge bosons more than standard model. This one extra fermion as

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[1]The supersymmetry model is yet needful in cosmology because it will be expected to interpret 90% dark matter

right-handed neutrino ν_R will be necessary if the neutrino mass is not zero. In order to unify the electromagnetism and the weak and the strong interactions for SO(10) model, the pattern of the spontaneous symmetry breaking (SSB) to be adopted must be as follows:

$$
\begin{aligned}
SO(10) \quad & \underline{M_G} \quad SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \\
& \underline{M_R} \quad SU(3)_c \times SU(2)_L \times U(1)_R \times U(1)_{B-L} \\
& \underline{M_{Z'}} \quad SU(3)_c \times SU(2)_L \times U(1)_Y \\
& \underline{M_Z} \quad SU(3)_c \times U(1)_{em}
\end{aligned} \tag{1}
$$

If we adopts the experimental values at $q^2 = M_Z^2$ of $\alpha_3 = 0.1134$ and $\alpha_2^{-1} = 29.7$, the two straight lines of the running coupling constant for SU(2) and SU(3) based on the renormalization group equations should meet at[5]

$$M_G \sim 10^{17}\ GeV \tag{2}$$

as illustrated in Fig.1. (That is Rosner's Fig.18(b)[5]). That is which the mass scale of the 30 extra non-neutral gauge bosons is about $10^{17}\ GeV$. They are too heavy and the accelerator can not arrive at the energy in the near future. So We will leave the 30 too heavy gauge bosons undone. The $U(1)_R$ must behave as shown in Fig.1 because we adopts a symmetry breaking scheme in which $SU(3)_c$ and $U(1)_{B-L}$ are unified at M_G scale. The $U(1)_R$ coupling strength becomes equal to that of $SU(2)_L$ at M_R mass scale

$$M_R \sim 10^9\ GeV \tag{3}$$

That is which the mass scale of the 2 extra non-neutral gauge bosons (that is $W_R^{\pm}$) is about $10^9\ GeV$. They are also very heavy and the accelerator is not easy to arrive at the energy in the near future. So we will not research the 2 heavy gauge bosons gauge in a hurry. The limit of the mass scale of Z' boson will be $m_Z < m_{Z'} < 10^9\ GeV$ in the SO(10) model. Because recent experiment gave Z' a limit of the mass, $m_{Z'} > 495\ GeV$[6], so the limit of the Z' mass scale ought to be corrected into 495 $GeV < m_{Z'} < 10^9\ GeV$. The Z' mass scale in the range of its limit is yet comparative heavy and that the accelerator arrive at the energy is not yet impossible for the future. In especial when Z' mass is not far away from low limit. Therefore Z' boson will be worthy of further research. The Z' boson has already been explored by a number of authors[7] over a decade ago. However the past research for Z' boson (included SO(10) model) was different from today. The common point of the past research was that their models have not been supported by experiment. so first assumed the existence of Z' boson then discussed their physical characters and quantities from theory or detailed comparisons between the data and theoretical predictions. Now the SO(10) model has been supported by the experiment that unify the three interactions and itself has natural included the Z' boson that is not again assumed by any persons. The running gauge coupling constants have completely determined except $g_c(m_{Z'}^2)$ for $U(1)_{B-L}$ and $g_{1R}(m_{Z'}^2)$ for $U(1)_R$

$$g_c^{-2}(m_{Z'}^2) = g_c^{-2}(M_G^2) - \frac{4}{(4\pi)^2}\, \ell n(\frac{m_{Z'}^2}{M_G^2}) \tag{4}$$

$$g_{1R}^{-2}(m_{Z'}^2) = g_{1R}^{-2}(M_G^2) \frac{4}{(4\pi)^2}\, \ell n(\frac{m_{Z'}^2}{M_G^2}) + \frac{1}{(4\pi)^2} \cdot \frac{10}{3} \cdot \ell(\frac{M_R^2}{M_G^2}) \tag{5}$$

The $g_c^{-2}(m_{Z'}^2)$ and $g_{1R}^{-2}(m_{Z'}^2)$ will be related to the coupling constant of Z' gauge boson and only depend on the Z' mass.

3. The Z' boson field and the B^0 gauge boson field of the $U(1)_Y$ group in the standard model are the translation of the group O(2) for C^0 boson field of the $U(1)_{B-L}$ group and W_R^0 boson field of the $U(1)_R$ group. If we use the notation and formula of Ref.[4,8] they may write as

$$\begin{pmatrix} B^0 \\ Z' \end{pmatrix} = O(2) \begin{pmatrix} W_R^0 \\ C^0 \end{pmatrix} \tag{6}$$

$$O(2) = \begin{pmatrix} \frac{(\frac{3}{2})^{1/2} g_c}{(g_{1R}^2 + \frac{3}{2} g_c^2)^{1/2}} & \frac{g_{1R}}{(g_{1R}^2 + \frac{3}{2} g_c^2)^{1/2}} \\ \frac{g_{1R}}{(g_{1R}^2 + \frac{3}{2} g_c^2)^{1/2}} & \frac{-(\frac{3}{2})^{1/2} g_c}{(g_{1R}^2 + \frac{3}{2} g_c^2)^{1/2}} \end{pmatrix} \tag{7}$$

The general form for the coupling of a Z' to fermions will be

$$\mathcal{L} = g\bar{f}[\gamma_\mu(V_f + A_f\gamma_5)]f \cdot Z'_\mu \tag{8}$$

If we use the formula of Ref.[4] then g, V_f and A_f may write as

$$\begin{cases} V_u = \frac{1}{2}(g_c^2 - g_{1R}^2) \\ A_u = \frac{1}{2}\, g_{1R}^2 \end{cases} \qquad for\ \ up\ \ quark \tag{9}$$

$$\begin{cases} V_d = \frac{1}{2}(g_c^2 + g_{1R}^2) \\ A_d = \frac{-1}{2}\, g_{1R}^2 \end{cases} \qquad for\ \ down\ \ quark \tag{10}$$

$$\begin{cases} V_\ell = \frac{1}{2}(-3g_c^2 + g_{1R}^2) \\ A_\ell = \frac{-1}{2}\, g_{1R}^2 \end{cases} \qquad for\ \ leptons \tag{11}$$

$$g = \frac{1}{2(g_{1R}^2 + \frac{3}{2}g_c^2)^{1/2}} \tag{12}$$

It is obvious that the running coupling constants of the Z' boson to fermions will only depend on the Z' mass. For this reason the width of the Z' decay into fermions depend only on the Z' mass too for SO(10) model.

$$\Gamma_{Z' \to f\bar{f}} = \frac{m_{Z'}}{12\pi}(1 - \frac{4m_f^2}{m_{Z'}^2})^{1/2}\{[\mid gV_f \mid^2 + \mid gA_f \mid^2] + \frac{2m_f^2}{m_{Z'}^2}[\mid gV_f \mid^2 - 2 \mid gA_f \mid^2]\} \tag{13}$$

If $m_{Z'}^2 >> 2m_f^2$, that is a very reasonable except for t quark, then eq.(13) may be reduced to

$$\Gamma_{Z' \to f\bar{f}} = \frac{m_{Z'}}{12\pi}[\mid gV_f \mid^2 + \mid gA_f \mid^2] \tag{14}$$

It is very easy to obtain the values and the curves of the width of the Z' decay if we use the formulas above mentioned. They display in the Table and the Figure 2. These values and curves will be inspected by further experiment though the SO(10) model has been supported by one experiment. It will be beneficial that the Z' boson of the SO(10) model compare with the Z' boson in other beyond SM before the Z' boson is not discovered by the experiment. Because there are many beyond standard models including Z' bosons (MIZ'), but they are not yet confirmed by the experiment. In these case, identifying the theoretical origin of the Z' boson from the numerous beyond SM will be profitable.

The strategy of Boudjema-Lynn-Renard-Verzegnassi[10] (BLRV) is very effective for identifying a theoretical origin of the Z' boson in a wide variety of models. The BLRV strategy is expressed by curves (or strips) of the $R_{5,6}$ versus $\Gamma_{Z'-\mu\bar{\mu}}/M_{Z'}$ plane, where $\Gamma_{Z'-\mu\bar{\mu}}$ is the partial width of the Z' decay into the muonic pair, where $\Gamma_{Z'-\mu\bar{\mu}}$ is the partial width of the Z' decay into the muonic pair.

$$\Gamma_{Z'-\sum_{i=1}^{5,6} q_i\bar{q}_i} \tag{15}$$

is the partial width of the Z' decay into the five or six known quarks pair, and

$$R_{5,6} \equiv \Gamma_{Z'-\sum_{i=1}^{5,6} q_i\bar{q}_i}/\Gamma_{Z'-\mu\bar{\mu}} \tag{16}$$

This strategy requires the preliminary measurement of the muonic pair width $\Gamma_{Z'-\mu\bar{\mu}}$ and of the ratio $R_{5,6}$ of the Z' resonance. They worked in the Born approximation, neglecting one-loop radiative corrections, whose the effect is smaller than the experimental errors of the various widths and ratios. In the $(R_{5,6},\ \Gamma_{Z'-\mu\bar{\mu}}/M_{Z'})$ plane the two MIZ' and the four alternative models belong to completely different regions except for their Z'_{V} boson. In order to differentiate among the three candidate models they further discussed longitudinal polarized asymmetries. The direct production of an Z' boson will be problematic both for future $p\bar{p}$ coliders and LEP for $M_{Z'} \geq 200\ GeV$. If the Z' is in the considered mass range, 495 GeV $\leq m_{Z'} \leq 1\ TeV$, then it will be possible to discover an Z' boson with a future e^+e^- collider with total energy up to 1 TeV and the measurement of its partial width including the top quark will also be possible.

The BLRV strategy has been applied to six MIZ' and has very well differentiated these models, so it will be very clean and convenient to make detailed comparison between the data and theoretical predictions. However, the SU(10) model as a MIZ' has not yet been identified using their strategy. It is worthwhile to make the analysis before mentioned, otherwise we will miss one candidate. Especially the SO(10) model has been supported by the experiment. $R_{5,6}$ and $\Gamma_{Z' \to \mu\bar{\mu}}$ in SO(10) model only depend on the Z' mass. Therefore it is very easy to obtain the curves of the $R_{5,6}$ versus $\Gamma_{Z' \to \mu\bar{\mu}}/M_{Z'}$ plane if we use the formulas before mentioned. They display in the Table and the Figure 3 and the Figure 4 when Z' masses are within 500 GeV to 10 TeV. In the Figure 3 and Figure 4 the scope of the variance of the curves are very small regardless of the $R_{5,6}$ or $\Gamma_{Z' \to \mu\bar{\mu}}/M_{Z'}$ that their scope of the variance of the values will not be over 0.1. These values may be considered as a determinate values and directly compared with experimental values if the errors of their experiments are not over 0.1. The Figure 5 (Figure 6) is a comparison between Figure 3 (Figure 4) and Figure 4 (Figure 5) of BLRV. It is very obvious in Figure 5 (Figure 6) that the curves in Figure 3 (Figure 4) and the strip of the left-right symmetric model (LRM)[11], the superstring inspired E_6 model[4] have not one common intersection. This result is consistent with the experiment that SO(10) model has been supported by the experiment, LRM and E_6 model has not. The Figure 7 (Figure 8) is another comparison between Figure 3 (Figure 4) and Figure 6 (Figure 7) of BLRV. Figure 6 and Gigure 7 of BLRV include other four MIZ' except for LRM and E_6. In the Figure 7 and 8 the curves of the SO(10) model and other four MIZ' have not also any common intersection. Because three composite models (Y, Y_L, Z^*) have not yet been differentiated in Figure 7 and 8 and are confused. BLRV used a polarized asymmetric method to eliminate the confusion of the composite models, so we will use the polarized asymmetric method for this SO(10) model in order to get rid of the confusion of the three composite models and compare with SO(10) model further. Based on the general method[10] of polarized asymmetries on Z' and equations (8)-(11), it will be very easy to get formulas for the three polarized asymmetries for this SO(10) model. Because the formulas of the asymmetry in $Z' \to f\bar{f}$ from the tree level are as follows (detailed in reference [8] and [13])

$$A_{LR}^{h,SO(10)} \equiv A_e^{SO(10)} = \frac{N_L - N_R}{N_L + N_R} \simeq \frac{2V_l A_l}{A_l^2 + A_l^2} \tag{17}$$

$$A_{FB}^{u,SO(10)} \equiv A_u^{SO(10)} = \frac{3}{2}\frac{V_u A_u}{V_u^2 + A_u^2} \ . \tag{18}$$

$$A_{FB}^{d,SO(10)} \equiv A_d^{SO(10)} = \frac{3}{2}\frac{V_d A_d}{V_d^2 + A_d^2} \tag{19}$$

It is very obvious from equation (8)-(11) and (4)-(5) that $A_e^{SO(10)}$, $A_u^{SO(10)}$ and $A_d^{SO(10)}$ will only depend on the Z' masses. It is very easy to calculate the values of the $A_e^{SO(10)}$, $A_u^{SO(10)}$ and $A_d^{SO(10)}$ between 500 GeV to 10 TeV of the Z' masses (Table). A comparison on the $A_{u,d}$ versus A_e plane between the SO(10) model and the six models of BLRV is given in Figure 9 and 10. It appears at a glance that the small line of $A_{u,d}^{SO(10)}$ versus $A_e^{SO(10)}$ with the six models has not any common intersections. These are consistent with Figure 5 to Figure 8. These results express that SO(10) model differntiate from other six model. The SO(10) model has been supported by the experiment, other six models have not.

4. The SO(10) model has been supported by the experiment that unifies the electromagnetism and the weak and strong interactions. Therefore the SO(10) model has simultaneous got some results as follows. i) The values of the mass scales of the SSB are $M_G \approx 10^{17}\ GeV$ and $M_R \approx 10^9\ GeV$. ii) The mass scale of the 30 gauge bosons is about $10^{17}\ GeV$. iii) The mass scale of the $w_R^{\mp}$ bosons is about $10^9\ GeV$. iv) The limit of the Z' mass is the $495\ GeV < m_{Z'} < 10^9\ GeV$. v) The values of the running gauge coupling constants have completely determined except $g_c(q^2)$ for $U(1)_{B-L}$ and $g_{1R}(q^2)$ for $U(1)_R$ and $g_c(m_{Z'}^2)$ and $g_{1R}(m_{Z'}^2)$ will only depend on the Z' mass. Therefore the running coupling constants and the width of the Z' decay into fermions will only depend on The Z' mass

We apply the strategy of BLRV for identifying a theoretical origin of Z' boson between SO(10) and other six models. The results are that SO(10) model with other six models has not any common intersection. In other words the SO(10) model is completely differentiated from other six models. We will believe SO(10) as compared with other six models. Because the SO(10) model has been supported by the experiment but other six models have not yet. Of course the final outcome will be future experiment.

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$m_{Z'}(GeV)$	500	600	700	800	900	1000	2000	3000	4000	5000	6000	7000	8000	9000	10000
$\Gamma_{Z'\to\mu\bar{\mu}}$	0.22	0.26	0.31	0.35	0.39	0.44	0.88	1.33	1.78	2.23	2.69	3.14	3.59	4.05	4.50
$\Gamma_{Z'\to d\bar{d},s\bar{s},b\bar{b}}$	0.41	0.49	0.57	0.65	0.74	0.82	1.65	2.49	3.34	4.18	5.03	5.88	6.74	7.59	8.44
$\Gamma_{Z'\to u\bar{u},c\bar{c}}$	0.21	0.14	0.16	0.19	0.21	0.23	0.47	0.71	0.96	1.20	1.44	1.69	1.94	2.18	2.43
R_5	6.645	6.654	6.661	6.663	6.660	6.658	6.675	6.683	6.687	6.692	6.695	6.797	6.699	6.701	6.702
R_6	7.173	7.182	7.190	7.194	7.192	7.189	7.209	7.218	7.223	7.229	7.232	7.235	7.237	7.240	7.242
$10^3\Gamma_{Z'\to\mu\bar{\mu}}/M_{Z'}$	0.436	0.437	0.437	0.438	0.438	0.439	0.442	0.444	0.446	0.447	0.448	0.449	0.449	0.450	0.450
A_d	-0.657	-0.657	-0.657	-0.658	-0.658	-0.658	-0.658	-0.658	-0.658	0.659	-0.659	-0.659	-0.659	-0.659	-0.659
A_u	-0.424	0.424	-0.425	-0.425	-0.425	-0.426	-0.428	-0.429	-0.430	-0.431	0.432	-0.432	-0.432	-0.433	-0.433
A_e	0.919	0.919	0.919	0.918	0.918	0.917	0.915	0.913	0.912	0.911	0.910	0.909	0.909	0.908	0.908

Table

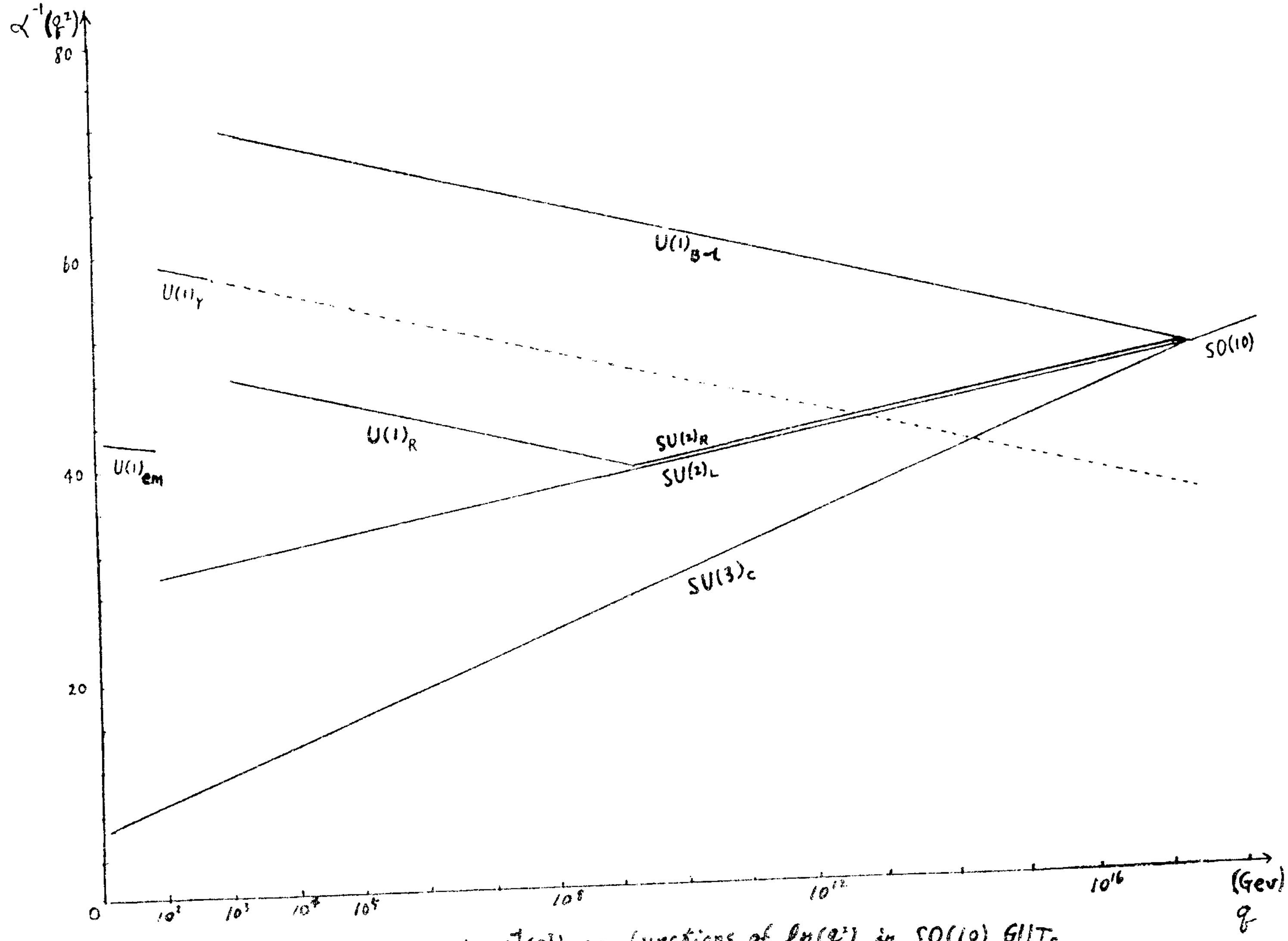

Figure 1. The coupling constant $\alpha^{-1}(q^2)$ as functions of $\ln(q^2)$ in SO(10) GUTs.

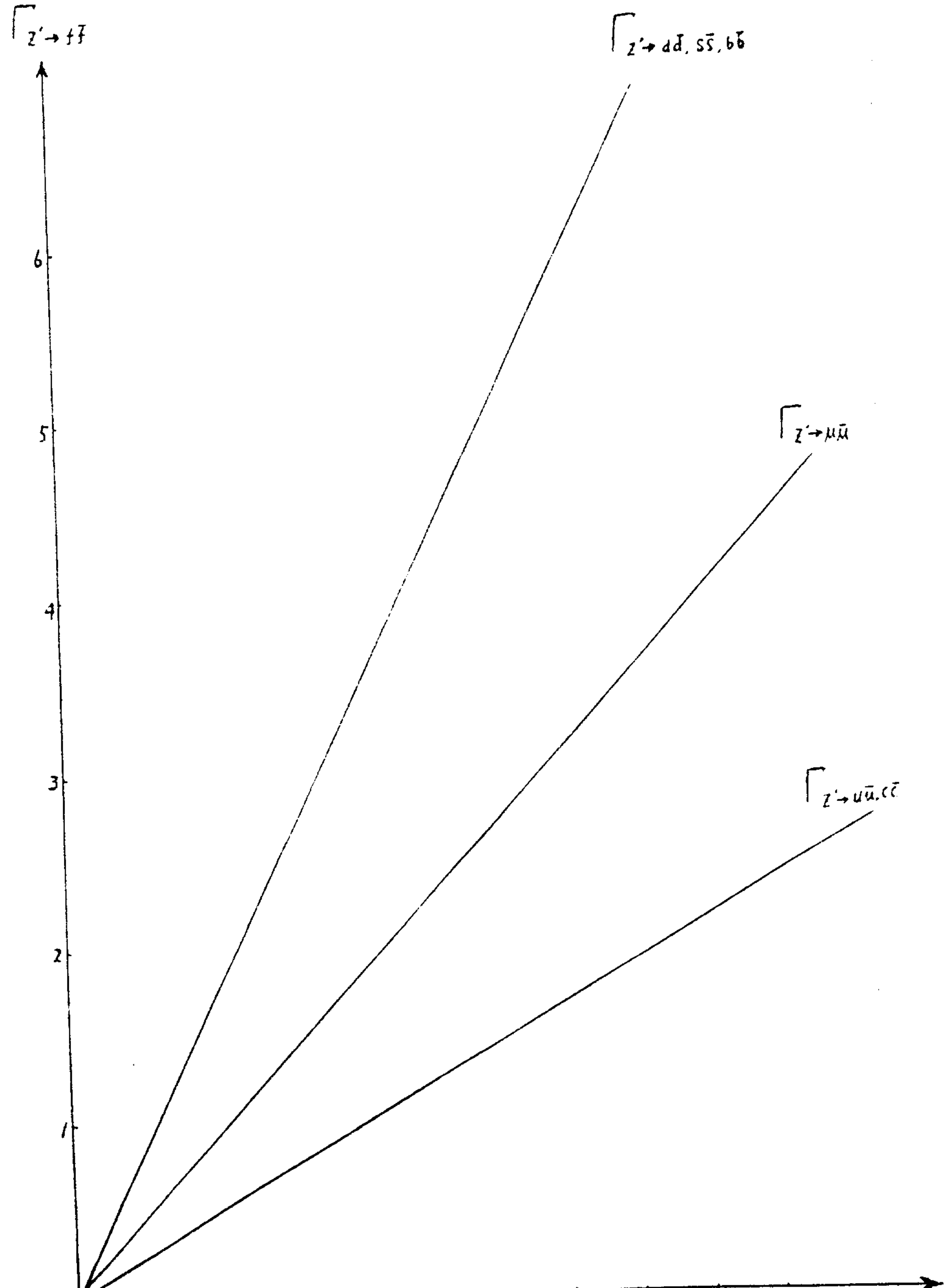

Figure 2. The $\Gamma_{Z'\to f\bar{f}}$ versus $m_{Z'}$ for SO(10) model

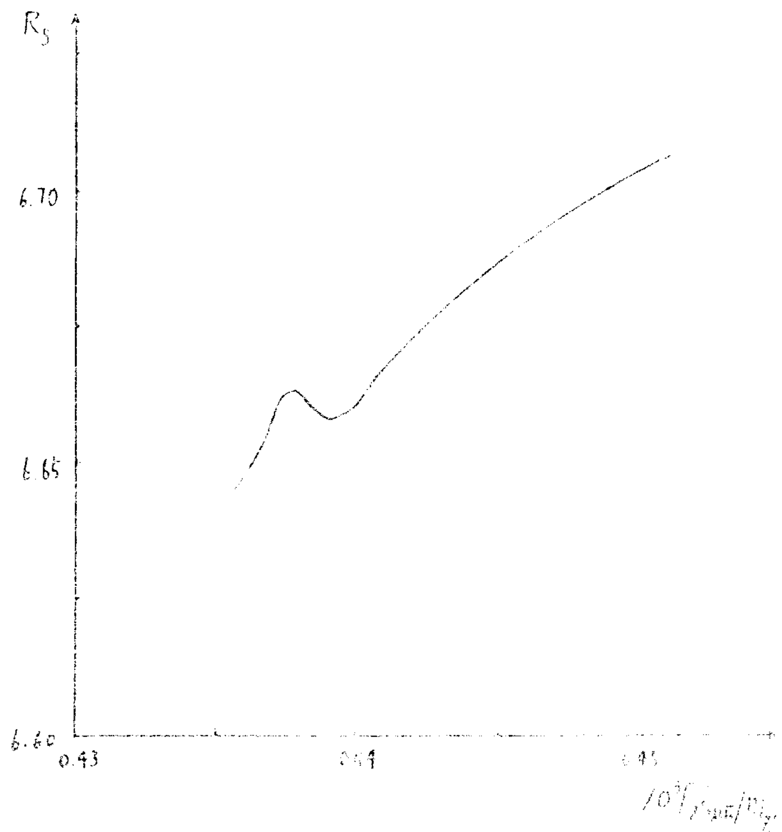

Figure 3. The ratio R_5 versus $\Gamma_{Z'\to\mu\mu}/m_{Z'}$ for SO(10) model

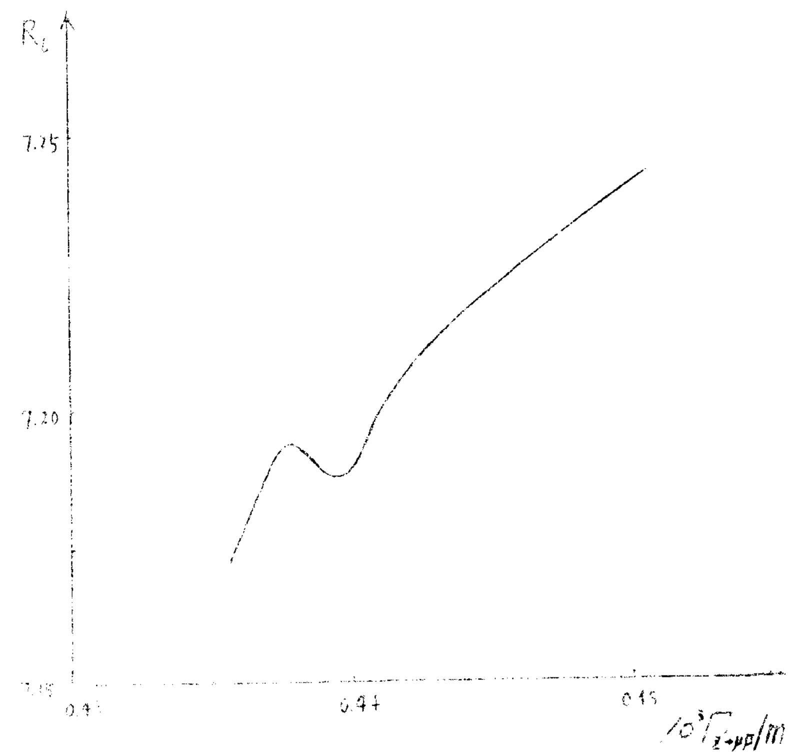

Figure 4. The ratio R_6 versus $\Gamma_{Z'\to\mu\mu}/m_{Z'}$ for SO(10) model

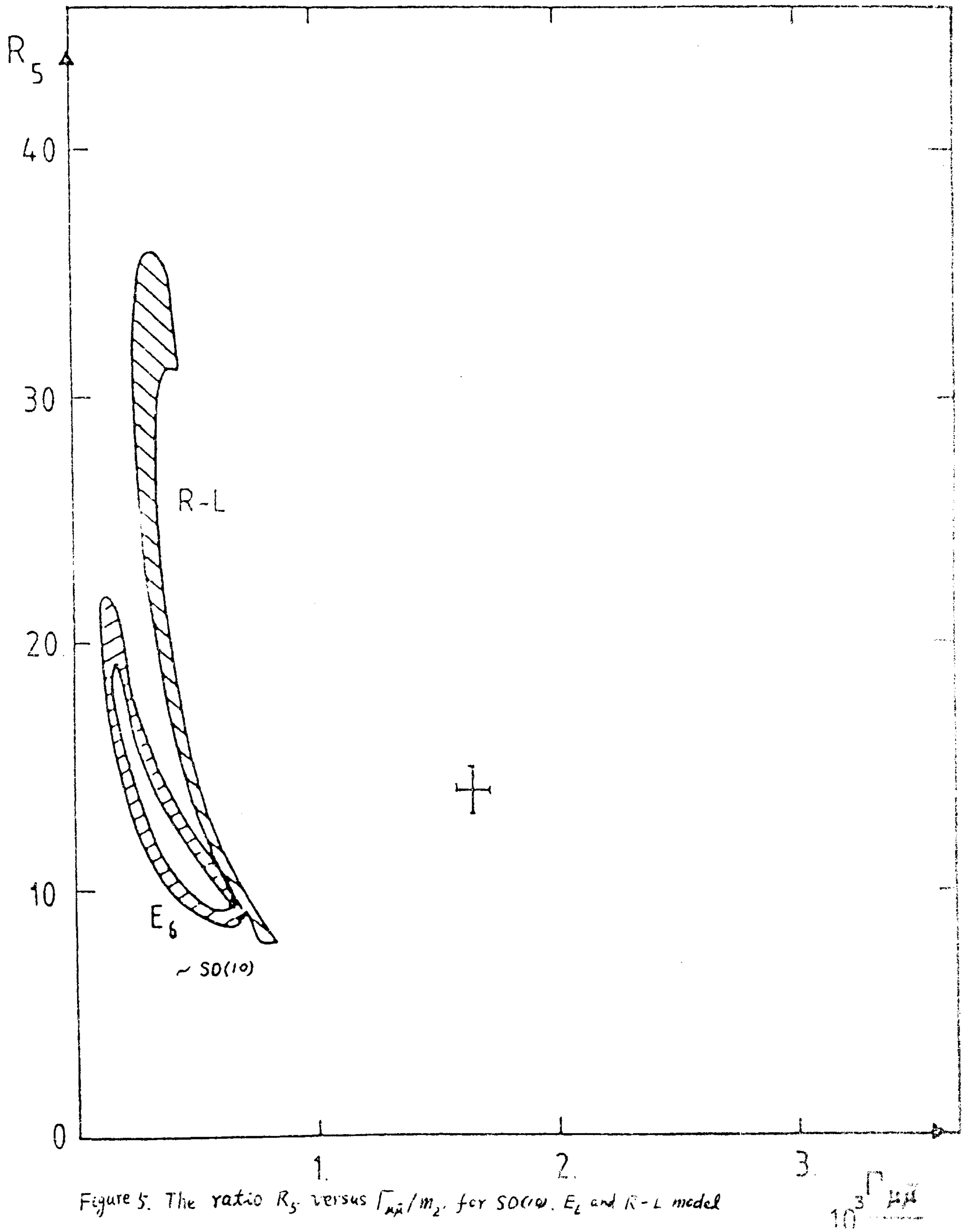

Figure 5. The ratio R_5 versus $\Gamma_{\mu\bar{\mu}}/m_Z$ for SO(10), E_6 and R-L model

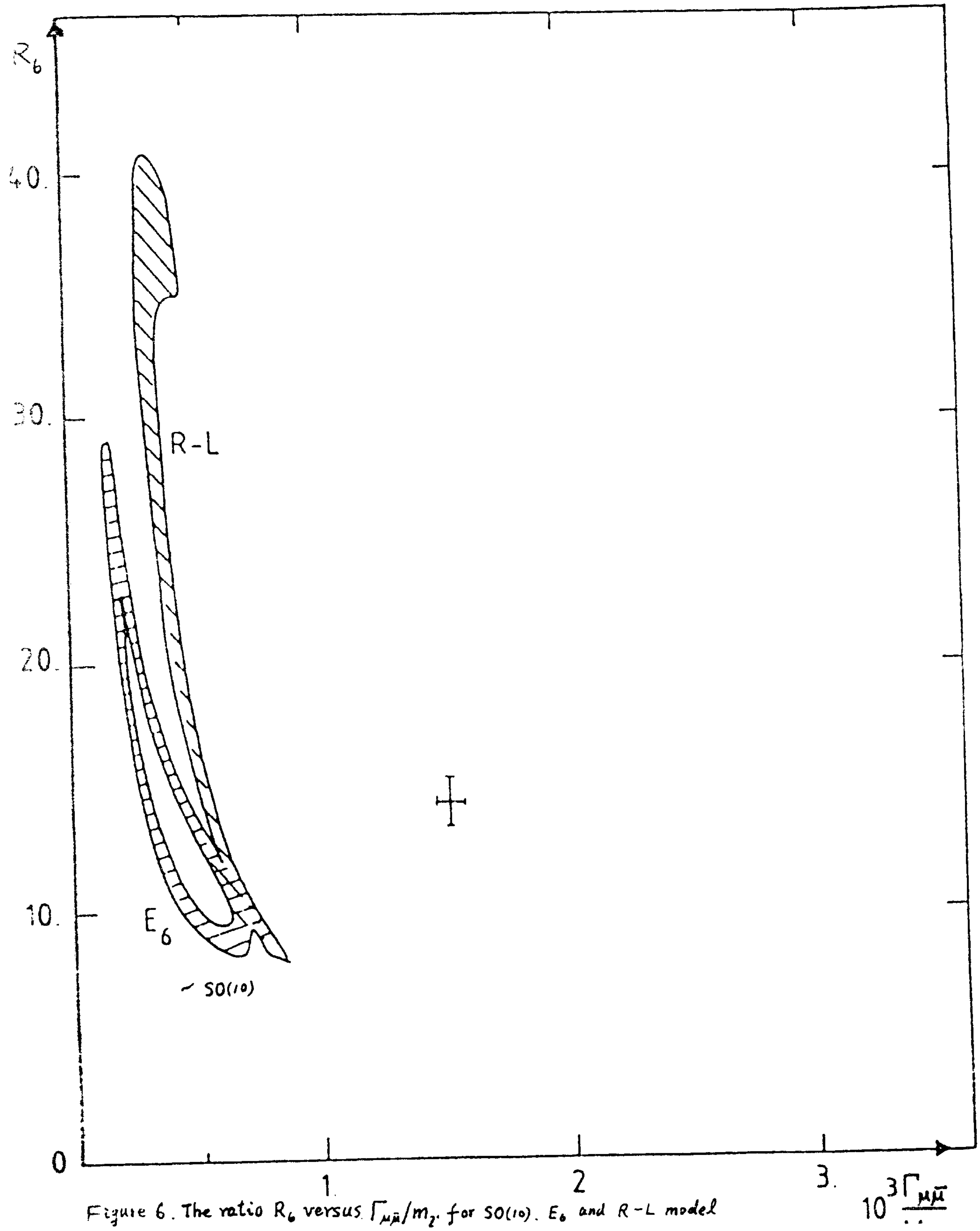

Figure 6. The ratio R_6 versus $\Gamma_{\mu\bar{\mu}}/m_{Z'}$ for SO(10), E_6 and R-L model

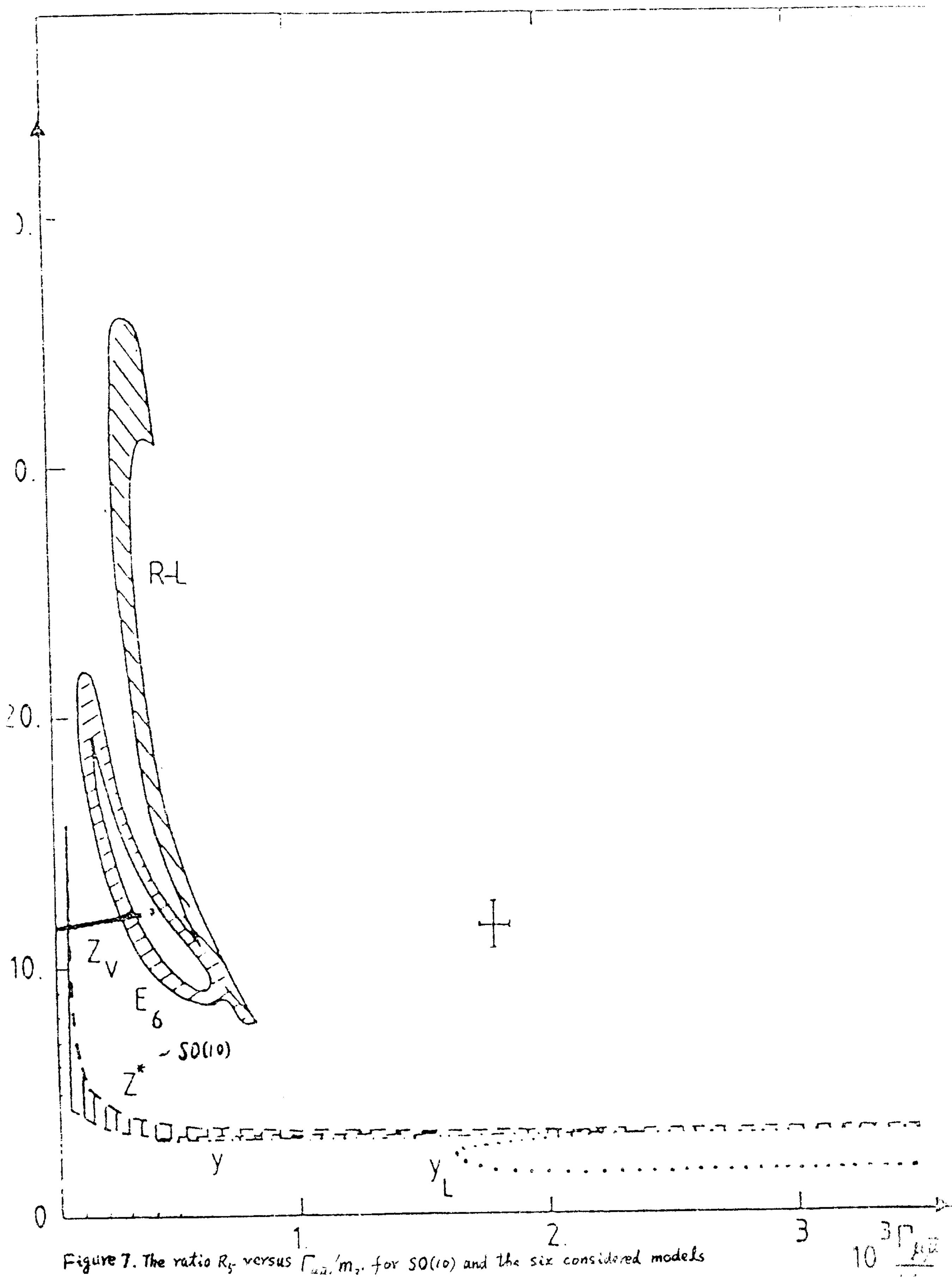

Figure 7. The ratio R_Z versus $\Gamma_{\mu\bar{\mu}}/m_Z$ for SO(10) and the six considered models

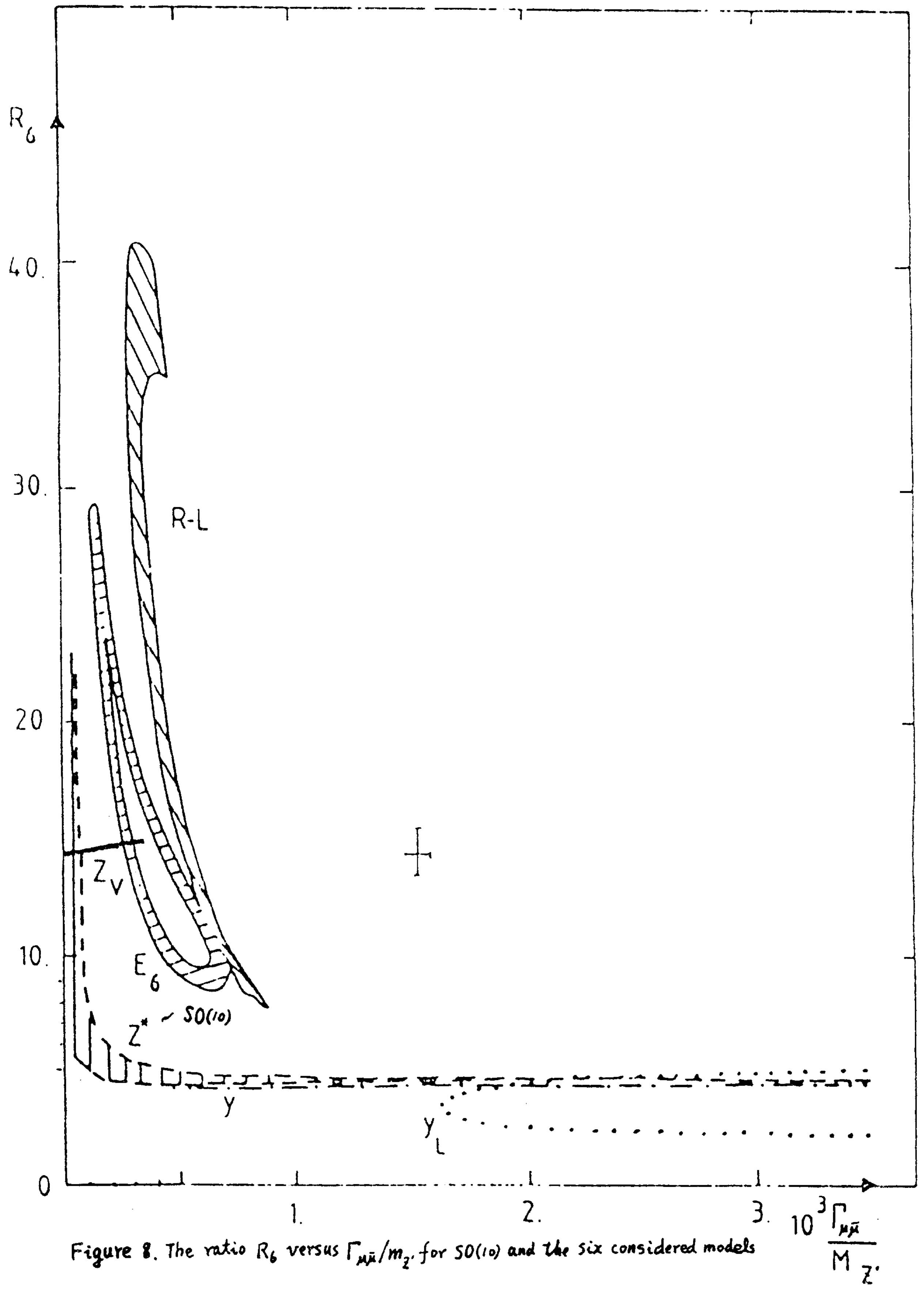

Figure 8. The ratio R_6 versus $\Gamma_{\mu\bar{\mu}}/m_{Z'}$ for SO(10) and the six considered models

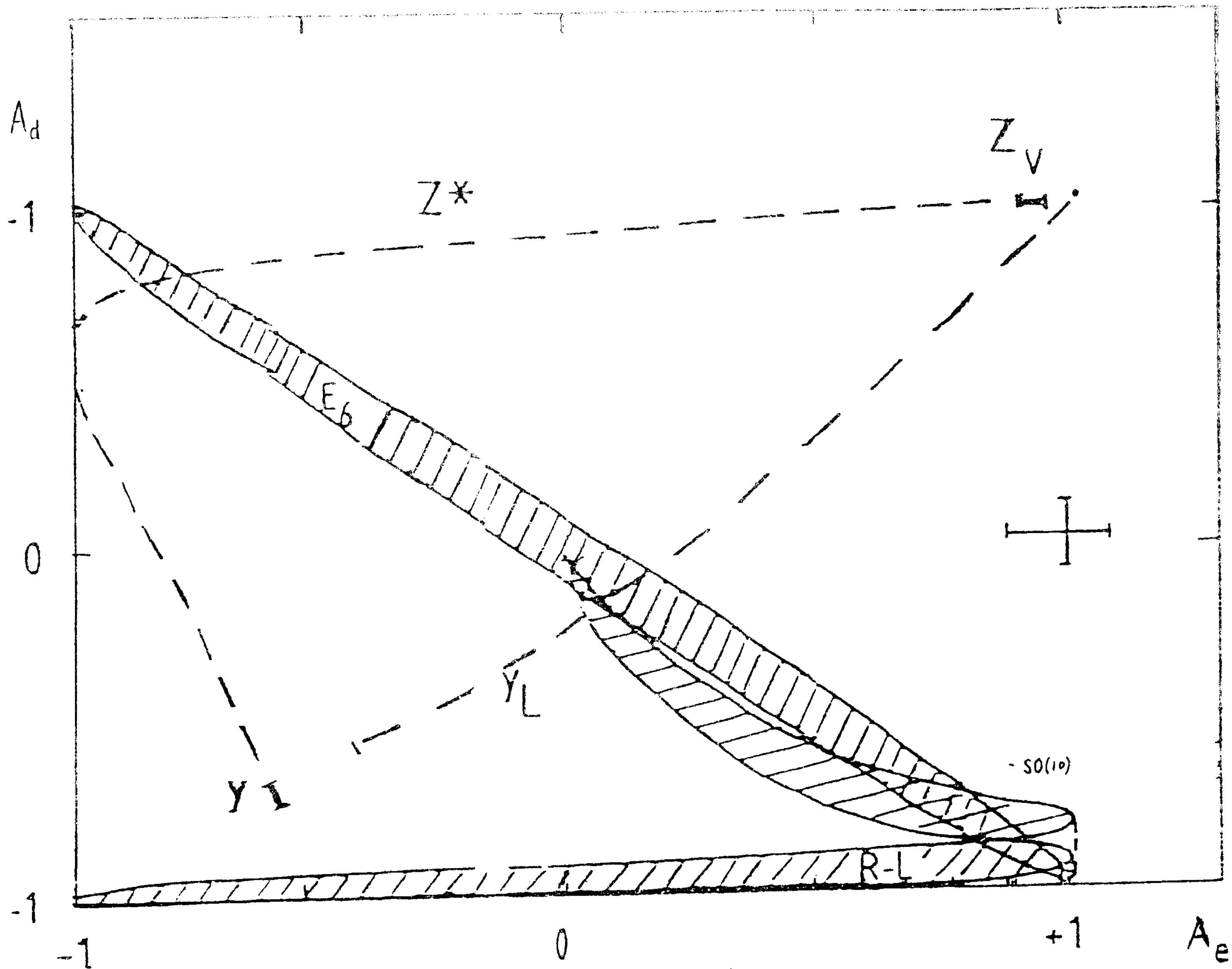

Figure 9. A_d versus A_e for SO(10) and the six considered models

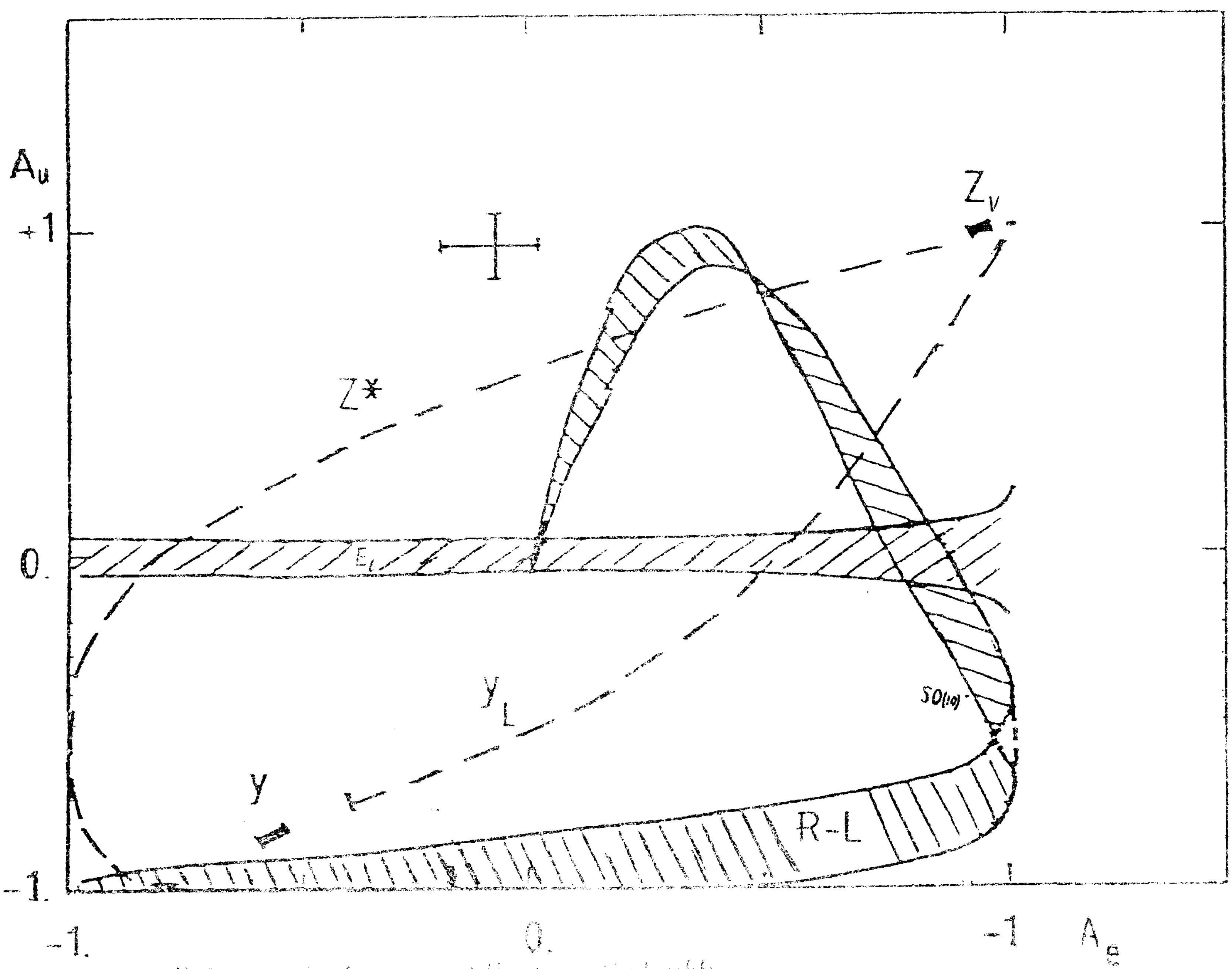

Figure 10. A_u versus A_e for SO(10) and the [illegible] constrained models.