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The $W^{\pm} \rightarrow l^{\pm}\nu$ Charge Asymmetry at Hadron Colliders

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Abstract

The $W^{\pm} \rightarrow l^{\pm}\nu$ charge asymmetry $A(y_l)$ as measured at the $p\bar{p}$ Tevatron is analyzed in detail in the perturbative LO and NLO with particular emphasis on treating the dominant heavy quark (charm) contribution according to the massive ($m_c \neq 0$) subprocesses $gq' \rightarrow W^{-}c \rightarrow l^{-}\bar{\nu}c$ with $q' = d$, s etc., instead of considering massless 'heavy' flavor constituents of the nucleon ($\bar{c}q' \rightarrow W^{-}$, etc.). It is furthermore demonstrated that most of the more recently suggested LO and NLO parton distributions are in very good agreement with present measurements of $A(y_l)$.

The $W^\pm$ charge asymmetry, measured by the CDF collaboration at the Fermilab's $p\bar{p}$ Tevatron collider [1], or more precisely the asymmetry of the rapidity distributions of the charged leptons from $W^\pm \to l^\pm \nu$ decays

$$A(y_l) = \frac{d\sigma(l^+)/dy_l - d\sigma(l^-)/dy_l}{d\sigma(l^+)/dy_l + d\sigma(l^-)/dy_l} \quad , \tag{1}$$

with $d\sigma(l^\pm)/dy_l$ being the differential $p\bar{p} \to W^\pm X \to l^\pm \nu X$ cross sections for producing $l^\pm$ leptons of rapidity y_l, is a sensitive probe of the difference between the u and d quark distributions in the (anti-)proton in the medium - x region ($x \simeq 0.1$) at a scale $Q^2 \simeq M_W^2$ [1, 2]. This asymmetry probes the d/u ratio, in particular its slope [2], since the dominant contribution to $W^+(W^-)$ production in $p\bar{p}$ collisions comes from the $u\bar{d}$ ($d\bar{u}$) annihilation process, i.e. $\sigma_{p\bar{p}}(W^+) \sim ud + \bar{u}\bar{d}$ etc., which is 'quadratic' in the (anti)quark distributions. Hence the $W^\pm$ asymmetry is a more direct and discriminating probe of the d/u ratio than measurements of the n/p structure function ratio, F_2^n/F_2^p, which is 'linear' in the small $\bar{q}/q$ corrections. In general one expects $A(y_l) > 0$ for $y_l > 0$ since the W^+ tends to follow the direction of the incoming proton and the W^- that of the antiproton because u quarks carry more momentum on the average than the d quarks.

We shall first show that almost all of the recently suggested leading (LO) and next-to-leading (NLO) order parton distributions are, in contrast to some claims in the literature [1], in excellent agreement with present measurements of the charge asymmetry $A(y_l)$. It will turn out that the heavy quark flavor (charm) contributions to $A(y_l)$ are sizeable (5 to 10%) and therefore not negligible according to the conventional naive approach [1-3] of treating heavy quarks (c, $b, \ldots$) as massless constituents of the nucleon which then contribute via $\bar{c}q' \to W^-$ with $q' = d$, s, etc. Since this naive massless treatment of heavy quarks is likely to be misleading and questionable [4], we shall pursue and analyze in detail the correct massive treatment of heavy quark contributions to $A(y_l)$, which is the main purpose of the present article. Here the heavy quark (charm) contributions result from the fully massive production subprocesses $gq' \to W^-c$ with $q' = d$, s, etc., which are initiated only by the light (massless) intrinsic constituents of the nucleon [4, 5], i.e. by the u, d, s and g distributions. The resulting contributions to $A(y_l)$ turn out to be roughly half as large as the naive massless 'zeroth order' one ($\bar{c}q' \to W^-$, etc.). The b-quark contribution with their strongly suppressed CKM couplings will and can be safely neglected throughout.

The differential LO $p\bar{p}$ production cross section, corresponding to the subprocess $q\bar{q}' \to W^+ \to l^+\nu$, is well known [6]

$$\frac{d\sigma}{dy_l}(p\bar{p} \to l^+X) = \sum_{\substack{q=u,c\\ q'=d,s}} \int_0^1 dx_p\, dx_{\bar{p}} \left[q(x_p, M_W^2)\, \bar{q}'(x_{\bar{p}}, M_W^2) \frac{d\hat{\sigma}^{\,q\bar{q}'\to l^+\nu}}{d\hat{y}_l} + \bar{q}'(x_p, M_W^2)\, q(x_{\bar{p}}, M_W^2) \frac{d\hat{\sigma}^{\,\bar{q}'q\to l^+\nu}}{d\hat{y}_l} \right] \tag{2}$$

with the rapidity being defined in the p - direction according to experiment and

$$\frac{d\hat{\sigma}^{\,{}^{q\bar{q}'}_{\bar{q}'q}\to l^+\nu}}{d\hat{y}_l} = \frac{1}{3}\left|V_{qq'}\right|^2 \left(\frac{G_F M_W^2}{4\sqrt{\pi}}\right)^2 \frac{\hat{s}}{(\hat{s} - M_W^2)^2 + (\Gamma_W M_W)^2} \left(\frac{1 \mp \tanh \hat{y}_l}{\cosh \hat{y}_l}\right)^2 \tag{3}$$

to be evaluated at $\hat{y}_l = y_l - \frac{1}{2}\ln(x_p/x_{\bar{p}})$ and $\hat{s} = x_p x_{\bar{p}} s$. For $p\bar{p} \to l^-X$ the summations in (2) extend obviously over antiquarks. The CKM matrix elements are as usual denoted by $V_{qq'}$ where for our purposes only the dominant 2×2 submatrix will be relevant, i.e. $q = u$, c and $q' = d$, s. For definiteness we shall use $V_{ud} = V_{cs} = 0.975$ and $V_{us} = V_{cd} = 0.22$, unless otherwise stated, and $M_W = 80.22\,\mathrm{GeV}$, $\Gamma_W = 2.08\,\mathrm{GeV}$. All calculations of $A(y_l)$ in (1) done so far [1, 2] treat the heavy flavor contributions in the same way as the massless u, d, s flavors, i.e. whenever q refers to a heavy quark flavor one uses, for calculating for example $\bar{c}s \to W^-$ transitions, $\bar{c}(x, Q^2) = c(x, Q^2)$ as radiatively generated from the massless evolution equations for $Q^2 \geq m_c^2$ [2, 3, 7]. Although these contributions are significantly smaller than the dominant ones stemming from the light (u, d, s) quarks in eq.(2), the charge asymmetry $A(y_l)$ in (1) is sensitive to it as will be shown below. (Without the massless charm contribution in eq.(2), the $f = 3$ light flavors alone give rise to an asymmetry which is typically about 10% larger.)

Treating heavy quarks (c, b, ...) as massless intrinsic constituents of the nucleon in the same way as the $f = 3$ light (u, d, s) flavors is certainly a physically doubtful and theoretically questionable procedure. It has been shown in ref.[4] that a consistent and perturbatively stable treatment of heavy quark contributions to lepton - hadron and hadron - hadron induced processes affords heavy flavors to be considered not as (intrinsic) massless partons of the nucleon, but instead to treat them in fixed - order (LO, NLO) perturbation theory, i.e. that heavy quarks are produced solely via the gluon and $f = 3$ light flavor u, d, s quarks of the nucleon. Furthermore, non - relativistic ($m_{c,b} \neq 0$) effects are non - negligible even far above heavy - quark production thresholds, in contrast to 'naive' expectations [4]. Therefore it appears to be more reasonable to use the simple LO expression (2) only for light quarks (i.e. the sum over q, q' extends only over light quarks (u, d, s) and antiquarks ($\bar{u}$, $\bar{d}$, $\bar{s}$)) as illustrated in fig. 1(a), and that the heavy quark (dominantly charm) contributions are calculated via the $q'g \to W^- c \to l^- \bar{\nu} c$, etc. fusion subprocesses depicted in fig. 1(b). The cross section for this $2 \to 3$ subprocess $q'g \to l^- \bar{\nu} c$ with $q' = d$, s, which can be straightforwardly calculated, is in an obvious notation given by

$$d\hat{\sigma}^{q'g \to l\bar{\nu}c} = \frac{1}{4\,(g \cdot q')} |\mathcal{M}|^2 \prod_{i=l,\nu,c} \left(\frac{d^3 k_i}{2E_i}\right) \frac{\delta^4(q' + g - l - \nu - c)}{(2\pi)^5} \tag{4}$$

where

$$\begin{aligned} |\mathcal{M}|^2 \;=\;& |V_{cq'}|^2 \frac{32\pi}{3} \alpha_s(M_W^2) \left(G_F M_W^2\right)^2 \left[\left(2\,l \cdot \nu - M_W^2\right)^2 + (\Gamma_W M_W)^2\right]^{-1} \\ \times\;& \Big\{ 2m_c^2\,(g \cdot c)^{-2} (\nu \cdot q')\,[l \cdot g - l \cdot c] + (g \cdot q')^{-1} (g \cdot c)^{-1} \big\{ m_c^2\,[(l \cdot q')(\nu \cdot q') \\ +\;& (l \cdot c)(\nu \cdot c)] + 2\,l \cdot \nu \left[(\nu \cdot q')^2 + (l \cdot c)^2 + m_c^2 (l \cdot c)\right] \big\} \Big\} \quad . \end{aligned} \tag{5}$$

This expression agrees with the appropriately crossed one given in ref.[6]. The differential LO $p\bar{p}$ production cross section in the $p\bar{p}$ c.o.m. frame, corresponding to the massive charm production subprocess $q'g \to l\bar{\nu}c$, is then given by

$$\begin{aligned} \frac{d\sigma_c}{dy_l}(p\bar{p} \to lX) = \frac{1}{(4\pi)^4} \sum_{q'} \int & dp_T^l\, dx_p\, dx_{\bar{p}}\, d\cos\theta_\nu\, d\varphi_\nu \frac{p_T^l}{(g \cdot q')} \frac{E_\nu^2}{|2\,g \cdot q' - 2\,g \cdot l - 2\,q' \cdot l - m_c^2|} \\ & \times q'(x_p, M_W^2)\; g(x_{\bar{p}}, M_W^2)\, |\mathcal{M}|^2 \;+\; (q' \leftrightarrow g) \end{aligned} \tag{6}$$

where the sum extends over d, s and $\bar{d}$, $\bar{s}$ for W^- and W^+ production, respectively, with the kinematic quantities given by

$$E_\nu = \frac{2\,g\cdot q' - 2\,g\cdot l - 2\,q'\cdot l - m_c^2}{\sqrt{s}\left[(x_{\bar p}+x_p)+(x_{\bar p}-x_p)\cos\theta_\nu\right] - 2E_l\left[1-\cosh^{-1}y_l\sin\theta_\nu\cos\varphi_\nu - \tanh y_l\cos\theta_\nu\right]}$$

$$\begin{aligned}
E_l &= p_T^l\cosh y_l \quad , & 2\,g\cdot q' &= x_p x_{\bar p} s \\
2\,l\cdot q' &= x_p E_l\sqrt{s}\,(1-\tanh y_l) \quad , & 2\,l\cdot g &= x_{\bar p}E_l\sqrt{s}\,(1+\tanh y_l) \\
2\,\nu\cdot q' &= x_p E_\nu\sqrt{s}\,(1-\cos\theta_\nu) \quad , & 2\,\nu\cdot g &= x_{\bar p}E_\nu\sqrt{s}\,(1+\cos\theta_\nu) \\
2\,l\cdot\nu &= 2E_lE_\nu(1-\tanh y_l\cos\theta_\nu - \cosh^{-1}y_l\sin\theta_\nu\cos\varphi_\nu)
\end{aligned}$$

and the remaining scalar products follow simply from energy - momentum conservation. The range of integration in (6) is restricted to $p_{T,min}^l \leq p_T^l \leq (s-m_c^2)/2\sqrt{s}\cosh y_l$ and

$$x_p \geq \frac{e^{y_l}\,p_T^l/\sqrt{s}+m_c^2/s}{1-e^{-y_l}\,p_T^l/\sqrt{s}} \quad , \quad x_{\bar p} \geq \frac{x_p e^{-y_l}\,p_T^l/\sqrt{s}+m_c^2/s}{x_p - e^{y_l}\,p_T/\sqrt{s}} \quad .$$

We have used the experimental cuts [1] $p_{T,min}^l = 25\,\text{GeV}$ and for the missing transverse energy $E_{\nu,T} = E_\nu\sin\theta_\nu \geq 25\,\text{GeV}$. The isolation of the final lepton as well as the cut on the transverse component of the energy of the c-jet, $E_c = (x_p + x_{\bar p})\sqrt{s}/2 - E_l - E_\nu$, not to exceed 20 GeV, turned out to be of minor importance for $A(y_l)$. Needless to say that eq.(6) has now to be added to eq.(2) with the latter being relevant only for the $f = 3$ massless u, d, s quarks (i.e. $q = u$, $q' = d$, s) and antiquarks. It should be noted that $A(y_l)$ as defined in (1) is not particularly sensitive to the specific choice of scales in eqs.(2), (5) and (6).

In fig. 2 we show the LO predictions for $A(y_l)$ according to the original (dynamical) GRV distributions [3]. The difference between the $f = 4$ (massless u, d, s, c contributions in (2)) and the $f = 3$ (only u, d, s contributions in (2)) results illustrates the above mentioned sensitivity of the asymmetry on the charm contribution. The more appropriate final result '$f = 3$ plus massive charm', where the charm contribution is calculated according to eq.(6), is shown by the dashed-dotted and solid curves for the original GRV '92 and the most recent GRV '94 distributions [5], respectively. The χ^2/dof of these latter predictions is 0.6 and 0.32, respectively, whereas the massless GRV '92 ($f = 4$) result in fig. 2 corresponds to 0.44. A somewhat less agreeable but still acceptable prediction, taking into account present experimental uncertainties, is obtained for the LO CTEQ2L

[8] densities ($\chi^2/\mathrm{dof} = 1.4$) which always refer to $f = 4$ massless u, d, s, c flavors. Since all previous calculations [1, 2] were performed by assuming simply a diagonal CKM unit-matrix $(V_{qq'}) = \mathrm{diag}(V_{ud} = 1, V_{cs} = 1)$, according to the use of the LO/NLO DYRAD program [9], we show in the insert of fig. 2 the changes of the massless $f = 4$ and the more appropriate '$f = 3$ plus massive charm' results due to this approximation. Future higher statistics data may require the correct CKM structure together with its additional off - diagonal contributions.

The NLO predictions are shown in fig. 3 which have all been calculated with the DYRAD program [9] based on the diagonal CKM approximation[1] $V_{ud} = V_{cs} = 1$. The massless $f = 4$ GRV '92 [3][2] and MRS-A [2] predictions compare very well with the CDF data [1],[3] corresponding to $\chi^2/\mathrm{dof} = 0.75$ and 0.3, respectively. The most recent GRV '94 distributions [5], which have been developed solely for the $f = 3$ light u, d, s quark flavors and the gluon, result in an equally good prediction ($\chi^2/\mathrm{dof} = 0.37$) for $A(y_l)$, with the (massive) charm contribution calculated more appropriately according to eq.(6). In order to illustrate that this more appropriate massive treatment of the charm contribution gives an equally acceptable prediciction for the charge asymmetry, we also show in fig. 3 the '$f = 3$ plus massive charm' result for the GRV '92 distributions (dashed - dotted curve) to be compared with the fully massless $f = 4$ result (dashed curve). A theoretically entirely consistent NLO calculation would clearly require the so far unknown NLO $\mathcal{O}(\alpha_s^2)$ contributions to eq.(6). In view of the fact that the massive $gq' \rightarrow Wc$ contributions in (6) are only a small correction to the dominant massless subprocesses $u\bar{d} \rightarrow W^+$, $u\bar{d} \rightarrow W^+g$, etc. [9], the $\mathcal{O}(\alpha_s^2)$ corrections to (6) are of little significance for the ratio of cross sections in eq.(1).

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[1] All our NLO results refer to this approximation. Because of the complexity of the NLO DYRAD Monte Carlo program, as described in [9], we refrain from implementing the full CKM structure with the additional subprocesses. We expect these contributions to increase our NLO predictions for $A(y_l)$ by about 5% around $y_l \simeq 1$.

[2] The GRV results presented in ref.[1] are incorrect.

[3] It should be noted that the NLO CTEQ2M fit [8] with the significantly larger d/u slope is ruled out [2] by these data. However, an even more recent fit (CTEQ3 [10]) reproduces essentially the MRS-A results [2].

As soon as high - statistics data for the charge asymmetry become available, the light (input) parton distributions have eventually to be (slightly) adjusted using the more appropriate massive treatment of charm contributions according to $q'g \to W^- c \to l^- \bar{\nu} c$ etc. in eq.(6), instead of considering heavy quarks as massless constituents ($\bar{c}q' \to W^-$ etc.) of the nucleon. For the time being, predictions based on most LO and NLO parton distributions recently suggested are in satisfactory agreement with present CDF data.

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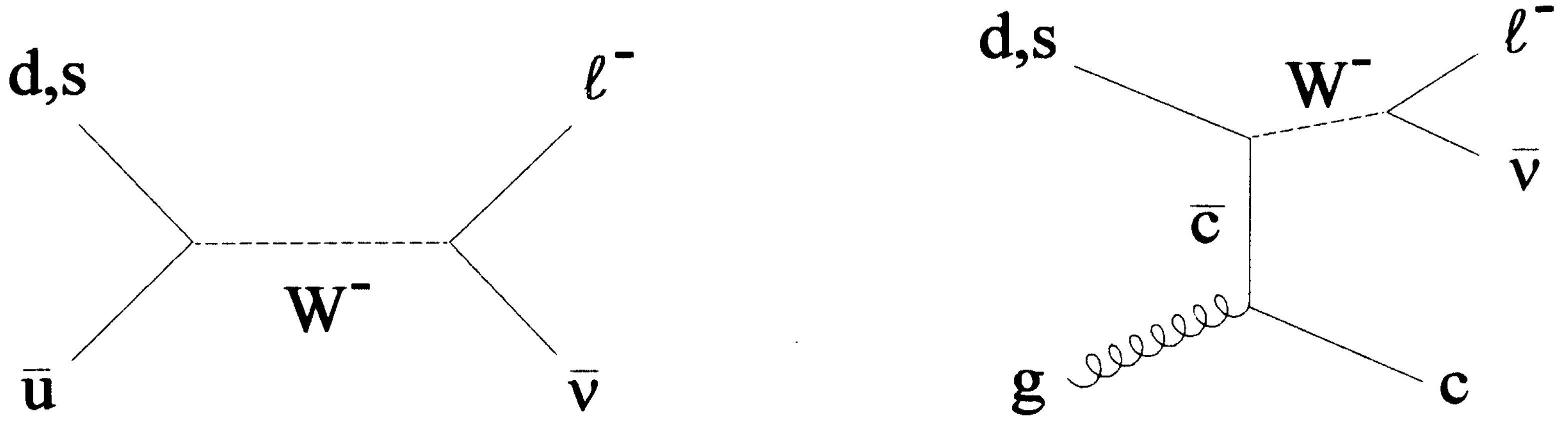
Figure Captions

Fig. 1 (a) The 'zeroth order' light quark fusion subprocess and (b) the LO heavy (massive charm) quark production subprocess which contribute to the charge asymmetry $A(y_l)$. Not shown are the charge - conjugated diagrams and the obvious crossed (s - channel) diagram.

Fig. 2 LO predictions for the asymmetry $A(y_l)$ of the rapidity distributions of the charged leptons from $W^\pm \to l^\pm \nu$ decays observed at Fermilab [1]. The GRV '92, GRV '94 and CTEQ2L distributions are taken from refs. [3], [5] and [8], respectively. Our notation '$f = 4$' refers to calculating $A(y_l)$ just with four massless u, d, s, c flavors in eq.(2); '$f = 3$' refers to the three light u, d, s flavors in eq.(2). The 'massive charm' contribution is calculated according to eq.(6) using $m_c = 1.5\,\mathrm{GeV}$. The curves in the insert refer to the GRV '92 distributions where two curves were, as indicated, calculated using approximately a diagonal CKM matrix with $V_{ud} = V_{cs} = 1$ in eqs.(2) and (6).

Fig. 3 As in Fig. 2 but in NLO. The MRS-A distributions are taken from ref. [2].



(a) (b)

Fig. 1

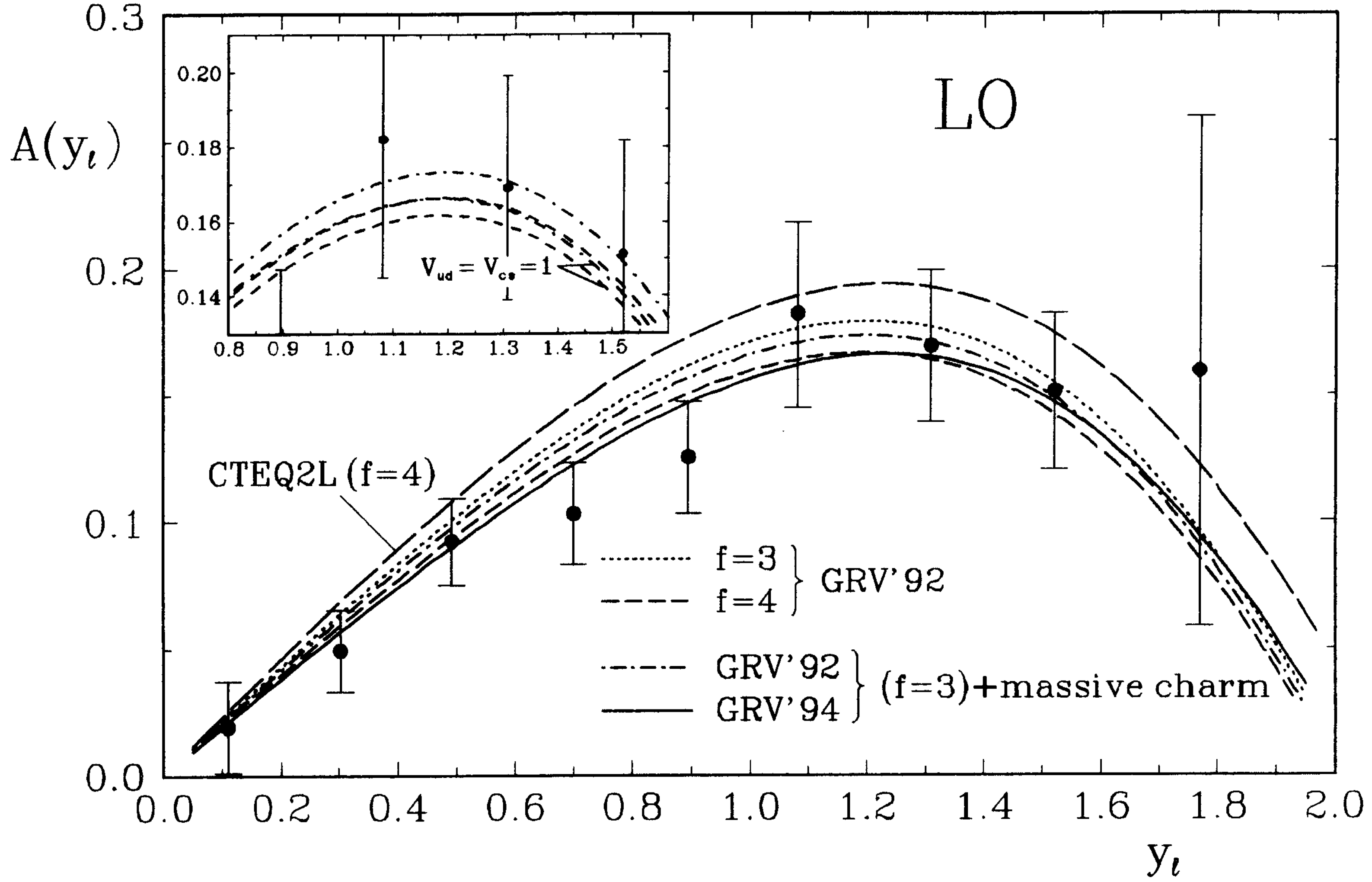

0.3
0.2
0.1
0.0
$A(y_\ell)$
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
y_ℓ
LO
CTEQ2L (f=4)
f=3
f=4
GRV'92
GRV'92
GRV'94
(f=3)+massive charm
0.20
0.18
0.16
0.14
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
$V_{ud} = V_{cs} = 1$

Fig. 2

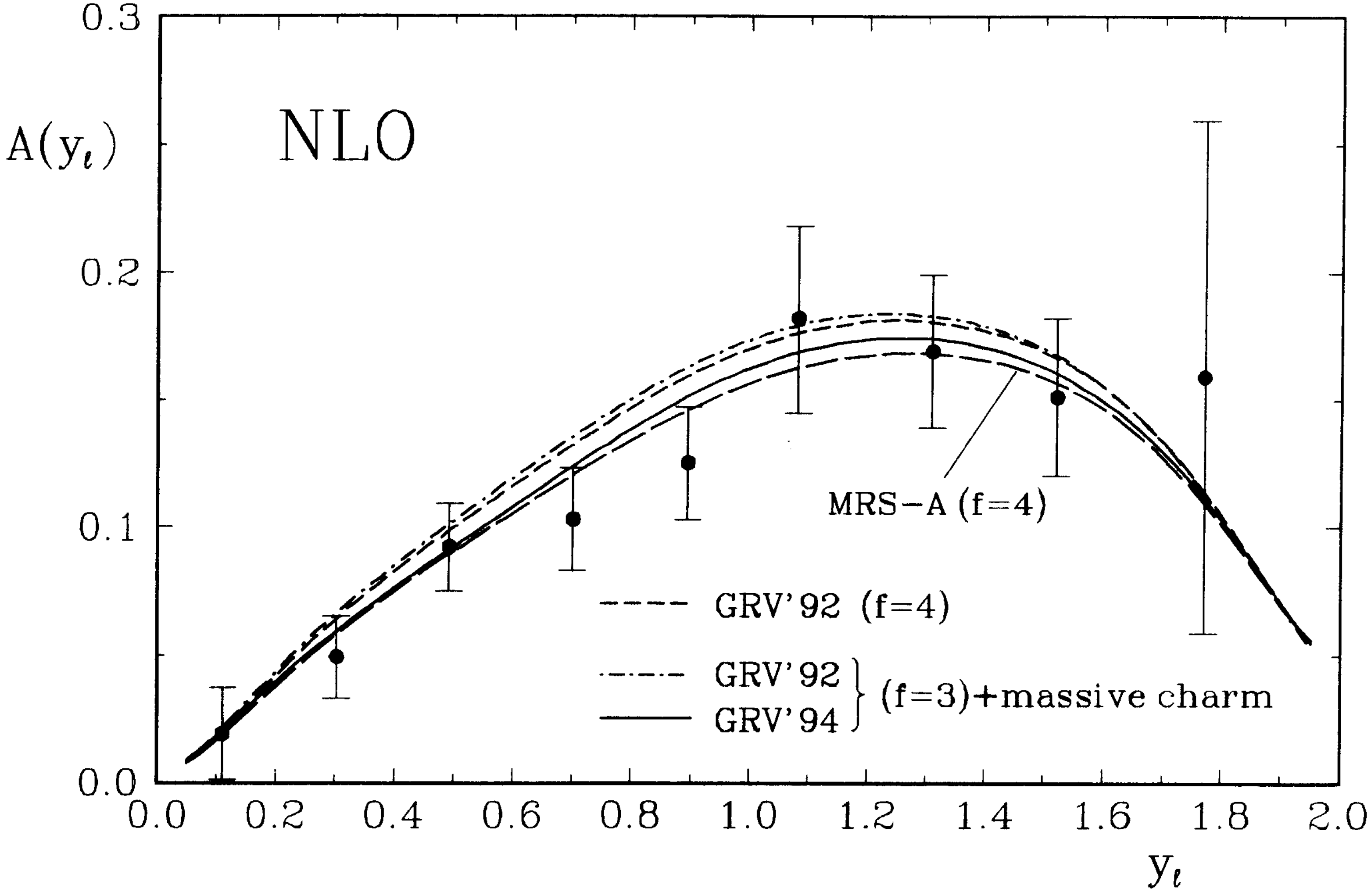

NLO
A(y_ℓ)
0.0
0.1
0.2
0.3
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
y_ℓ
MRS−A (f=4)
GRV'92 (f=4)
GRV'92
GRV'94
(f=3)+massive charm

Fig. 3