Revised QCD effects on the Z → $b\bar{b}$ forward-backward asymmetry

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Abstract

The forward-backward (FB) asymmetry of b quarks in e^+e^- collisions at the Z pole measured at LEP, $A_{\mathrm{FB}}^{0,b} = 0.0992 \pm 0.0016$, remains today the electroweak precision observable with the largest disagreement (2.4σ) with respect to the Standard Model prediction, $(A_{\mathrm{FB}}^{0,b})_{\mathrm{th}} = 0.1030 \pm 0.0002$. Beyond the dominant statistical uncertainties, QCD effects, such as b-quark showering and hadronization, are the leading sources of $A_{\mathrm{FB}}^{0,b}$ systematic uncertainty, and have not been revised in the last twenty years. We reassess the QCD uncertainties of the eight original $A_{\mathrm{FB}}^{0,b}$ LEP measurements, using modern parton shower PYTHIA 8 and PYTHIA 8 + VINCIA simulations with nine different implementations of soft and collinear radiation as well as of parton fragmentation. Our analysis, combined with NNLO massive b-quark corrections independently computed recently, indicates total propagated QCD uncertainties of ~0.7% and ~0.3% for the lepton-charge and jet-charge analyses, respectively, that are about a factor of two smaller than those of the original LEP results. Accounting for such updated QCD effects leads to a new $A_{\mathrm{FB}}^{0,b} = 0.0996 \pm 0.0016$ average, with a data-theory tension slightly reduced from 2.4σ to 2.1σ. Confirmation or resolution of this long-term discrepancy requires a new high-luminosity e^+e^- collider collecting orders-of-magnitude more data at the Z pole to significantly reduce the $A_{\mathrm{FB}}^{0,b}$ statistical uncertainties.

Keywords: e^+e^- collisions, Z boson, bottom quarks, QCD, weak mixing angle, forward-backward asymmetry

1. Introduction

In the Standard Model (SM), the fermions are arranged in weak-isospin doublets for left-handed particles and weak-isospin singlets for right-handed particles. The Z boson mediates weak neutral current interactions between fermions of the same generation, and it couples to both left- and right-handed fermions with different strengths depending on their weak-isospin I^f and electric charge Q^f. The vector and axial-vector Z couplings for a fermion of type f are $g_V^f = (g_L^f + g_R^f) = I_3^f - 2Q^f \sin^2\theta_{\mathrm{W}}$ and $g_A^f = (g_L^f - g_R^f) = I_3^f$ respectively, where I_3^f is the third component of the weak isospin of the fermion, related to the electric charge via its hypercharge Y^f: $Q^f = I_3^f + Y^f/2$, and where $\sin^2\theta_{\mathrm{W}}$ is the weak mixing angle that controls the γ–Z mixing and connects the coupling

constants of the electroweak theory: $g \sin \theta_W = g' \cos \theta_W = e$. For $m_W = 80.379$ GeV and $m_Z = 91.1876$ GeV, one has $\sin^2 \theta_W \equiv 1 - m_W^2/m_Z^2 = 0.22301$ in the on-shell electroweak scheme [1]. The $g_{V,A}^f$ expressions above describe the varying strengths of the Z-fermion couplings in the $(\nu_e, \nu_\mu, \nu_\tau)$, (e, μ, τ), (u, c, t), and (d, s, b) lepton/quark families. The mixed vector and axial-vector Z couplings not only affect the total $e^+e^- \to f\bar{f}$ cross section $\sigma_{f\bar{f}}^{\text{tot}}$, but also induce asymmetries $\mathcal{A}_f$ in the angular distributions of the final-state fermions produced. The Born-level differential cross section for Z exchange alone, as a function of the scattering angle θ of the outgoing fermion with respect to the direction of the incoming e^- beam, summed over final-state helicities and assuming e^+e^- unpolarized beams, is: $\frac{d\sigma}{d\cos\theta} = \frac{3}{8}\sigma_{f\bar{f}}^{\text{tot}}\left((1 + \cos^2\theta) + 2\mathcal{A}_f \cos\theta\right)$. The angular asymmetries $\mathcal{A}_f$ in the $e^+e^- \to f\bar{f}$ final-state are thereby ultimately driven by Q^f and $\sin^2\theta_W$:

$$\mathcal{A}_f = \frac{(g_L^f)^2 - (g_R^f)^2}{(g_L^f)^2 + (g_R^f)^2} = 2\,\frac{g_V^f/g_A^f}{1 + (g_V^f/g_A^f)^2}\,, \quad \text{with} \quad \frac{g_V^f}{g_A^f} = 1 - 4|Q^f|\sin^2\theta_{\text{eff}}^f\,, \tag{1}$$

where $\sin^2\theta_{\text{eff}}^f$ is the effective fermion mixing angle that includes electroweak corrections beyond the tree level. The $e^+e^- \to f\bar{f}$ forward-backward (FB) asymmetry can be determined from the ratio of the difference over the sum of the number of forward N_F and backward N_B events, where forward (backward) indicates that the outgoing fermion (antifermion) is produced in the hemisphere defined by the direction of the electron (positron) beam:

$$A_{\text{FB}}^f = \frac{N_F - N_B}{N_F + N_B}, \quad \text{where} \quad F = \int_0^1 \frac{d\sigma}{d\Omega}d\Omega, \;\; B = \int_{-1}^0 \frac{d\sigma}{d\Omega}d\Omega\,. \tag{2}$$

Experimentally, the FB asymmetries are usually derived from fits of the differential distribution of events as a function of the polar angle θ between $e^\pm$ and b-quark directions, which is a procedure statistically slightly more powerful than the simple event counting given by Eq. (2). From the measured FB asymmetries of the produced f and $\bar{f}$ fermions fitted at various e^+e^- c.m. energies around the Z resonance, one derives the relevant asymmetry parameter at the Z pole ($A_{\text{FB}}^{0,f}$).

Among the FB quark asymmetries in $e^+e^- \to Z \to q\bar{q}(g)$ at $\sqrt{s} \approx m_Z$, the b-quark one (A_{FB}^b) is the most accurately measured at LEP, with a 1.6% precision [2]. This is so because b-quark jets are the easiest ones to be properly identified (through the secondary vertex associated with their leading B hadrons), and also because the large Z coupling to down-type quarks leads to a $\sim$30% larger branching fraction for $Z \to d\bar{d}, s\bar{s}, b\bar{b}$ ($\mathcal{B} = 15.6\%$) compared to $Z \to u\bar{u}, c\bar{c}$ ($\mathcal{B} = 11.6\%$) [1]. The value $A_{\text{FB}}^{0,b} = 0.0992 \pm 0.0016$, obtained from the combination of eight measurements at $\sqrt{s} = 91.21$–91.26 GeV using two different (lepton- and jet-charge based) methods [2], shows today the largest tension (2.4σ) with respect to the theoretical SM prediction, $(A_{\text{FB}}^{0,b})_{\text{th}} = 0.1030 \pm 0.0002$, derived from the combined fit to all precision electroweak observables [1]. Such a discrepancy also propagates to the effective weak mixing angle derived from it, via Eq. (1), $\sin^2\theta_{\text{eff}}^f = 0.23221 \pm 0.00029$, compared with the $\sin^2\theta_{\text{eff}}^f = 0.23122 \pm 0.00004$ world-average [1].

The experimental uncertainties of the individual $A_{\text{FB}}^{0,b}$ extractions include 3–11% (1–6%) statistical (systematic) components. Among the systematic uncertainties, those that have an origin in quantum chromodynamics (QCD) effects, such as those related to angular decorrelations induced in the event axis due to hard and/or soft and collinear parton radiation and/or parton hadronization, were estimated twenty years ago combining next-to-leading-order (NLO) perturbative corrections with Monte Carlo (MC) parton shower simulations [3]. A recent theoretical study [4] has reduced the perturbative QCD (pQCD) uncertainties by calculating next-to-next-to-leading order (NNLO)

corrections with non-zero b-quark mass, but the impact of parton shower (PS) and fragmentation effects remains untested since LEP times. Future high-luminosity e^+e^- machines operating at the Z pole, such as the FCC-ee [5] with 10^5 times more data collected than at LEP, will feature negligible statistical uncertainties, and the latter QCD corrections will dominate the systematics of the $A_{\mathrm{FB}}^{0,b}$ measurement. Our understanding of QCD effects has significantly improved since the work of [3] thanks to theoretical and experimental progress that has been incorporated into modern PS + hadronization models. The main purpose of this work is to reanalyze the original measurements with up-to-date MC parton-shower tools to see if the existing data-theory $A_{\mathrm{FB}}^{0,b}$ discrepancy could be (partly) reduced by a reevaluation of the impact of QCD effects.

The paper is organized as follows. In Section 2, we succinctly review the LEP $A_{\mathrm{FB}}^{0,b}$ measurements and their uncertainties. In Section 3, we describe the various modern parton shower MC event generators used here to simulate the LEP measurements and to reassess their associated QCD uncertainties. In Section 4, the eight LEP measurements are updated with the reevaluated QCD effects, a new $A_{\mathrm{FB}}^{0,b}$ average value is derived, and perspectives at a future e^+e^- collider are outlined. In Section 5, we conclude with a summary.

2. Experimental measurements and uncertainties

Eight measurements of $A_{\mathrm{FB}}^{0,b}$ were performed in e^+e^- collisions at LEP around the Z pole. All of them start by clustering the e^+e^- final-state particles into jets, mostly with the JADE algorithm [6] and jet resolution parameter $y_{\mathrm{cut}} = 0.01$ or 0.02, and by identifying its quark flavour (b tagging) to determine the thrust axis of the event, used as a proxy of the b-quark direction. Secondly, the b and $\bar{b}$ are separated through charge-tagging methods that rely on either lepton- or jet-charge measurements. On the one hand, in four measurements (ALEPH at $\sqrt{s} = 91.21$ GeV [7], DELPHI at $\sqrt{s} = 91.26, 91.23$ GeV [8], L3 at $\sqrt{s} = 91.26$ GeV [9], and OPAL at $\sqrt{s} = 91.25, 91.24$ GeV [10]), the original b ($\bar{b}$) quark is identified from the negative (positive) charge of the leading lepton $\ell^{\pm}$ inside each jet (i.e. that with the largest momentum perpendicular to the jet axis, through the fragmentation(s) $b \to B, b \to c \to D$, and subsequent $B, D \to \ell^{\pm}$ semileptonic decay). The raw FB asymmetry $(A_{\mathrm{FB}}^b)_{\mathrm{obs}}$ is then obtained by fitting the corresponding distribution of polar angles θ between the e^- and the thrust axis to the following expression

$$\frac{d\sigma}{d\cos\theta} = \sigma_{b\bar{b}}^{\mathrm{tot}} \left(\frac{3}{8}(1 + \cos^2\theta) + (A_{\mathrm{FB}}^b)_{\mathrm{obs}}(1 - 2\chi_B)\cos\theta \right), \tag{3}$$

where $\chi_B \approx 0.125$ is the $B^0\overline{B^0}$ effective mixing parameter, measured by each experiment *in situ*, encoding the probability that a semileptonically decaying b quark is reconstructed as a $\bar{b}$ quark. In the other four measurements (ALEPH at $\sqrt{s} = 91.23$ GeV [11], DELPHI at $\sqrt{s} = 91.23$ GeV [12], L3 at $\sqrt{s} = 91.24$ GeV [13], and OPAL at $\sqrt{s} = 91.26$ GeV [14]), the b-quark charge is reconstructed from the constituent charged particles of the jet via $Q_{\mathrm{jet}} = \sum p_L^{\kappa} Q / \sum p_L^{\kappa}$ (where p_L is the longitudinal momentum of the final-state particles, with individual charge Q, with respect to the thrust axis, and the power κ varies between 0.4 and 0.6 depending on the experiment). The $(A_{\mathrm{FB}}^b)_{\mathrm{obs}}$ asymmetry is then extracted by fitting the $\cos\theta$ distribution to the expression

$$\frac{\langle Q_F - Q_B \rangle}{\langle Q_b - Q_{\bar{b}} \rangle} = \frac{8}{3}(A_{\mathrm{FB}}^b)_{\mathrm{obs}} \frac{\cos\theta}{1 + \cos^2\theta}, \tag{4}$$

where Q_F (Q_B) is the jet charge of the forward (backward) hemisphere, Q_b ($Q_{\bar{b}}$) is the jet charge in the hemisphere containing the b ($\bar{b}$) quark.

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The measured $(A^b_{\text{FB}})_{\text{obs}}$ values are further corrected for different QCD effects, including higher-order perturbative QCD effects (hard gluon real radiation and virtual exchanges), as well as for further smearing due to b and $(b \to)c$ soft and/or collinear radiation and hadronization, that lead to a difference between the result obtained with the reconstructed thrust axis (T) and the b-quark direction. This is done through the correction factor

$$A^b_{\text{FB}} = (A^b_{\text{FB}})_{\text{obs}}/(1 - C_{\text{QCD}})\,, \text{ with } C_{\text{QCD}} = s_b \cdot C^{\text{T}}_{\text{QCD}}\,, \qquad (5)$$

where the full QCD effects are decomposed into the product of two coefficients: (i) the $C^{\text{T}}_{\text{QCD}}$ term including perturbative gluon radiation plus the non-perturbative conversion of partons into hadrons, determined originally at next-to-leading order (NLO) accuracy including hadronization effects, $C^{\text{T,had}}_{\text{QCD}} = (3.54 \pm 0.63)\%$ [2, 3], and known today at the partonic level at NNLO [15] including b-quark mass effects: $C^{\text{T,parton}}_{\text{QCD}} = (3.92 \pm 0.25)\%$ [4], and (ii) an extra experiment-dependent scaling factor, $s_b \approx 0.5, 1$ for the lepton- and jet-charge-based extractions respectively, that accounts for the convolution of the detector response with any biases introduced in the event topology by the data selection and analysis method of each experiment. In practice, the overall QCD correction C_{QCD} and its uncertainty, were determined from the spread of the differences between the parton- and hadron-level results obtained with various tunes of the PYTHIA 5/JETSET 7.408 [16] parton shower simulations [3], after confirming that the latter (at the parton level) were consistent with the $C^{\text{T,parton}}_{\text{QCD}}$ value obtained analytically at NLO. Lastly, the fitted asymmetries require a final correction for Quantum Electrodynamics (QED) and electroweak effects, including γ radiation and exchange, Z-γ interference, and a shift to the pole $m_{\text{Z}} = 91.1876$ GeV mass. Accounting for the latter effects increases the observed asymmetry by 1.9% with negligible uncertainties [17].

$$A^{0,b}_{\text{FB}} = A^b_{\text{FB}} + \delta A^b_{\text{FB}}, \text{ with } \delta A^b_{\text{FB}} = +1.9\%. \qquad (6)$$

Table 1 lists the eight individual $A^{0,b}_{\text{FB}}$ measurements with the breakdown of their uncertainties. The statistical uncertainties dominate, being about twice bigger than the systematic ones, while the QCD uncertainties, fully correlated among all measurements, account for about half of the latter. The QCD uncertainties include in quadrature all sources of QCD-related effects extracted from the original measurements by the $A^{0,b}_{\text{FB}}$ averaging study [2]. The combination of the eight measurements gives $A^{0,b}_{\text{FB}} = 0.0992 \pm 0.0016$, with 1.5% statistical uncertainty, and 0.7% systematic uncertainty dominated by $\sim$0.5% QCD sources (taken as fully correlated in the final average).

3. Monte Carlo simulations

In order to replicate in simulation each one of the eight A^b_{FB} extractions at LEP, and reassess the size of the QCD systematic uncertainties with more modern MC tools, we use the PYTHIA 8.226 [18] event generator, with seven different sets of e^+e^- parameters ("tunes") to model the showering and hadronization, as well as the PYTHIA 8.210 MC generator combined with two versions of the alternative VINCIA parton shower [19, 20]. The QCD evolution in the PYTHIA 8 MC generator is based on Dokshitzer–Gribov–Lipatov–Altarelli–Parisi (DGLAP) LO splittings [21] with the option to use the Catani–Marchesini–Webber (CMW) rescaling of the strong coupling α_{s} [22] to account for NLO corrections to soft gluon emissions, approximating soft and collinear radiation beyond the leading logarithm in an effective way. The b and c quark fragmentation is modeled according to the Lund string model [23] combined with the Bowler [24] function or, at LEP times, to the

Table 1: LEP measurements of $A_{\mathrm{FB}}^{0,b}$ and associated relative statistical, QCD-related and total systematic uncertainties. The last row lists the LEP combined result.

| Measurement: | $(A_{\mathrm{FB}}^{0,b}) \pm \delta(\text{stat}) \pm \delta(\text{syst})$ | relative uncertainties | | |
Experiment		stat.	QCD syst.	total syst.
Lepton-charge based:				
ALEPH (2002) [2, 7]	$0.1003 \pm 0.0038 \pm 0.0017$	3.8%	0.7%	1.7%
DELPHI (2004–05) [2, 8]	$0.1025 \pm 0.0051 \pm 0.0024$	5.0%	1.2%	2.3%
L3 (1992–99) [2, 9]	$0.1001 \pm 0.0060 \pm 0.0035$	6.0%	1.8%	3.5%
OPAL (2003) [2, 10]	$0.0977 \pm 0.0038 \pm 0.0018$	3.9%	1.1%	1.8%
Jet-charge based:				
ALEPH (2001) [2, 11]	$0.1010 \pm 0.0025 \pm 0.0012$	2.5%	0.7%	1.2%
DELPHI (2005) [2, 12]	$0.0978 \pm 0.0030 \pm 0.0015$	3.1%	0.7%	1.5%
L3 (1998) [2, 13]	$0.0948 \pm 0.0101 \pm 0.0056$	10.6%	4.3%	5.9%
OPAL (1997,2002) [2, 14]	$0.0994 \pm 0.0034 \pm 0.0018$	3.4%	0.7%	1.8%
Combination [2]	$0.0992 \pm 0.0015 \pm 0.0007$	1.5%	0.5%	0.7%

Peterson function [25] . The VINCIA parton shower is based on the dipole, or antenna, picture of QCD radiation [26] that follows a $2 \to 3$ branching evolution (e.g. $q\bar{q} \to qg\bar{q}$), rather than the standard $1 \to 2$ splitting functions typical of collinear factorization (PYTHIA 8 combines a $1 \to 2$ splitting probability with a $2 \to 3$ phase-space mapping). By considering colour dipoles, i.e. by considering particle emissions as stemming from colour-anticolour pairs, the VINCIA shower effectively incorporates coherent (wide-angle) emissions in the DGLAP formalism to leading power in $(1/N_c^2)$, where N_c is the number of colours. Compared to PYTHIA 8 stand-alone, the VINCIA dipole shower effectively includes suppressed unordered branchings that cover the hard region of phase space, as well as systematic "next-to-leading-colour" corrections [19]. In the non-perturbative sector, VINCIA uses the fragmentation model of PYTHIA 8 but, given the aforementioned differences between the two parton evolutions that the hadronization model does not reabsorb automatically in the soft region, its parameters (in particular the b-quark fragmentation) are retuned. Such differences effectively lead to modified b-jet thrust axis reconstruction, and to variations in different regions of the (refitted) $b \to B$ fragmentation [27]. The two VINCIA 1.1 [19] and 2.2 [20] versions used, differ basically in the fact that the former is tuned to reproduce e^+e^- data alone, whereas the latter includes also the latest results from proton-proton collisions at the LHC.

Details of the seven PYTHIA 8 and the two VINCIA tunes used are listed in Table 2. Most of them, except PYTHIA 8 Tune-7 and VINCIA 2.2 that also use hadron-collider data, employ exclusively e^+e^- results to constrain the parameters of the underlying showering and hadronization models. The closest parameter set to that used in the original LEP studies of the QCD corrections and uncertainties of the FB asymmetry [3] is PYTHIA 8 Tune-1*, where the '*' superscript indicates that we have modified the heavy-quark fragmentation from the default Lund-Bowler to the Peterson function to better match the LEP times settings. The whole MC setup effectively corresponds to nine different modelings of the underlying QCD effects: parton radiation, and heavy-quark, light-quark and gluon hadronization.

The eight original LEP analyses of $e^+e^- \to Z(b\bar{b})$ data and associated $A_{\mathrm{FB}}^{0,b}$ extractions have been implemented in a MC event simulation based on PYTHIA 8.226, stand-alone or with VINCIA,

Table 2: Details of the nine different sets of MC parameter settings in the PYTHIA 8 stand-alone and PYTHIA 8+VINCIA parton-shower simulations used in this work.

PYTHIA 8.226:	
Tune-1*	First PYTHIA 8 parameter set (2006), based on 1990s LEP-1 studies with JET-SET [16], modified ('*' mark) to use the Peterson fragmentation, rather than the default Lund-Bowler, function to match closely the original LEP QCD studies [3].
Tune-2	2007 tune to LEP-1 particle composition. Bowler b-frag. parameter: 0.58
Tune-3	2009 tune of hadronization and timelike-shower parameters using a large set of LEP-1 data with RIVET+PROFESSOR [28]. Bowler b-frag. parameter: 0.8
Tune-4	2013 tune of shower and hadronization parameters with LEP data. CMW α_s scale.
Tune-5	2013 flavour-composition tune based on ALEPH+DELPHI event shapes, ALEPH identified hadron spectra, PDG multiplicities, and ALEPH b fragmentation.
Tune-6	Second update of Tune-5.
Tune-7	2013 Monash tune [29] using e^+e^-, pp, p$\bar{\text{p}}$ data. Bowler b-frag. parameter: 0.855
VINCIA:	
1.1	2013 default tune to LEP-1 event shapes, jet rates, multiplicities, inclusive spectrum. 1-loop CMW α_s evolution. Bowler b-fragmentation parameter: 0.9.
2.2	2016 update of 1.1 tunes. 2-loop CMW α_s running. Bowler b-frag. parameter: 1.1

with the nine different tunes listed in Table 2. One-hundred million $e^+e^- \to Z(b\bar{b})$ events are generated at $\sqrt{s} = 91.24$ GeV with QED radiation switched on, in order to mimic closely the original experimental conditions, and analyzed as done in the original experiments by implementing as close as possible the analysis flow and selection criteria of the lepton-based and jet-charge based analyses of each one of the eight measurements. First, all final-state particles (except neutrinos) are clustered into jets using the JADE algorithm, interfaced via FASTJET [30], with resolution parameter $y_{\text{cut}} = m_{\text{jet}}^2/E_{\text{vis}}^2 = 0.01$ (DELPHI) or 0.02 (rest of experiments). The thrust axis of the event is then computed as a proxy of the original b-quark direction. The same original jet invariant mass $m_{\text{jet}} > 6$ GeV selection criteria (ALEPH and L3), as well as the $\kappa = 0.4$ (L3), 0.5 (OPAL and ALEPH), 0.6 (DELPHI) exponents (for the jet-charge extraction), and the (transverse) momenta (p_T) p cuts (above 2, 2.5, or 4 GeV) on the final electron and muons (for the lepton-based analyses), are applied. The resulting charged-lepton and jet-charge angular distributions, with the same $\cos\theta$-binning as used in the original measurements, are then fitted to Eqs. (3) and (4) respectively. Examples of the fitted $\cos\theta$ distributions are shown in Fig. 1 for lepton-based (left) and jet-charged-based (right) with $e^+e^- \to Z(b\bar{b})$ events generated with PYTHIA 8 (tune 1*). In the lepton-based analyses, the extracted $(A_{\text{FB}}^b)_{\text{obs}}$ fit values are corrected by the specific $\chi_B = 0.1177$–0.1318 B-mixing values directly measured by each experiment (the ALEPH and OPAL χ_B values are updated by half-a-percent, following the final corrections discussed in [2]).

4. Results and discussion

Through the procedure describe above, nine different MC values of $(A_{\text{FB}}^b)_{\text{obs}}$ are extracted for each one of the eight experimental setups simulated, which we compare among themselves as well as against the experimental data in Fig. 2 (lepton-based extractions) and Fig. 3 (jet-charge based analyses). We recall, first, that the experimental $(A_{\text{FB}}^b)_{\text{obs}}$ values plotted differ slightly from
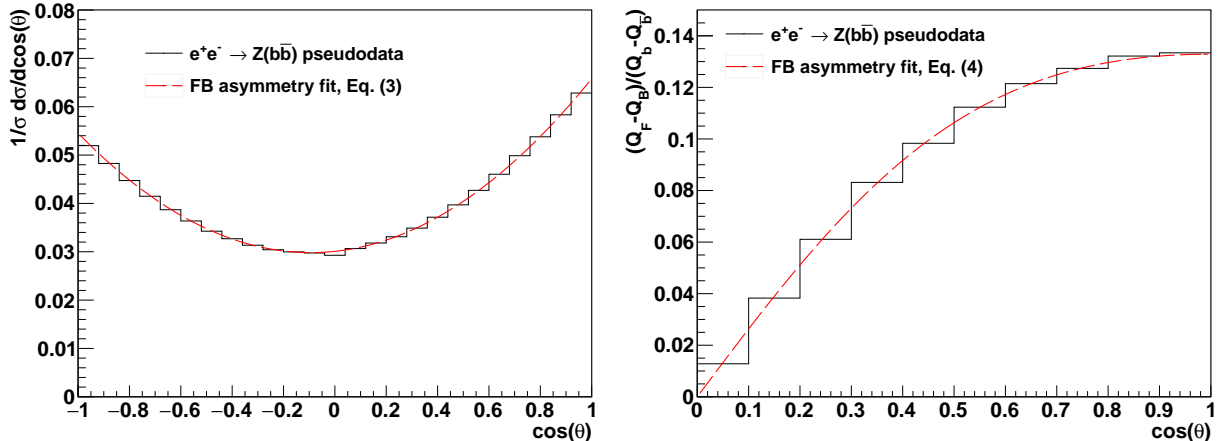
Figure 1: Examples of simulated $\cos\theta$ distributions in $e^+e^- \to \mathrm{Z}(b\bar{b})$ events generated with PYTHIA 8 (tune 1*). Left: Asymmetry reconstructed from the b-quark lepton charge fitted to Eq. (3) for a L3-like analysis. Right: Asymmetry reconstructed from the b-quark charge, for events passing the OPAL analysis flow, fitted to Eq. (4).

the corresponding $A_{\mathrm{FB}}^{0,b}$ values quoted in Table 1 since, as aforementioned, the latter are further corrected for electroweak effects and shifted to the pole m_Z mass. Since we are interested in comparing the uncertainties in the data and MC simulation, with the latter estimated from the *relative* differences among the $(A_{\mathrm{FB}}^b)_{\mathrm{obs}}$ values extracted using different PS + hadronization models, we do not correct for the latter effects. The large MC data samples generated (100 million events, of which between 20–30 million events pass the analysis cuts depending on the experiment) lead to very small statistical fluctuations in the simulated asymmetries, and any relative differences among them is in principle just related to intrinsic differences in the physics modelling and settings of each tune.

The FB asymmetries plotted in Figs. 2 and 3 show that, within the (comparatively larger) experimental uncertainties, data and simulation results agree well for all systems. The overall LEP QCD uncertainty of $(A_{\mathrm{FB}}^b)_{\mathrm{obs}}$ was derived from the average of the differences between the QCD corrections at hadron- and parton-level based on the results of the baseline MC JETSET 7.408 tune (equivalent to the leftmost PYTHIA 8 tune-1* MC point in both figures), four other JETSET variations, and one HERWIG 5.1 [31] simulation. Here, we similarly estimate the PS + hadronization uncertainties of the bottom-quark asymmetry from the spread of the $(A_{\mathrm{FB}}^b)_{\mathrm{obs}}$ results obtained from the nine MC tunes used. This is shown as an orange band around the MC points, which corresponds to the 95% (68%) confidence-level standard deviation of the predictions for lepton- (jet-) charge-based analysis, which we take as indicative of the associated overall uncertainty for each system. For the lepton-based analyses, we take a conservative 2σ error, rather than the standard 1σ used for the jet-charge case, to cover the comparatively lower asymmetry value derived for the tune-1*. The so-derived $(A_{\mathrm{FB}}^b)_{\mathrm{obs}}$ uncertainties amount to $\sim$0.7% and $\sim$0.05% for the lepton- and jet-charge-based measurements. The fact that the results of the jet-based analyses have much smaller spread than those from the lepton-based ones is not unexpected: Although, in principle, leptonic final-states allow for a better b-quark tagging and more accurate charge-sign determination than the pure hadronic final-state analyses, they are also prone to larger QCD-related systematic uncertainties because of a larger sensitivity to the b- and c-quark fragmentation details compared to the jet-charge
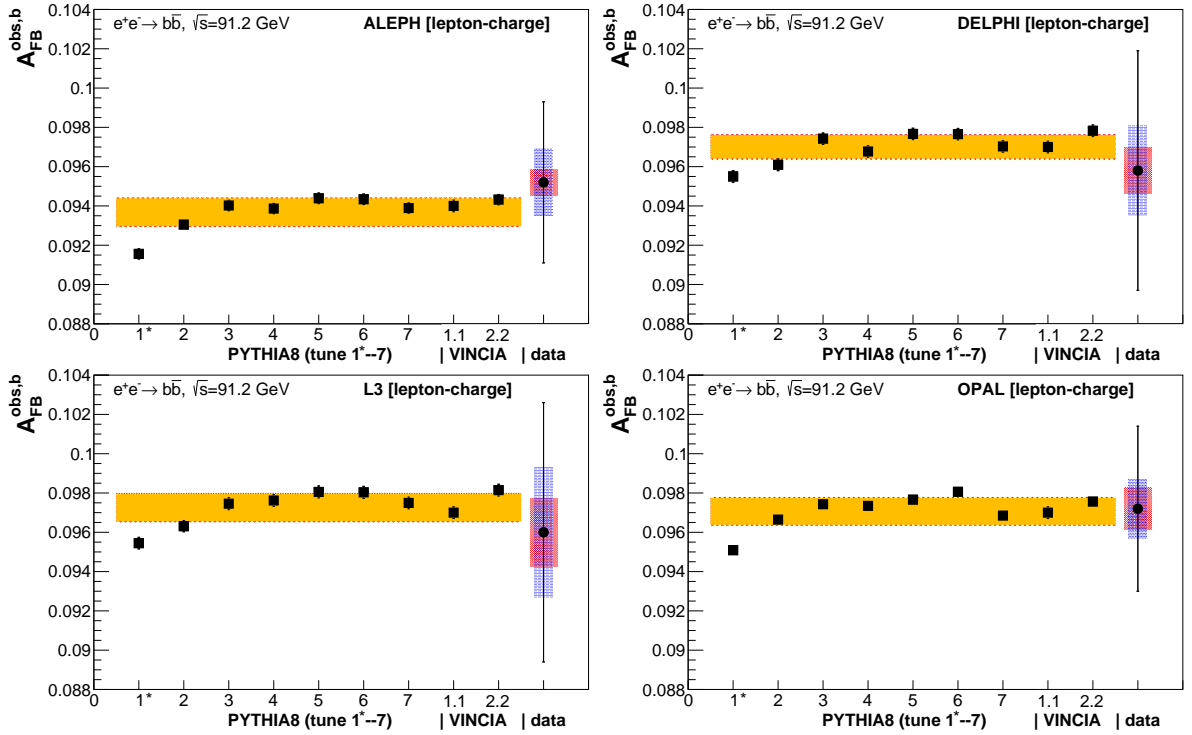
Figure 2: Values of the *b*-quark forward-backward asymmetry extracted from lepton-charge analyses of $e^+e^- \to \mathrm{Z}(b\bar{b})$ simulations based on seven PYTHIA 8 and two PYTHIA 8+VINCIA tunes (black squares), compared to the corresponding experimental results (rightmost data point with statistical uncertainties indicated by the error bar, and QCD, in magenta, and total, in blue, systematic uncertainty boxes) measured by ALEPH (top left) [7], DELPHI (top right) [8], L3 (bottom left) [9], and OPAL (bottom right) [10]. The orange band around the MC points indicates their overall assigned uncertainty and the outer hatched red band includes the NNLO pQCD uncertainty ($\pm 0.25\%$) added in quadrature, as explained in the text.
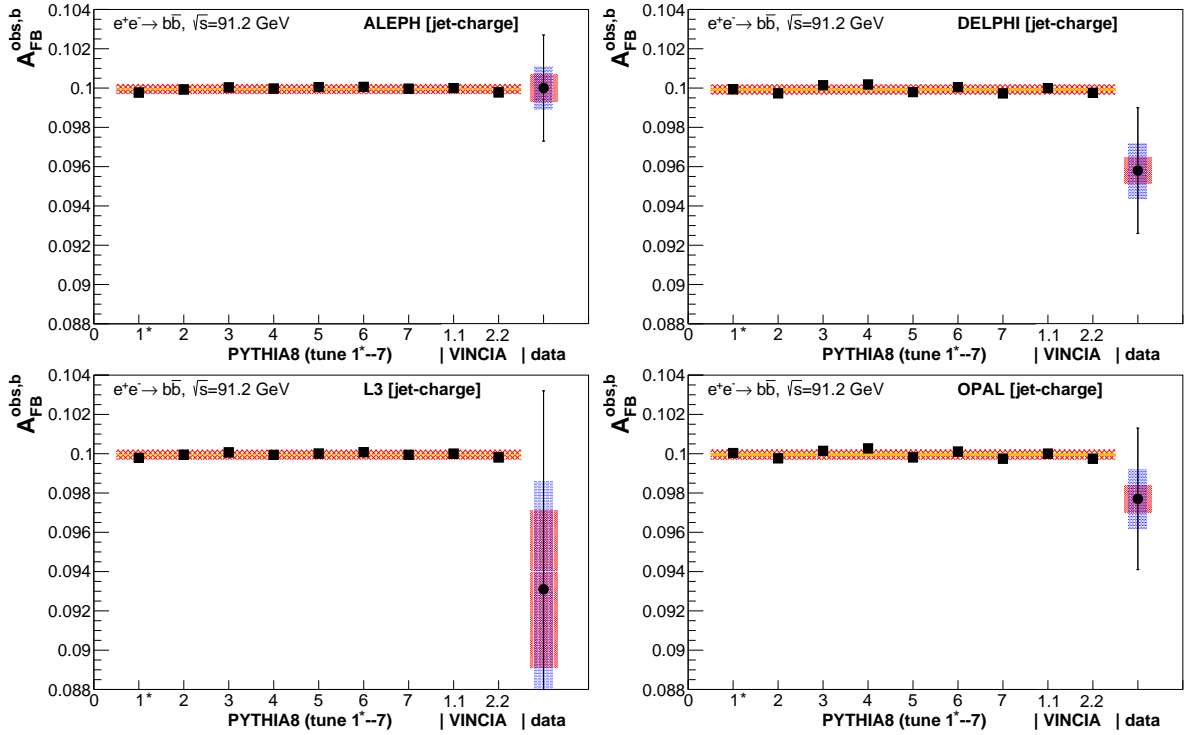
Figure 3: Values of the b-quark forward-backward asymmetry extracted from jet-charge analyses of $e^+e^- \to Z(b\bar{b})$ simulations based on seven PYTHIA 8 and two PYTHIA 8+VINCIA tunes (black squares), compared to the corresponding experimental results (rightmost data point with statistical uncertainties indicated by the error bar, and QCD, in magenta, and total, in blue, systematic uncertainty boxes) measured by ALEPH (top left) [11], DELPHI (top right) [12], L3 (bottom left) [13], and OPAL (bottom right) [14]. The orange band around the MC points indicates their overall assigned uncertainty and the outer hatched red band includes the NNLO pQCD uncertainty ($\pm 0.25\%$) added in quadrature, as explained in the text.

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extractions. The LEP data points plotted in Figs. 2 and 3 show full QCD uncertainties (magenta boxes) that include also a 0.63% contribution from the NLO perturbative $C_{\mathrm{QCD}}^{\mathrm{T,had}}$ correction factor given by Eq. (5). In order to make a more accurate comparison of the original and the new QCD uncertainties, we add quadratically to the parton-shower orange band the $\pm 0.25\%$ partonic NNLO uncertainty [4]. Our final reevaluated full QCD uncertainty is shown with the outer hatched red band around the MC points. The impact of the extra pQCD uncertainty is barely visible for the lepton-based analyses, dominated by the $\sim 0.7\%$ PS + hadronization uncertainties, but it becomes now the dominant source for the jet-based studies. Table 3 lists the updated QCD (and, correspondingly, total) systematic uncertainties for each one of the LEP A_{FB}^{b} extractions.

Table 3: Updated values of the $A_{\mathrm{FB}}^{0,b}$ asymmetries extracted at each LEP experiment (listed in Table 1) with the central value scaled by 1.004 to account for NNLO pQCD corrections [4], and with the QCD-related (and, accordingly, total) systematic uncertainties updated in this work. The last row lists the average result derived combining the individual estimates as described in the text.

| Extraction: | $(A_{\mathrm{FB}}^{0,b}) \pm \delta(\mathrm{stat}) \pm \delta(\mathrm{syst})$ | relative uncertainties | | |
Experiment		stat.	QCD syst.	total syst.
Lepton-charge based:				
ALEPH (2002)	$0.1007 \pm 0.0038 \pm 0.0017$	3.8%	0.7%	1.7%
DELPHI (2004–05)	$0.1029 \pm 0.0051 \pm 0.0021$	5.0%	0.7%	2.0%
L3 (1992–99)	$0.1005 \pm 0.0060 \pm 0.0030$	6.0%	0.7%	3.0%
OPAL (2003)	$0.0981 \pm 0.0038 \pm 0.0014$	3.9%	1.1%	1.6%
Jet-charge based:				
ALEPH (2001)	$0.1014 \pm 0.0025 \pm 0.0010$	2.5%	0.3%	1.0%
DELPHI (2005)	$0.0982 \pm 0.0030 \pm 0.0013$	3.1%	0.3%	1.3%
L3 (1998)	$0.0952 \pm 0.0101 \pm 0.0039$	10.6%	0.3%	4.0%
OPAL (1997,2002)	$0.0998 \pm 0.0034 \pm 0.0017$	3.4%	0.3%	1.7%
Combination	$0.0996 \pm 0.0015 \pm 0.0006$	1.5%	0.3%	0.6%

As a last exercise, we can try to assess the impact that our revised QCD uncertainties, combined with the updated massive NNLO pQCD correction of [4], would have on the final LEP $A_{\mathrm{FB}}^{0,b}$ average. If one takes into account the complete massive NNLO correction, rather than the NLO one used at LEP times, the central value of the each extracted FB asymmetry would increase slightly by +0.4%. If, in addition, we use the new QCD uncertainties estimated as explained above, of order $0.7\% \otimes 0.25\%$ (for the lepton-charge analysis) and $0.05\% \otimes 0.25\%$ (for the jet-charge extraction), we obtain the new individual A_{FB}^{b} values listed in Table 3. To obtain a single final asymmetry from all these results, we use the Best Linear Unbiased Estimate (BLUE) method as implemented in the BLUE (v.2.1.1) code [32], combining all individual $A_{\mathrm{FB}}^{0,b}$ values with their (newly reassessed) QCD uncertainties taken as fully correlated among experiments, and their statistical and remaining systematics as fully uncorrelated. Such a procedure increases the combined b-quark FB asymmetry by +0.0004 (with a total uncertainty, dominated by statistical uncertainties, basically unmodified) compared to the average derived with the BLUE method applied on the original LEP asymmetries listed in Table 1. The corresponding new result $A_{\mathrm{FB}}^{0,b} = 0.0996 \pm 0.0016$ would slightly diminish the tension between the b-quark asymmetry and the SM fit from 2.4σ to 2.1σ. Such a reduction of

the data-theory discrepancy would also propagate into the weak mixing angle derived from $A_{\mathrm{FB}}^{0,b}$, decreasing by an equal amount the existing pull between $\sin^2\theta_{\mathrm{eff}}^b$ and the current $\sin^2\theta_{\mathrm{eff}}^f$ world average.

The results presented here indicate that further improving the theoretical calculations and reducing the associated QCD systematic uncertainties alone will never solve this long-term discrepancy, because the experimentally measured $A_{\mathrm{FB}}^{0,b}$ precision is ultimately limited by a dominant 1.5% statistical uncertainty. Eventual confirmation or resolution of this long-term discrepancy requires therefore a new high-luminosity e^+e^- collider collecting orders-of-magnitude more data at the Z pole. At the FCC-ee, with 10^{12} Z bosons expected (compared to 10^7 Z's at LEP), the $A_{\mathrm{FB}}^{0,b}$ measurement will have negligible statistical uncertainties ($\sim$300 times smaller than at LEP, i.e. amounting to about $\pm 0.005\%$), and the experimental systematics will be reduced to the permille level thanks to an optimized control of the detector acceptances, efficiencies, and resolutions [5]. The results of our study (Figs. 2 and 3, and Table 3) indicate that the jet-charge-based analyses have today QCD systematics uncertainties in the 3-permille range, at least twice better than the lepton-charge based extraction. Even with the expected progress in the modeling of soft/collinear and higher-order QCD effects, the latter will still contribute to the systematics uncertainty budget of any $A_{\mathrm{FB}}^{0,b}$ measurement. In order to further reduce the impact of such QCD effects, it has been proposed to impose tighter acollinearity criteria between the reconstructed b jets [33]. In any case, improvements of a factor of ten or more in our theoretical numerical control of b-, c-quark showering and hadronization and higher-order pQCD corrections —as an integral part of a wider QCD-related experimental program [34]— will be needed to precisely measure electroweak observables such as A_{FB}^b at any future e^+e^- collider running at the Z pole. Such improved QCD understanding is key in the quest for indirect signals of new physics through high-precision measurements of multiple SM observables at the FCC-ee [35].

5. Summary and conclusions

The forward-backward asymmetry (A_{FB}^b) of bottom quarks produced in $e^+e^- \to b\bar{b}(g)$ processes, measured around $\sqrt{s} = m_{\mathrm{Z}}$ at LEP, remains one of the few experimental measurements not fully consistent with the SM theoretical predictions. We have studied to what extent the QCD developments in the last twenty years have improved our understanding of the uncertainties related to hard gluon radiation, parton showering, and hadronization corrections in the original A_{FB}^b experimental extractions. We implemented the eight original LEP A_{FB}^b analyses —four of them reconstructing the (leading) lepton-charge and four the jet-charge— in a PYTHIA 8 simulation with nine different models of parton radiation and fragmentation, including the alternative VINCIA shower model. The parton shower and hadronization uncertainties have been estimated from the spread of the $(A_{\mathrm{FB}}^b)_{\mathrm{obs}}$ values derived from the nine different MC simulations. The total QCD uncertainties, including NNLO corrections, of the extracted asymmetries appear to be about a factor of two smaller than those of the original LEP results. The final propagated QCD uncertainties are 0.7% and 0.3% for the lepton-charge- and jet-charge-based analyses, respectively. Combining all eight LEP measurements, with their revised QCD uncertainties and with the massive b-quark NNLO theoretical pQCD corrections, would lead to a new b-quark FB asymmetry changing slightly from $A_{\mathrm{FB}}^{0,b} = 0.0992 \pm 0.0016$ to 0.0996 ± 0.0016, with a 0.4% increase of its central value but basically unchanged uncertainties, dominated by statistical errors. This result diminishes the tension between the forward-backward b-quark asymmetry and the SM fit from 2.4σ to 2.1σ. We have also shown that the QCD systematic uncertainties will ultimately impact the maximum precision attainable

of any future $A_{\mathrm{FB}}^{0,b}$ measurement, and associated $\sin^2\theta_{\mathrm{eff}}^b$ determination, at future high-luminosity e^+e^- facilities running at the Z pole with orders of magnitude smaller statistical uncertainties, and have pointed out the importance of dedicated studies to minimize them.

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