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Can an extra HOM-like impedance explain the high octupole threshold at $Q' \sim 0$ in the LHC?

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Keywords: LHC, Impedance, Octupole threshold

Summary

The 2015 LHC octupole threshold measurement campaign revealed a higher Landau octupole current between the measured octupole threshold and impedance model predictions for chromaticities below few units. The discrepancy was recently reduced by revisiting the emittances of both transverse planes but still exceeds 300 A in some measurements (i.e. 400 A instead of 100 A), although excesses of $50 - 100$ A were also measured at multiple Q' between -10 and -5, where the expected octupole threshold should be negligibly small. One possible cause of the difference might be an HOM unaccounted for in the impedance model and recently, after having analyzed other options, it was considered likely. In this note we investigate this possibility via analytical and semi-analytical estimates and compare the findings with existing experimental evidence.

Contents

1 Introduction

Measurements of octupole threshold in LHC normally confirm predictions of its impedance model [1]. The level of agreement is typically quite good, at the level of tens of percent to $\sim 50\%$. A notable exception is the situation near chromaticity $Q' = 0$ at Flat-Top. A dedicated measurement campaign of 2015 [2] followed by an additional analysis of 2018 [3] indicated an excess of $100 - 300$ A (Fig. 1) with respect to what one would expect based on the best available knowledge of machine impedance, related beam stability, and the measured beam parametres. Additionally, a consistent excess of about 50 A was observed at a number of negative chromaticities.

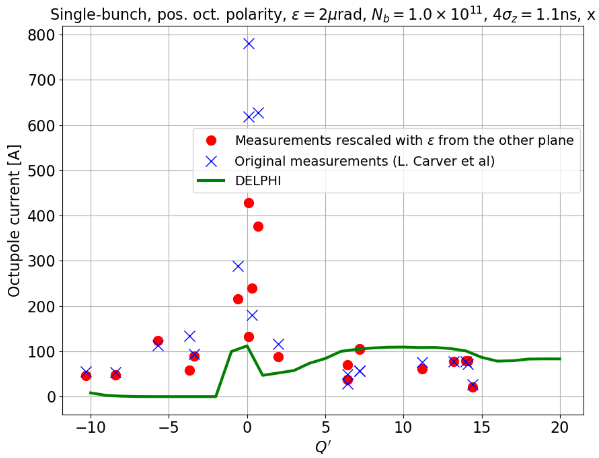


Figure 1: Measurements of the LHC octupole threshold at the top energy ($E = 6.5$ GeV) as a function of chromaticity. Solid line depicts the expectation based on the present impedance model [3] in the presence of an ideal 100-turn transverse feedback, computed with the DELPHI Vlasov solver for the most critical horizontal (x) plane.

The higher octupole current could be explained, for example, by an overall larger machine impedance. Indeed, other studies, such as measurements of tune shift with intensity [4–6] and instability rise time with chromaticity [7], indicated a potentially up to 50% larger impedance in certain planes [5, 6]. This assumption, however, contradicts the measurement data at the operational chromaticity of $Q' = 15$ [8] and around it, provided there are no fundamental differences in the Landau damping from the octupoles between the large and the small chromaticities. A non-perfect transverse feedback was considered to attack the discrepancy $Q' < 0$ but it was found to be insufficient [9]. The destabilizing effect of the transverse feedback, including the effect of mode coupling on the octupole threshold, was also considered at some point but found to be insufficient to explain the observations (see for example [10, 11]). Similarly, a small change in the octupole threshold may arise from a change of the longitudinal distribution [12]. Hardware noise, leading to instabilities with high latency times [13, 14], could provide an explanation of the observation, although a quantitative model of this effect for LHC is still under development. Recently, S. Berg [15] reminded that the discrepancy could be explained by a modification of the impedance model, namely an isolated high order mode (HOM). This note investigates this possibility. The goal is to find a range of parameters (Q-factor, frequency, shunt impedance) that are consistent with the observations. The scope of this report is limited to beam dynamics only, questions related to how realistic the parameters are or what hardware might cause such an impedance could be considered separately.

2 Analytical Considerations

In order to begin the analysis it is useful to know which mode is crossing the stability threshold, leading to the excess current. Let us start the analysis by assuming an unknown impedance is acting primarily on the most unstable mode thus enhancing its impact. From numerical simulations with the PyHEADTAIL macroparticle code [16, 18] and semi-analytical computations [19] it is clear that under normal conditions at $Q' \sim 0$ the most unstable head-tail mode is the one with an azimuthal mode number -1 (which corresponds to a mode with a negative tune shift of one synchrotron tune, see Fig. 2 for details). This mode is the most prominent in the presence of the transverse feedback at small negative and small positive chromaticities, whereas at larger positive chromaticities mode -2 dominates. Thus, if one affects mode -1 without affecting mode -2 one can increase the thresholds at $Q' \leq 0$, while keeping the thresholds around $Q' = 10 - 20$ unchanged.

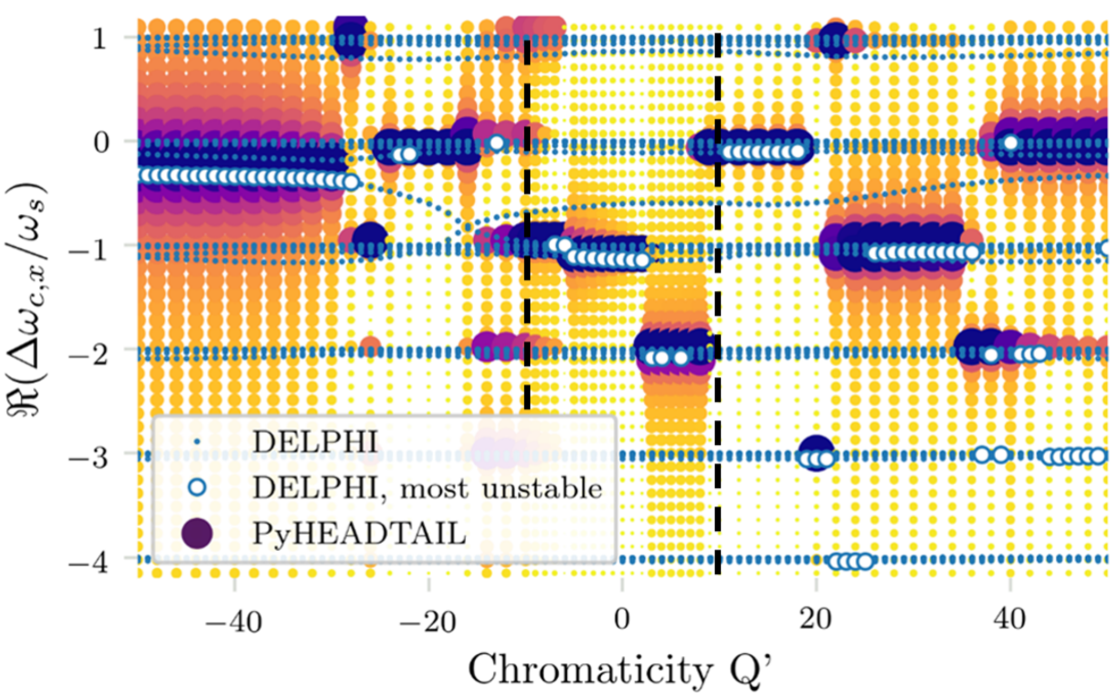


Figure 2: In the presence of a feedback mode -1 is the most unstable in the range of chromaticities of interest below and around $Q' = 0$, as seen in DELPHI and PyHEADTAIL simulations [20]. Dashed black lines highlight the region of interest $-10 \leq Q' \leq 10$. In the top side of this range mode -2 is also excited.

Another hint comes from an analytical air-bag model which captures qualitatively all the physics, despite being somewhat unrealistic. In the air-bag model the complex tune shift of a coherent mode Ω produced by a transverse impedance $Z_1(\omega)$ is governed by the equation [21]:

$$\Omega - \omega_\beta - l\omega_s = -i\frac{N_b r_0 c}{2\gamma T_0^2 \omega_\beta} \sum_{p=-\infty}^{+\infty} Z_1(\omega') J_l^2(\omega'\tau - \chi), \tag{2.1}$$

where ω_β and ω_s are the betatron and synchrotron angular frequencies respectively, γ stands for the relativistic γ-factor, r_0 is the classical proton radius, N_b is the number of particles in a bunch, c – the speed of light, l is the azimuthal mode number and J_l – the corresponding Bessel function, χ is the conventional head-tail phase, τ is the air-bag radius, $T_0 = 2\pi/\omega_0$ is the revolution period and ω_0 – the corresponding angular frequency. The summation is taken over the single-bunch spectral lines:

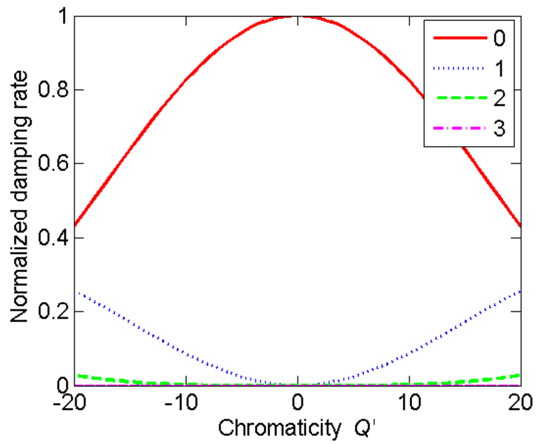


Figure 3: Ideal flat feedback is inefficient at damping azimuthal modes 2 or higher and modes -2 or lower in the range of chromaticities $|Q'| < 20$. Colored lines depict the fraction of the overall damping rate, acting on a head-tail mode with particular modulus of azimuthal number. Calculation performed for an air-bag beam [22].

$\omega' = p\omega_0 + \omega_\beta$, where p is an integer. A narrow-band impedance would sample the beam spectrum, given by the squares of Bessel functions, at its frequency corrected for the chromatic frequency shift. An ideal flat feedback has an equivalent impedance proportional to $\delta(\omega)$ and thus samples mode spectra at the chromatic frequency. Its efficiency against different coherent modes at any given chromaticity is given by the squares of the appropriate Bessel functions. It is easy to check that in the range of chromaticities between -10 and 10 an ideal flat feedback is only effective against the lowest azimuthal modes (Fig. 3). Therefore the unknown impedance can only excite modes 0 and -1, and not -2 or higher, which are effectively not damped by the feedback. An impedance acting on mode -1 seems even more plausible since the feedback is essentially inefficient against this mode at low chromaticities.

Based on the above considerations it makes sense to consider two separate cases: 1) a high-Q narrow-band impedance acting on mode -1; and 2) a low-Q broadband impedance acting on modes 0 and -1. First estimates can be obtained using Eq. (2.1) but the quantitative assessment will be done using the DELPHI Vlasov solver [19], which solves for the complex frequencies of the modes without the air-bag simplification given an arbitrary impedance model. For this purpose a Flat-Top impedance model of 2017 will be used [23]; a list of machine and beam parameters can be found in Tab. 1.

Table 1: Key parameters used for the study

Parameter	Value
Beam energy	6.5 TeV
Beam intensity	1×10^{11} ppb
Number of bunches	1
Norm. emittance, rms	2.5 μm
Bunch length, $4\sigma_{rms}$	1 ns
Fractional tunes: x, y, z	0.275, 0.295, 0.005
Feedback gain	1/100 turn^{-1}
Chromaticity Q'	$-20 ... 20$

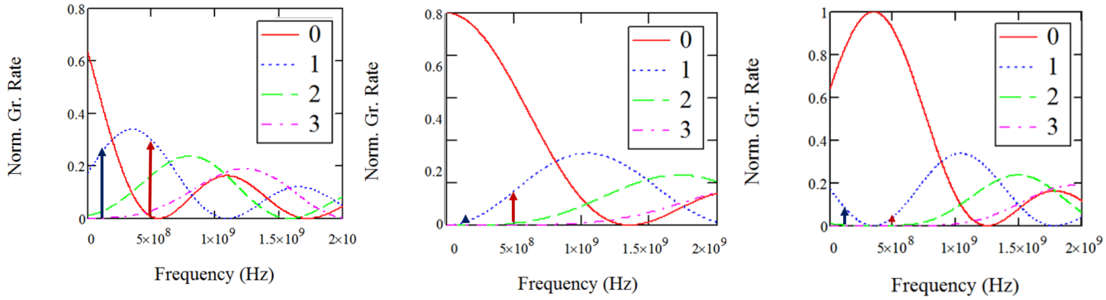


Figure 4: Mode spectra of the first 4 azimuthal modes of an air-bag beam. Left – $Q' = -15$, center – $Q' = 0$, right – $Q' = 15$. Blue arrows show an example of a narrow band HOM acting at 100 MHz, red – at 500 MHz.

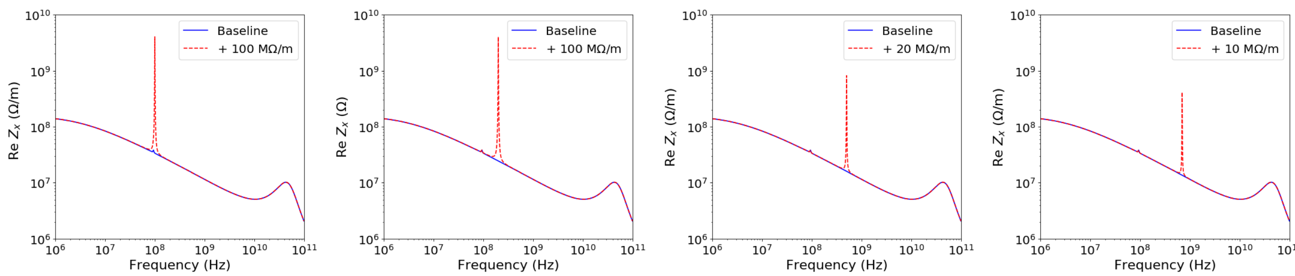


Figure 5: Examples of high-Q HOMs considered for this study and their impact on the overall machine impedance at the frequencies of (from left to right) 100, 200, 500, and 700 MHz. Plots show the real part of a dipolar horizontal impedance; HOM shunt impedance is indicated on the graphs.

3 High-Q HOM

The first option to check is a high-Q HOM with a quality factor around $100 - 1000$. Let us assume the mode is narrow enough and acts on only one line in the beam spectrum. This would yield an increase in the imaginary part of the coherent modes. One can estimate the extra impedance needed from the discrepancy of the octupole current using Eq. (2.1) and assuming a δ-like impedance located around a frequency f and put boundaries on the frequency range from the consideration that it is mode -1 that has to be the most affected by the extra impedance. For example, a 10 MHz HOM that would produce the necessary extra growth rate of mode -1 would also significantly affect mode 0. The growth rate of mode 0, which relates to the growth rate of mode -1 as $J_0^2(\omega)/J_1^2(\omega)$, would then be too large to be suppressed by a realistic feedback. On the other hand, an HOM above 1 GHz would also significantly excite mode -2 as shown in Fig. 4. Based on these considerations it is likely that the mode in question should have a frequency of a hundred to several hundred MHz. Its shunt impedance would need to be the higher the smaller its frequency, up to 100 MΩ/m for the 100 MHz case. Please note that while some figures presented in this analysis might seem unrealistic, such a constraint has not been considered here. Moreover, depending on its location the impedance might be greatly magnified by the local β-function, as explained in App. A.

3.1 Impact on the octupole threshold

In order to check the hypothesis a simulation was performed in the DELPHI code. Figure 5 shows some examples of impedances considered in the study. Despite the high shunt impedance of the mode the results show no significant increase of the octupole threshold around $Q' = 0$ for an HOM with

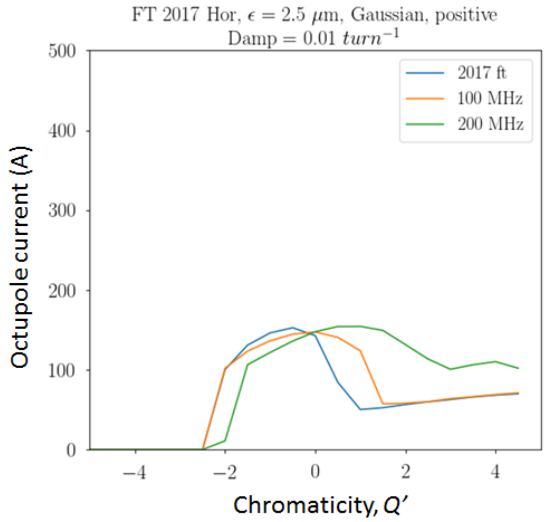


Figure 6: Octupole current as a function of chromaticity with additional high-Q modes at 100 and 200 MHz with R_s = 100 MΩ/m. Instead of the desired 'bump' at $Q' = 0$ the stability at positive chromaticities is affected.

$f = 100$ MHz (Fig. 6). Similar results are obtained at higher frequencies. One may expect that increasing the mode frequency would raise the mode -1 growth rate and hence the octupole threshold at $Q' = 0$, but in reality mode -2 starts getting affected, increasing the threshold at positive chromaticities.

As an exercise one may keep increasing the mode frequency to 500 MHz and further. From a simple estimate, see Eq. (2.1), we may expect the maximum of the mode -1 spectra to be around 600-700 MHz. Numerical simulations in DELPHI show that, as one may expect, higher azimuthal modes get excited in this case leading to large octupole thresholds in a broad range of chromaticities (Fig. 7).

To sum it up, a high-Q mode seems to be an unlikely candidate as it would not increase the octupole threshold at $Q' = 0$ significantly without increasing the threshold at higher chromaticities, which contradicts the observations.

4 Low-Q HOM

A low-Q mode with Q-factor of the order of 1–5 primarily would cause a significant real tune shift of the coherent modes. For a given impedance the sign and magnitude of the shift depend on the frequency of the HOM and the azimuthal number of the mode in question. For example, a $Q = 1$ HOM at 100 MHz drives the most unstable coherent modes inside the stability diagram, while a similar HOM at 1 GHz drives them outside of the stability diagram thus increasing the octupole threshold. Figure 8 shows the movement of the most critical head-tail modes as the HOM impedance is increased. Please note that the magnitude of the shunt impedance is so far irrelevant, what is important for this matter is the direction of the tune shift: positive or negative.

Equation (2.1) allows estimating what frequency of the impedance is needed in order to produce a negative shift of mode -1. As seen in Fig. 9 a negative mode -1 shift and hence an increase of the octupole current in the range of $Q' \leq 0$ is attained when the HOM frequency is above 500 MHz – lower than that it creates a positive mode frequency shift.

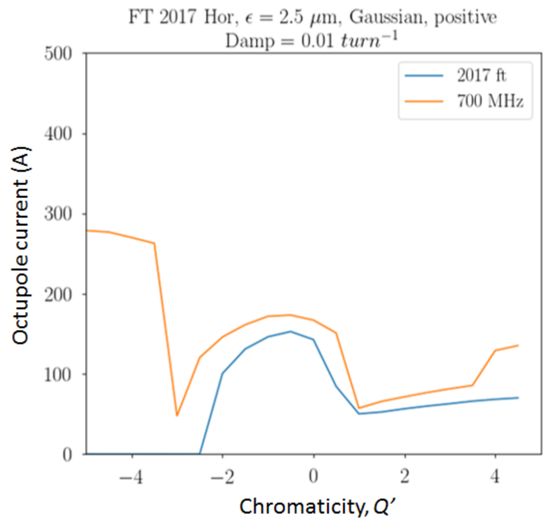


Figure 7: Octupole current as a function of chromaticity with additional high-Q modes at 700 MHz with a shunt impedance of 10 MΩ/m. A broad effect is seen at negative chromaticities in simulation.

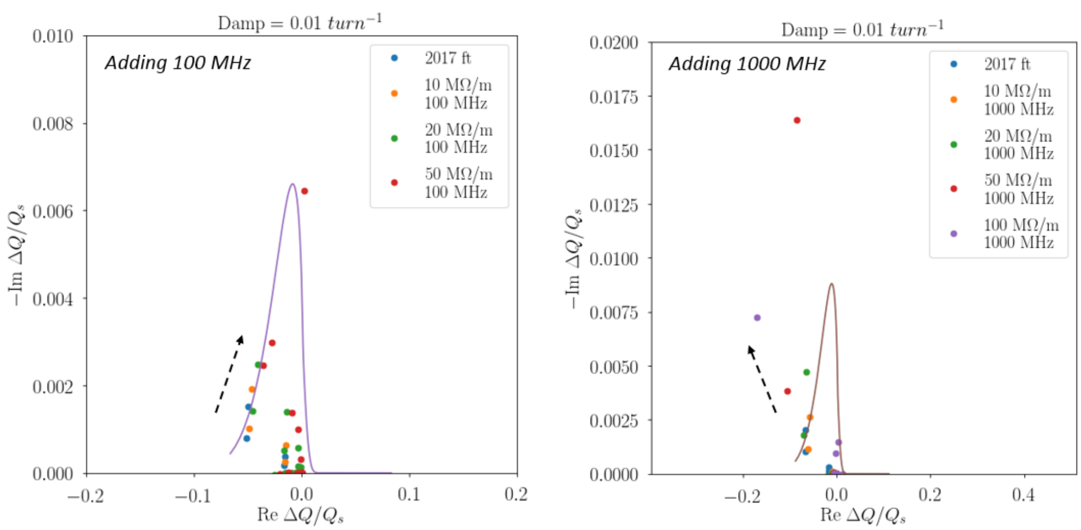


Figure 8: A low-Q HOM at 100 MHz drives the most unstable mode (mode -1) inside the stability diagram as its shunt impedance increases, while a mode at 1000 MHz drives it outside of the diagram. The most unstable head-tail modes obtained via a numerical simulation in DELPHI are shown as coloured dots. Each impedance model has several head-tail modes with different azimuthal and radial mode numbers, that are unstable, i.e. Im $\Delta Q < 0$. The mode that is the 'furthest' away from the stability diagram would ultimately dominate the dynamics as it would overcome the others thanks to the exponential growth rate. Flat-Top, $E = 6.5$ TeV, $Q' = 0$, 100-turn feedback. The solid line represents the stability diagram for a 100 A octupole current and a Gaussian beam of 2.5 μm normalized emittance. Dashed arrows guide the eye in the direction of the mode complex frequency shift as the HOM impedance increases. In both cases $Q = 1$ and R_s is increased gradually from negligibly small values. Synchrotron tune $Q_s = 2.1 \times 10^{-3}$.

4.1 Impact on the octupole threshold

Let us assume for the following analysis a mode with a frequency of 1000 MHz. Assuming a $Q = 1$ and scanning the mode shunt impedance one can reproduce the right 'bump' in the octupole threshold

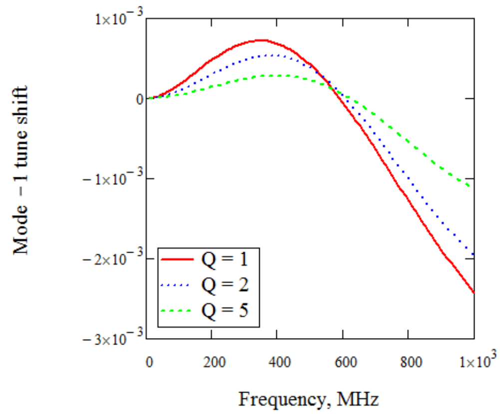
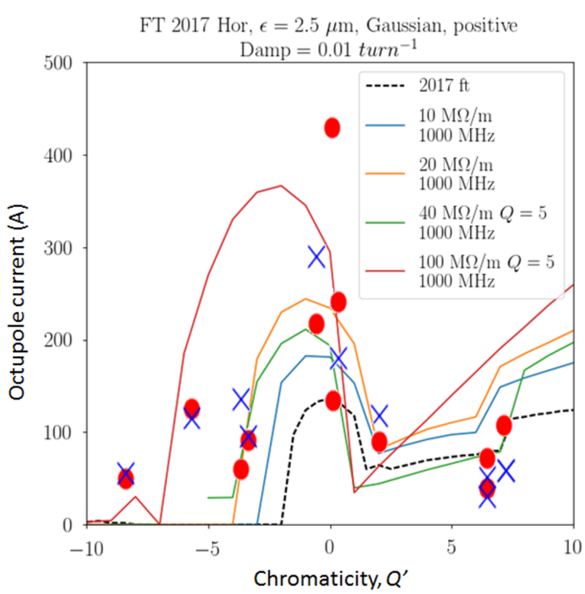


Figure 9: mode -1 frequency shift Re ΔQ created by a 100 MΩ/m HOM at $Q' = 0$.

at $Q' = 0$ in DELPHI (Fig. 10). One can also attain a similar result with a slightly larger $Q = 5$ and a larger shunt impedance. Tuning the mode impedance and its quality factor one can approach the model threshold at positive Q', an excess of about 100 A around $Q' = 0$, and a small additional octupole current at $Q' < 0$. Similar results can also be obtained with slightly smaller frequencies, i.e. 700 MHz.

In all the studied cases some discrepancy still remains at higher positive chromaticites. This discrepancy is normally not observed in the measurements, although it is safe to assume that octupole threshold measurements might have a significant error bar, coming for example from the uncertainties of optics, impedance model, and beam distribution. While the curves in Fig. (10) do not exactly reproduce the measurements, one can still consider them a good enough fit for the first attempt to explain the measurements with an HOM-like impedance.



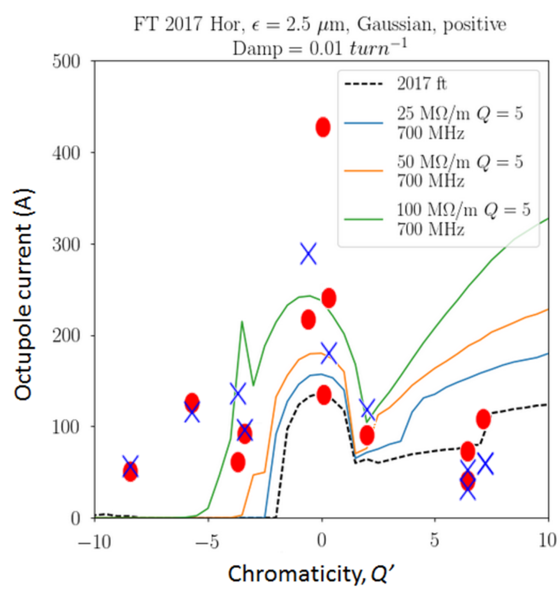


Figure 10: Simulated octupole threshold for several low-Q modes. One can reproduce qualitatively the behaviour observed in the measurements at negative and small positive chromaticities, as well as at 0. Left – HOM frequency 1000 MHz, right – HOM frequency 700 MHz. $Q = 1$ if not stated otherwise. Some discrepancy remains at larger chromaticites for the studies cases. Dots and crosses represent the measured data points from Fig. 1.

4.2 Comparison to other measurements

A low-Q HOM high-frequency impedance would generate an additional negative tune shift of the dipolar mode (mode 0). Numerical simulations in DELPHI suggest that this extra shift can be as large as tens of percent of the overall mode 0 tune shift for the parameters considered above (Fig. 11). This is in line with the observations of tune shift with intensity performed during the TMCI MD [4–6]. Both in Beam 1 and in Beam 2 the measured negative tune shifts of mode 0 were consistently larger than what was predicted by the impedance model (Fig. 12). In Beam 1 the discrepancy was observed for 2 different collimator settings in IR-7 and in Beam 2 – for 4 different settings, suggesting the presence of an additional impedance not related to primary and secondary collimators in IR-7, which are expected to dominate the machine's impedance at Flat-Top.

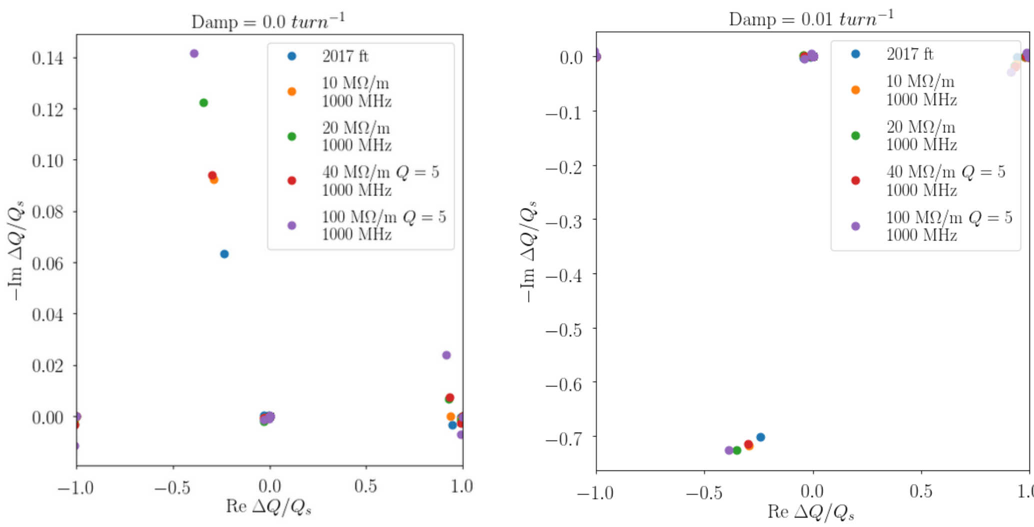


Figure 11: Mode 0 (in the center) tune shift is negative when adding an HOM impedance at 1000 MHz. For the parameters in question the shift increases by ~ 10 % at $Q' = 5$. Modes -1 and 1 can also be seen at -1 and 1 Q_s real tune shifts respectively. Left figure – no feedback, right – with a 100-turn feedback. All the radial modes are plotted for each azimuthal mode number: -1, 0, and 1.

Another hint can come from the measurements of the growth rate at $Q' < 0$ [7]. Such measurements were performed with single bunches at the injection energy in 2018. Measurement of the growth rate in the horizontal plane in the absence of the feedback shows around a factor $2-3$ excess when compared to the model in the range of chromaticities between -5 and -20. An HOM with a Q-factor between 1 and 5 and a frequency between 700 and 1000 MHz would be consistent with such a discrepancy (Fig. 13). It has to be noted though that the measurements were performed at the injection energy (450 GeV), where the ring's horizontal impedance is significantly smaller than at the top energy (Fig. 14). Such a difference is likely caused by the specific tight collimator setting at the Flat-Top. Another factor that could have played a role though at the injection energy is the space charge; its effect on mode growth rates lies beyond of the scope of this note. In the future a dedicated measurement of growth rate at Flat-Top, where the space charge does not have a large impact, would be beneficial for further benchmarking the impedance model.

5 Summary and Outlook

In conclusion, the present numerical study has considered several possibilities for an additional resonator-like impedance to explain the inconsistencies in the octupole threshold at $Q' \sim 0$ and other measurements.

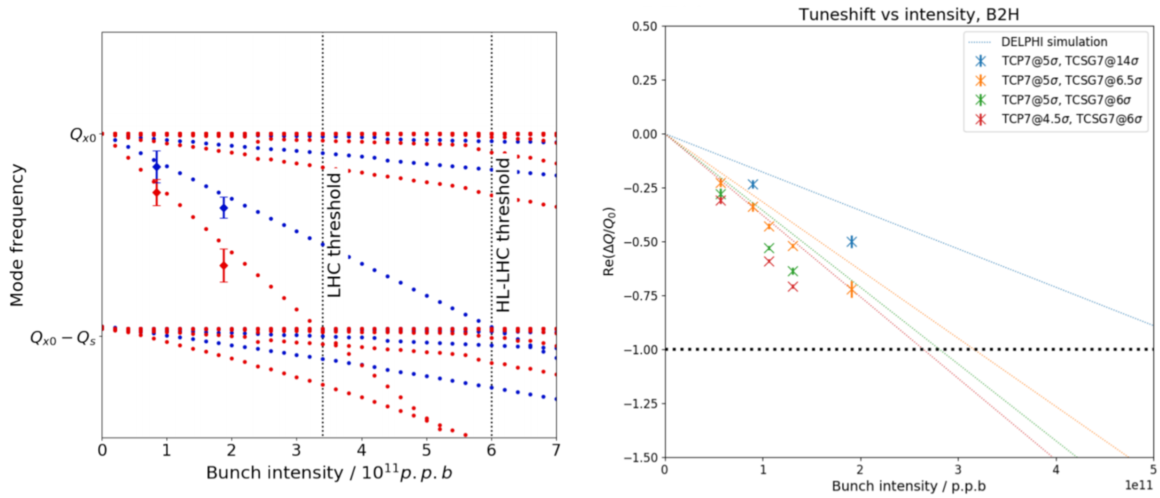


Figure 12: Mode 0 tune shift with intensity measured during the TMCI MD: left – Beam 1 [5]; right – Beam 2 [6]. Lines represent DELPHI predictions of mode frequency shifts for particular collimator settings based on the present understanding of the LHC impedance, while error bars – the measured frequency shifts, which are systematically lower than the theoretical predictions. Flat-Top ($E = 6.5$ GeV), $Q' \sim 5$.

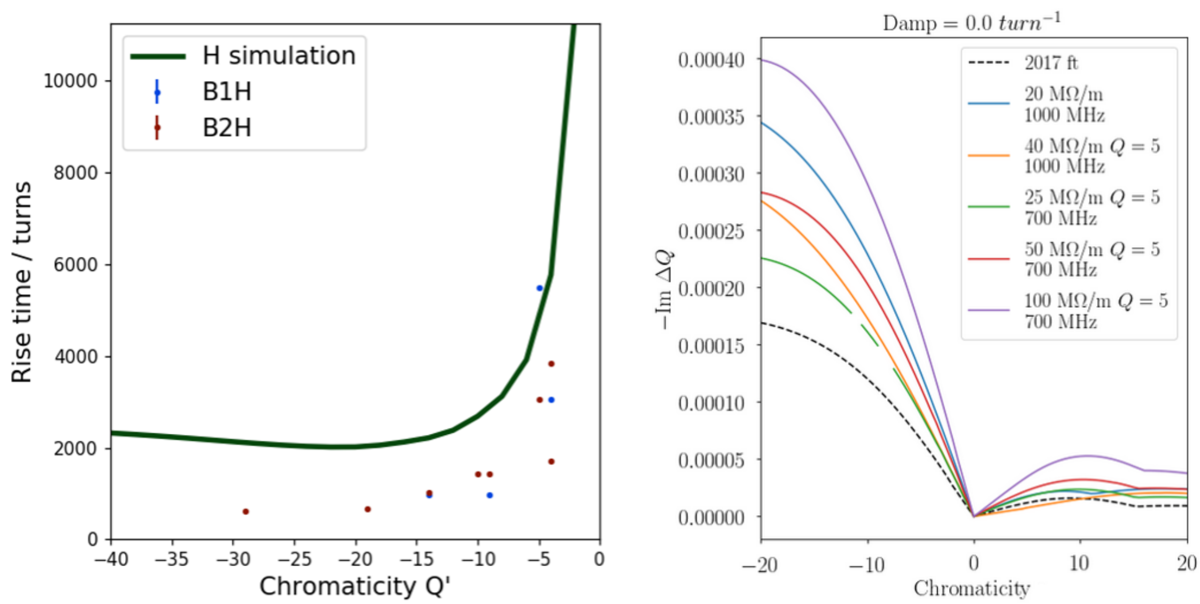


Figure 13: Measurements of the instability growth rate at negative chromaticities [7] at injection (450 GeV) on the left and a DELPHI simulation with several different extra HOMs on the right at the Flat-Top (6.5 TeV).

It has excluded any high-Q impedance in the whole frequency range up to 2 GHz as such an impedance would lead to a distortion of the octupole current at the chromaticites where it has not been observed. It has also excluded any broadband resonator impedance with a central frequency below 500 MHz as it would not be able to provide a negative tune shift of the most unstable mode at $Q' \sim 0$ and thus would not detrimentally affect the octupole threshold. After narrowing down the parameter space a plausible candidate has been found: a mode with a frequency of about 1 GHz, shunt impedance of $20 - 40$ MΩ/m and a quality factor of $1 - 5$.

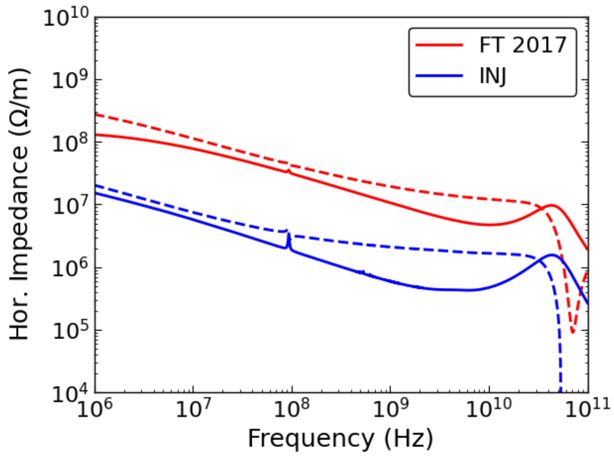


Figure 14: Present impedance model [23] predicts the machine's transverse impedance to be significantly larger at Flat-Top than at Injection. Solid lines depict the real part of impedance and dashed – its imaginary part.

Since this study study limited itself to the situation around $Q' \sim 0$ a possible next step could be to examine in detail the situation at the operational chromaticity, where plenty of measurement data has been obtained over the years. Another logical step would be to investigate if there could be a realistic impedance source for the found HOM candidates. Further experimental studies, such as growth rate vs chromaticity at Flat-Top, octupole threshold vs chromaticity with the opposite polarity, and measurements of the threshold at different points during the β^* squeeze, could shed more light on this problem. It must be noted that the measurements of 2015 [2] display a large spread of octupole current threshold, which is cannot be explained from an impedance point of view alone. Nevertheless, it seems plausible that an unaccounted HOM-like impedance could be a part of this puzzle.

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A Possible large HOM impedance from places with high β-function

An effect of a transverse impedance is proportional to the β-function at its location. Thus it can get significantly multiplied if the β-function is large. There are several points in the machine, in particular close to the main IPs, where β can reach kilometers as the β^* is squeezed. For example, Tab. 2 lists β values at the tertiaries in IP1,5. As the mean β-function is $\beta_{avg} = 70$ m, the impedance located there can get multiplied by up to a factor 30 compared to the same impedance located elsewhere in the ring; the situation may become even more critical in HiLumi. Note that in IP's 2,8 the maximum β-functions are lower. A series of systematic tests of octupole threshold vs chromaticity performed at the same energy but different β^* could indicate whether or not the missing impedance is located in one of such high β locations.

Table 2: Maximum beta functions at tertiary collimators at different points during the machine cycle

Machine State	β-function
Injection	160 m
Flat-Top	700 m
Physics	2 km