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Coulomb coefficient and volume-symmetry coefficient of nucleus incompressibility in the σ-ω-ρ model with derivative scalar coupling

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Abstract

With aid of the scaling model, the coulomb coefficient and the volume-symmetry coefficient of nucleus incompressibility are calculated in the mean field theory based on σ-ω-ρ model with derivative scalar coupling. These results are in good agreement with the empirical analysis of the giant monopole resonance data.

One way to determine the incompressibility K_{nm} of nuclear matter from the giant monopole resonance (GMR) data is using the leptodermous expansion[1] of nucleus incompressibility $K(A,Z)$ as follows.

$$K(A,Z) = K_v + K_{sf}A^{-1/3} + K_{vs}I^2 + K_cZ^2A^{-4/3} + \cdots \quad ; \quad I = 1 - 2Z/A, \qquad (1)$$

where the coefficients K_v, K_{sf}, K_{vs} and K_c are the volume coefficient ($= K_{nm}$), surface term coefficient, volume-symmetry coefficient and Coulomb coefficient respectively. We have omitted higher terms in eq. (1). Though there is uncertainty in the determination of these coefficients by using the present data, Pearson [2] pointed out that there is a strong correlation among K_v, K_c and the skewness coefficient, i.e., third order derivative of nuclear saturation curve. Similar observations are done by Shlomo and Youngblood [3].

According to this context, Rudaz et al. [4] studied the relation between K_v and the skewness coefficient using the generalized version of the relativistic Hartree approximation [5]. Recently, both of compressional and surface properties are studied by Von-Eiff et al. [6][7][8] in the frame work of the mean field approximation of the nonlinear σ-ω-ρ model.

In previous paper[9], we have studied the relation between K_v and the skewness coefficient in detail using the mean field theory of the nonlinear σ-ω model[10]. However, the four parameters in the nonlinear σ-ω model can not be determined uniquely. In this paper, we study the coefficients K_v, the K_c and K_{vs} using the model which is proposed by Zimanyi and Moszkowski [11](referred to as the ZM model), and modified by Song, Qian and Su [12] to include the effect of ρ meson. It is well-known that the ZM model shows smaller incompressibility K_{nm} than the original Walecka model [13], although the ZM model has only two parameters as well as in the Walecka model. The smaller K_{nm} in the ZM model is consistent with the nonrelativistic analysis of GMR data[1].

The extended ZM model (referred to as the ZM-II model according to ref. [12]) is based on the lagrangian density which consists of four fields, the nucleon ψ, the σ-meson ϕ, the ω-meson V_μ and the ρ-meson $\mathbf{b}_\mu$ and has derivative scalar coupling, i.e.,

$$L = -\bar{\psi}M\psi + \left(1 + \frac{g_\sigma\phi}{M}\right)\bar{\psi}(i\gamma_\mu\partial^\mu - g_\omega\gamma_\mu V^\mu - \frac{g_\rho}{2}\gamma_\mu\tau\cdot\mathbf{b}^\mu)\psi$$

$$+\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m_\sigma^2\phi^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_\omega^2V_\mu V^\mu - \frac{1}{4}\mathbf{L}_{\mu\nu}\cdot\mathbf{L}^{\mu\nu} + \frac{1}{2}m_\rho^2\mathbf{b}_\mu\cdot\mathbf{b}^\mu$$

$$; F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu, \quad \mathbf{L}_{\mu\nu} = \partial_\mu \mathbf{b}_\nu - \partial_\nu \mathbf{b}_\mu. \tag{2}$$

By rescaling the fermion wave function

$$\psi \to \left(1 + \frac{g_\sigma \phi}{M}\right)^{-1/2} \psi, \tag{3}$$

the eq. (2) can be written as

$$L = \bar{\psi}(i\gamma_\mu \partial^\mu - M^* - g_\omega \gamma_\mu V^\mu - \frac{g_\rho}{2}\gamma_\mu \tau \cdot \mathbf{b}^\mu)\psi$$

$$+\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m_\sigma^2\phi^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_\omega^2 V_\mu V^\mu$$

$$-\frac{1}{4}\mathbf{L}_{\mu\nu}\cdot\mathbf{L}^{\mu\nu} + \frac{1}{2}m_\rho^2 \mathbf{b}_\mu \cdot \mathbf{b}^\mu, \tag{4}$$

where the effective nucleon mass M^* is given by

$$M^* = m^* M, \qquad m^* = \left(1 + \frac{C_\sigma m_\sigma \phi}{M^2}\right)^{-1}, \tag{5}$$

where $C_\sigma = g_\sigma M/m_\sigma$. In the mean field approximation, M^* is given by

$$M^* = m^* M, \qquad m^* = \left(1 + \frac{C_\sigma m_\sigma < \phi >}{M^2}\right)^{-1}, \tag{6}$$

where $< \phi >$, the mean field of scalar meson ϕ, is given by

$$< \phi > = \frac{g_\sigma}{m_\sigma^2} m^{*2} < \bar{\psi}\psi > . \tag{7}$$

For the explicit expression of the scalar density $< \bar{\psi}\psi >$, see eq. (10) in ref. [12]. The energy density of the system is given by

$$\epsilon = \epsilon_{fp}(M^*) + \epsilon_{fn}(M^*) + \frac{1}{2}C_\omega^2 \frac{\rho_B^2}{M^2} + \frac{1}{2}m_\sigma^2 < \phi >^2 + \frac{1}{8}\frac{C_\rho^2}{M^2}(\rho_p - \rho_n)^2;$$

$$C_\omega = \frac{g_\omega M}{m_\omega}, \qquad C_\rho = \frac{g_\rho M}{m_\rho}, \tag{8}$$

where ρ_B, ρ_p and ρ_n are the baryon density, the proton density and the neutron density respectively. The $\epsilon_{fp}(M^*)$ and the $\epsilon_{fn}(M^*)$ are defined so that $\epsilon_{fp}(M)$ and $\epsilon_{fn}(M)$ are

equal to the energy density of free proton and that of free neutron, respectively. We do not show the details of this model. See the ref. [12] for the ZM-II model and the refs. [11] and [14] for the original ZM model.

We calculate K_v, K_c and K_{vs} using the equations of state (8) with aid of the scaling model[1]. The K_v is given by

$$K_v = K_{nm} = 9\rho_{B0}^2 \frac{\partial^2 e}{\partial \rho_B^2}|_{\rho=\rho_{B0}} \tag{9}$$

where $e = \epsilon/\rho_B$. The coefficients K_c and K_{vs} are given by [7]

$$K_c = \frac{3q_{el}^2}{5r_0}\left(\frac{K_3}{K_v} - 8\right), \tag{10}$$

and

$$K_{vs} = K_{sym} + L\left(\frac{K_3}{K_v} - 6\right), \tag{11}$$

where q_{el} is the electric charge of proton, $r_0 = (3/(4\pi\rho_{B0}))^{1/3}$,

$$K_3 = -27\rho_{B0}^3 \frac{\partial^3 e}{\partial \rho_B^3}|_{\rho_B=\rho_{B0}}, \tag{12}$$

$$L = 3\rho_{B0}\frac{\partial a_4}{\partial \rho_B}|_{\rho_B=\rho_{B0}}; \quad a_{sym} = \frac{1}{2}\rho_B \frac{\partial^2 \epsilon}{\partial \rho_3^2}|_{\rho_3=0}; \quad \rho_3 = \rho_p - \rho_n \tag{13}$$

$$K_{sym} = 9\rho_{B0}^2 \frac{\partial^2 a_{sym}}{\partial \rho_B^2}|_{\rho_B=\rho_{B0}}. \tag{14}$$

Table 1

Fig. 1(a),(b)

In numerical calculations, we put $\rho_{B0} = 0.15\text{fm}^{-3}$, $\epsilon_0/\rho_{B0} - M = -15.75\text{MeV}$. We summarize the results in table 1. The results in the ZM-II model are compared with the QHD-II model (the mean field calculation based on the linear σ-ω-ρ model) [15][16] and the fully relativistic Hartree calculation [17] including the mean field contribution of ρ-meson [15][16]. (We call the latter model the RHA-II.) The ZM-II model has smaller K_v, larger K_c and larger K_3 than those in the QHD-II and the RHA-II models. These facts shows a strong correlation with the large M^* in the ZM-II model. In fig. 1(a), we show the K_v-K_c relations in the ZM-II, the QHD-II and the RHA-II models, comparing them with the result obtained by using the mean field theory based on the non-linear σ-ω-ρ model (referred to as the NL-II model)[18][6][7][8] which has a cubic-plus-quartic potential of ϕ. In the NL-II model, except for the ρ-nucleon coupling, the other four parameters are determined to satisfy the saturation condition and to realize the indicated K_v and M^*. (We remark that there is no ρ-meson contribution in M^*, K_v, K_3 and K_c under the mean field approximation.) The result of the ZM-II model has a good agreement with the NL-II results because the ZM-II model has also the higher-order terms of ϕ[11], i.e., the ZM-II result in which M^* is equal to $0.85M$ lies between the result with $M^* = 0.8M$ and the result with $M^* = 0.9M$ in the NL-II model. The RHA-II result in which $M^* = 0.73M$ lies near the NL-II result with $M^* = 0.7M$. Naturally, the QHD-II result is obtained by dropping nonlinear terms in the NL-II model. From fig. 1(a), it is clear the smaller K_v with the larger M^* gives larger K_c and, from eq. (10), the larger K_c needs the larger K_3 or the smaller K_v. In fig. 1(a), we also show the analysis of GMR data by Pearson[2] and Shlomo-Youngblood[3]. According to their conclusions, we show only the results with $K_v \approx 150 \sim 350$MeV. In the ZM-II model, not only K_v but also K_c has a good agreement with the empirical ones.

The a_4, K_{vs} and L depend on the value of C_ρ. We use $C_\rho^2 = 54.71$ [15] in the QHD-II and the RHA-II models, and $C_\rho^2 = 54.71/m^{*2}$ in the ZM-II model [12]. The a_4 in the ZM-II model is smaller than that in the QHD-II model but larger than that in the RHA-II model, while the K_v in the ZM-II model is the smallest among the three models. The a_4 in the ZM-II model is smaller than the empirical value($\approx$ 30MeV). We also determine C'_ρ which realizes $a_4 = 30$MeV, and calculate K'_{vs} and L'. The $K_{vs}(K'_{vs})$ in the ZM-II model is much larger than those in the QHD-II and the RHA-II models. On the other hand, the $L(L')$ in the ZM-II model is positive and is not much different from those in the QHD-II and the RHA-II models. The K_{sym} in the ZM-II model is smaller than those in the other two models and the absolute value of them are very small. From the eq. (11), it is reasonably concluded that the large $K_{vs}(K'_{vs})$ in the ZM-II model is originated from the small value of K_v and the large value of K_3 which do not depend on $C_\rho(C'_\rho)$. In fig. 1(b), we compare these results with the results of non-linear σ-ω-ρ model and the empirical data[2][3]. From the empirical data, it is seen that the smaller K_v needs the larger K_{vs} and the larger K_v needs the smaller K_{vs}. The result in the ZM-II model has a good agreement with the empirical data, i.e., it has small K_v and large $K_{vs}(K'_{vs})$. Furthermore, comparing the results in the NL-II model with those in the other three models, we see that the smaller K_v and the larger M^* in the ZM-II model give the larger value of $K_{vs}(K'_{vs})$ as well as K_c. It is also interesting that not only the empirical data but also the results of the ZM-II, the RHA-II and the QHD-II models have an approximately linear relation of K_v and $K_{vs}(K'_{vs})$.

In summary, we have calculated K_c and K_{vs} in the ZM-II model and compared the results with the results of the QHD-II, the RHA-II, the NL-II models and the empirical data. We found that the ZM-II model has the good agreement with empirical values, because of the smaller K_v and the larger M^*. In this paper, we have restricted our discussions to K_v, K_c and K_{vs}, because they can be calculated in the framework of infinite nuclear matter. The other properties must be studied. Using the nonlinear σ-ω-ρ model, Von-Eiff et al. [6][8] studied the surface properties in the framework of the semi-infinite nuclear matter and found that a low incompressibility ($K_{nm} \approx 200$MeV) and a high effective nucleon mass ($0.70 \leq M^*/M \leq 0.75$) are favorable[8]. On the other hand, using the same model, Bodmer and Price [18] found that the experimental spin-orbit splitting in light nuclei supports $M^* \approx 0.60M$. The result of the generator coordinate calculations for breathing-mode GMR by Stoitsov, Ring and Sharma [19] suggests that $K_{nm} \approx 300$MeV. It is very interesting to study these problems in the ZM-II model.

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Table and Figure Captions

Table 1

The coefficients of leptodermous expansion and another properties of nuclear matter at the saturation density calculated by using the QHD-II, the RHA-II and the ZM-II models. Except for dimensionless parameters, they are shown in MeV.

Fig. 1 (a)The K_v-K_c relations. (b) The K_v-K_{vs} relations. :

The open circle, open square and open triangle are results in the QHD-II, the RHA-II and the ZM-II models respectively. The solid circle, solid square and the solid triangle in (b) are results in the QHD-II, the RHA-II and the ZM-II models with the parameter C'_ρ. The dotted lines are results by the NL-II model with indicated M^*. The crosses with error bars are the results in table 3 in ref. [2] and the small solid inverse-triangles are the results in table IV in ref. [3]. For simplicity of the figures, we have omitted the error bars in the latter data.

Table 1

	QHD-II	RHA-II	ZM-II
C_σ^2	355.3	226.7	178.6
C_ω^2	271.9	146.7	64.01
M^*/M	0.54	0.73	0.85
K_v	544.5	452.5	221.3
K_c	-8.834	-5.870	-3.874
K_3	-2152	25.45	610.3
K_{vs}	-854.4	-401.9	-234.4
K_{sym}	93.14	-13.17	-21.26
a_4	28.2	23.9	25.4
L	95.20	65.40	65.75
$C_\rho^{2\prime}$	65.62	92.06	103.9
K'_{vs}	-907.4	-510.2	-279.4
L'	100.5	83.62	79.64

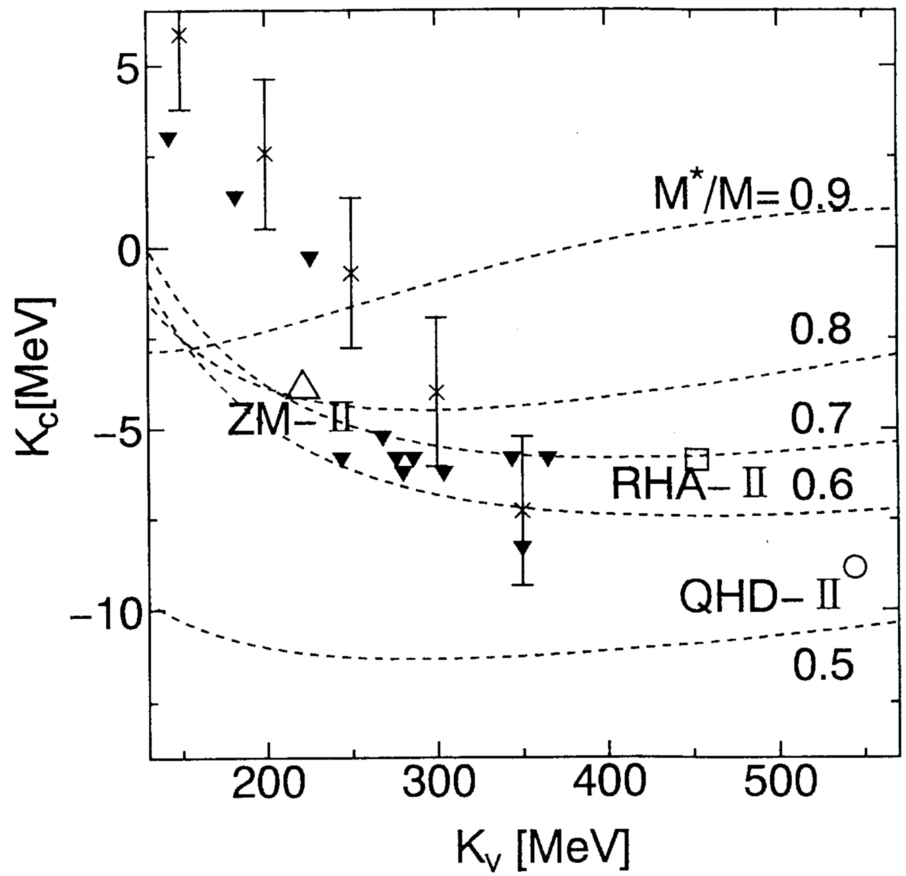
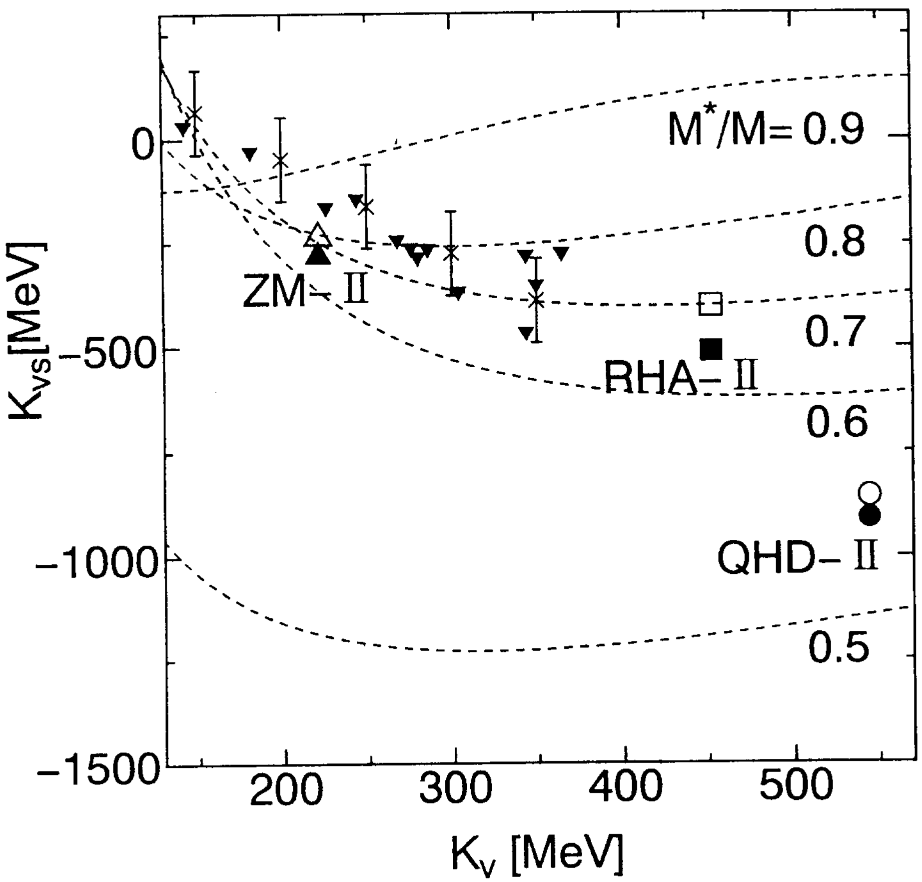


Fig.1(a)



M*/M= 0.9
0.8
0.7
0.6
0.5
ZM- II
RHA- II
QHD- II
0
−500
−1000
−1500
200
300
400
500
K_vs[MeV]
K_v [MeV]

Fig.1(b)