Scaling Property of $W^{\pm}$ boson from the $\bar{p}p$ collider at CERN

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An analysis of the single-particle distribution of inclusive $\bar{p} + p \rightarrow W^{\pm} + \cdots$ at $\sqrt{s} = 540$ GeV in terms of the covariant Boltzmann factor indicates that the Feynman-Yang scaling holds, the temperature of W being ~ 0.188 GeV. The specific central particle density of W is found to be the same as π and K^{o} production at the same energy.

1. Introduction

One of the characteristic features of multiparticle production by high energy hadronic collisions is the scaling, a property implied by QCD in the context of the renormalization group approach. It has been found that the $x = 2P_{\parallel}/\sqrt{s}$ distributions of various secondary hadrons follow practically the same exponential law of the well-known Feynman-Yang scaling. The question arises: How is this scaling in the case of the production of the intermediate vector boson W and Z, which is of a quite different nature, especially as far as the mass of the secondary is concerned?

We present in this paper the results of investigation of this important property of the Feynman-Yang scaling for the $W^{\pm}$ production by the $\bar{p}p$ collider at CERN of the UA 1 Collaboration [1]. We will use the covariant Boltzmann factor to derive an exponential law for the x-distribution and estimate its characteristic slope, which turns out to be consistent with that of hadron production in general.

An analysis of the angular distribution of W in the c.m.s. of the colliding $\bar{p}$ and p of the UA1 data [1] shows a strong peak near $\cos\theta = 1$. This characteristic feature, reminiscent of the bremsstrhlung, will be investigated using an appropriate formula derived from the Boltzmann factor. We find another important property that the specific particle density h of the W production is comparable to those of π and K^o production at the same energy of another CERN experiment by the UA 5 Collaboration [2], in spite of the tremendous difference in the production cross-section, namely $\sigma_W/\sigma_\pi \approx 10^{-7}$. The energy dependence of h behaves like $E_{cm}^{-1/3}$, just like the case of hadron production by e^+e^- annihilations, indicating the universality of the scaling of the specific particle density.

A remark will be made on the application of the scaling property of the

the angular distribution of secondaries to investigate hadronic collisions of very high energy.

2. Scaling of the x-distribution

Consider the production of secondary particles by $\bar{p}p$ collisions at a given energy $\sqrt{s}$. We use the covariant Boltzmann factor to describe its folded single-particle distribution in the c.m.s., as follows

$$f \sim e^{-(E - \beta^* P_{\parallel})/T} \tag{1}$$

where E and P_0 are the energy and the longitudinal momentum of the secondary of mass m, β^* is the velocity of the rest frame of the secondaries with respect to the c.m.s of collision and T is the temperature. The longitudinal momentum distribution according to (1) is, in the high energy limit,

$$\frac{dn}{dP_{\parallel}} \sim \int_0^{\infty} f \cdot dP_{\perp} = T^2(1 + P_{\parallel}/T)\, e^{-(E - \beta^* P_{\parallel})/T} \tag{2}$$

As $P_{\parallel} >> T$, therefore, we may rewrite this expression in terms of $x = 2P_{\parallel}/\sqrt{s}$ as

$$\frac{dn}{dx} \sim x \cdot e^{-ax} \tag{3}$$

where

$$a \equiv \frac{(1 - \beta^*)\sqrt{s}}{2T}. \tag{4}$$

We now use (3) to analyze the UA 1 data of inclusive $\bar{p} + p \rightarrow W^{\pm} + \cdots$ at $\sqrt{s}$ = 540 GeV. Their data are shown by the semi-log plot of $\frac{1}{x}\frac{dn}{dx}$ vs. the Feynman variable x. The straight line represents a least-squares fit with (3), the slope parameter being

$$a_W = 9.99 \pm 0.15. \tag{5}$$

A comparison of the fit with the data indicates that the agreement is satisfactory. Note that this slope is comparable to that for π production by high energy hadronic collisions in general, namely $a \approx 10$.

As regards the scaling property, we recall that for π^- production by $\bar{p}p$ collisions at $P_{lab} \geq 20$ GeV/c, $1 - \beta^* \rightarrow 2/\gamma_{cm}$, where $\gamma_{cm} = \sqrt{s}/2m_p$ is the Lorentz factor of the cms of pp or $\bar{p}p$ collisions [3]. Therefore, in the case when the scaling holds, (4) becomes

$$a = \frac{2m_p}{T}. \tag{6}$$

From the parameter a_W we find for the temperature of W:

$$T_W = \frac{2m_p}{a_W} = 0.188 \pm 0.003 \text{ GeV}, \tag{7}$$

compared to the π temperature

$$T\pi = 0.168 \pm 0.004 \text{ GeV}, \tag{8}$$

assuming $<P_\perp> = 0.418 \pm 0.004$ GeV/c measured by the UA 1 experiment [4].

The kinetic energy of the W boson produced by the CERN collider is $3T_W/2 = 0.282$ GeV $<< m_W$, corresponding to a velocity ≈ 0.084, extremely small compared to other relativistic secondaries such π's, K's, ... of the same reaction. This indicates that the W boson is dragged on by other secondaries in the mass motion of the secondaries along the collision line with the velocity β^* as is determined by the scaling $\beta^* = 1 - 2/\gamma_{cm}$.

3. The transverse momentum of W

Turn next to the plot of the transverse momentum distribution of W in Fig. 2, as has been reported by the UA 1 Collaboration. We see that here, $P_\perp$ is in general very large compared to the case of π production. Indeed, we find

$$<P_\perp> = 7.70 \pm 0.41 \text{ GeV/c}, \quad <P_\perp^2> = 122.2 \pm 6.4 \text{ (GeV/c)}^2. \tag{9}$$

This is because, here, we are dealing with the hard process of $u\bar{d} \rightarrow W^+$ or $\bar{u}d$

$\rightarrow W^-$ of quark-antiquark interaction a la Drell-Yan.

None-the-less, $<P_\perp> \ll m_W$; therefore, we may use the Maxwell distribution in a restricted region, e.g. $\lesssim 2<P_\perp>$ in order to get another estimate of the temperature using instead the $P_\perp$ distribution. We find $T \approx 0.135 \pm 0.021$ GeV, consistent with the previous estimate using the x-distribution, within about 2 and 1/2 standard deviations.

The appropriate formula for large $P_\perp$ distribution as we are now dealing with, is a modification of the exponential form for small $P_\perp$ with a power law fall-off for large $P_\perp$, as is inspired by QCD. The formula due to Hagedorn [5] is as follows:

$$\frac{dn}{dP_\perp^2} = \frac{C}{(1 + P_\perp/P_o)^n} \tag{10}$$

where P_o and n are two parameters and C is the normalization coefficient.

The moments of $P_\perp$ of order ν according to (10) is

$$<P_\perp{}^\nu> = P_o{}^\nu . \frac{\Gamma(n - \nu - 2)\Gamma(\nu + 2)}{\Gamma(n - 2)} \tag{11}$$

where Γ is the gamma function. The first and the second moment of $P_\perp$ are rather simple:

$$<P_\perp> = \frac{2P_o}{n - 3}, \quad <P_\perp{}^2> = \frac{6P_o{}^2}{(n - 3)(n - 4)} . \tag{12}$$

Therefore, we may use the experimental values of $<P_\perp>$ and $<P_\perp{}^2>$ as mentioned above (9), to compute the parameters P_o and n of (10) by inverting Eqs. (12), namely,

$$n = 4 + \frac{1}{2<P_\perp{}^2>/3<P_\perp>^2 - 1}, \tag{13-a}$$

and

$$P_o = \frac{<P_\perp>}{2} \cdot \left(1 + \frac{1}{2<P_\perp^2>/3<P_\perp>^2 - 1}\right). \quad (13\text{-b})$$

This will enable us to perform a no-free-parameter fit to the UA 1 data.

We find using (9)

$$n = 6.64, \quad P_o = 14.19\ \mathrm{GeV/c} \quad (14)$$

The fit is shown by the solid curve in Fig. 2. A comparison with the data indicates that the fit thus made is very satisfactory, especially in the region of $P_\perp < 20$ GeV/c, i.e. $\lesssim 3 <P_\perp>$ and the position of the maximum of $P_\perp$ at

$$(P_\perp)_{max} = \frac{P_o}{n - 1} = 2.08\ \mathrm{GeV/c} \quad (15)$$

is well determined as is seen from the figure.

That the $P_\perp$ distribution in Fig 2 is Maxwellian near the maximum, i.e. for $P_\perp \lesssim 9$ GeV/c, as mentioned before, indicates that the long tail of the $P_\perp$ distribution, which extends to ~ 40 Gev/c, about 20 times $(P_\perp)_{max}$ is likely due to the gluon emission accompanying the W production. The proportion of such events is estimated to be ~ 21 %, their average $P_\perp$ is ~ 14.6 GeV/c.

4. The angular distribution

In Fig. 3, we plot the UA 1 data of the cms angular distribution of W [1]. The sharp peak near $\cos\theta \approx 1$ resembles the bremsstrahlung, as has been observed in the π^- production by the pp collision and the e^+e^- annihilation [6]. We now analyze this characteristic feature using a formula derived from the covariant Boltzmann distribution (1).

Let $\mu \equiv |\cos\theta|$, we find in the high energy limit

$$\frac{dn}{d\mu} \sim \int_0^\infty e^{-(E-\beta^* P_\parallel)/T} P^2 dP = \frac{N}{[1 - (1 - \lambda^2)\mu^2]^{3/2}} \quad (16)$$

where, for simplicity, we have set $\lambda \equiv 1 - \beta^*$ and $N \sim T^3$ is the normalization coefficient.

We note that the moments of μ are such that $<\mu^\nu> = o$ for even ν, in view of the symmetry of the angular distribution with respect to the origin of the cms, whereas for odd ν

$$<\mu^\nu> = \frac{1}{(1+\nu)^2} \cdot {}_2F_1\left(\frac{\nu}{2}, 1; \frac{\nu+3}{2}; 1-\lambda^2\right) \tag{17}$$

where ${}_2F_1$ is the ordinary hypergeometric function. It follows that

$$<\mu> = \frac{1}{1+\lambda}, \quad <\mu^3> = \frac{1}{(1+\lambda)^2} \tag{18}$$

so that the condition

$$<\mu^3>/<\mu>^2 \equiv 1 \tag{19}$$

may serve as a validity test of the Boltzmann distribution (16).

From the UA 1 data shown in Fig. 3, we get

$$<\mu> = 0.726 \pm 0.038, \quad <\mu^3> = 0.492 \pm 0.054,$$

and

$$<\mu^3>/<\mu>^2 = 0.933 \pm 0.081$$

consistent with 1 within ~ 1 standard deviation.

We therefore proceed to fit the angular distribution data of UA 1 in Fig. 3 with (16). Noting that the end point of the distribution is seemingly under valued and that the shape of the peak is rather critical to the estimation of the parameter λ in (16), we therefore carry out two trials of fit, one with and another without the end point, as shown by the solid and the dashed curves in Fig. 3, respectively; the corresponding parameters are :

$$1 - \lambda^2 = 0.719 \pm 0.109 \quad \text{for } \mu < 1$$

$$= 0.985 \pm 0.036 \quad \text{for} \ \mu \leq 0.9.$$

It is to be noted that the parameter λ in (16) has a geometrical meaning related to the area A covered by the angular distribution in the entire domain of μ [-1, +1] . Indeed, we get by integrating (16)

$$\mathbf{A} = 2 \int_0^1 \frac{dn}{d\mu} d\mu = \frac{2N}{\lambda}. \tag{20}$$

It follows that λ is related to the specific height of the angular distribution as follows

$$h = \left(\frac{dn}{d\mu} \right)_0 / \mathbf{A} = \frac{\lambda}{2}. \tag{21}$$

From the estimates of $\lambda = 0.530$ and 0.122 of the two fits to the data we have tried, as shown in Fig. 3, we compute the values of h, then take the average to get:

$$1/h = 10.0 \pm 6.3.$$

An important feature of the specific height thus defined (21) is the scaling property, namely, it is rather a geometrical parameter. Therefore, h should be independent of the mass of the secondary particle under consideration, as will be discussed in the next section.

5. Comparison with π and K^o production

We now consider the inclusive $\bar{p} + p \rightarrow \pi, K^o$ at the same energy $\sqrt{s}$= 540 GeV of another CERN experiment by the UA 5 Collaboration [2]. Their angular distributions, Fig. 4, are expressed in terms of the pseudorapidity η, which is related to $\mu = \cos\theta$ as follows

$$\eta = \frac{1}{2} \ln \frac{1 + \mu}{1 - \mu}. \tag{22}$$

Clearly, the *specific* height as is defined by Eqs. (20) and (21) applies to the psudorapidity distribution by substitution $\mu \rightarrow \eta$. It is to be noted that h thus defined is identical to the central particle density $\rho_o=(dn/d\eta)_o$ commonly used in the literature, if and only if, the η-distribution is normalized to 1, namely $\int (dn/d\eta)d\eta = 1$.

We note that as regards the estimation of h, there is advantage to use the η distribution rather than transform it to the cos θ distribution, in this way we avoid the distortion of the transformed distribution because of the singularity of the Jacobian $d\eta/d\mu = 1/(1 - \mu^2)$ near the end of the μ-distribution.

We therefore make use of the pseudorapidity distributions of UA 5 to estimate h of π and K^o without changing the variable. The curves in Fig. 4 represent the fits according to the partition temperature model of Chou, Yang and Yen [6]:

$$\frac{dn}{d\eta} = \frac{K}{(\alpha + \frac{1}{T_p}\cosh(\eta - \eta^*))^2} \qquad (22)$$

where $\alpha = 2/\langle P_\perp \rangle$ is a parameter corresponding to the exponential cut-off of the $P_\perp$ distribution, T_p is the partition temperature, η^* is the peak shift parameter and K is the normalization coefficient. The parameters of the fits are summarized in Table I, together with the computed area **A** and the estimate of the height h using the parameters of the fits.

A comparison of the values of h for π and K with that of the W mentioned above indicates that they are practically equal within quoted errors. As the mass of W is much heavier than π and K, we are therefore led to assert:

$$h_W \approx h_\pi \approx h_K. \qquad (24)$$

This indicates the similarity property of the phase-space of the secondaries described either by the angular distribution in terms of $\mu = \cos\theta$ or by the pseudorapidity η distribution. We note that this geometrical property of *h* has also been found in the case of hadron production by pp collisions and e^+e^- annihilations as reported before [7].

6. Energy dependence of *h*

We now investigate the energy dependence of the specific height of the phase-space of secondaries. For this purpose, we have presented in Fig. 5 the semi-log plot of some values of $1/h$ for π, K and $\bar{\Lambda}$ produced by $\bar{p}p$ collisions at $\sqrt{s} = 7.86$ to 1800 GeV [8].

The straight line represents a lest-squares fit assuming a power law behaviour, namely

$$1/h = c.s^{a/2} \qquad (25)$$

with

$$a = 0.32 \pm 0.04, \quad c = 1.32 \pm 0.36$$

Note that the W point in Fig. 5, which is not included in the fit is actually very close to those of π and K at the same energy $\sqrt{s}$ =540 GeV, indicating that the scaling law for *h*, (25), holds also for W.

Finally, we note that $a \approx 1/3$, as has also been found for *h* in the case of hadrons produced by pp collisions and e^+e^- annihilations [7]. This indicates the universality of the scaling law (25).

7. Conlusions

From an analysis of the $x = 2P_{\parallel}/\sqrt{s}$ distribution of the inclusive $\bar{p} + p \to W^{\pm}$ at $\sqrt{s}$ = 540 GeV of the UA 1 Collaboration [1] in terms of the covariant Boltzmann factor, we find that the Feynman-Yang scaling holds just like the case of secondary hadrons from hadronic collisions in general. This indicates that the W boson participates in the same mass motion as other secondary particles. However, in contrast with the relativistic π's, its velocity in the rest frame of secondaries is rather small, as its kinetic energy determined by its temperature amounts to only ~ 0.282 GeV much less than its mass.

The angular distribution of W in the c.m.s. of the colliding $\bar{p}$ and p has this remarkable scaling invariance property, namely its specific height *h* is found to be mass independent, just like other secondary hadrons, π , K, from the $\bar{p}p$ collisions at the same energy of the UA 5 experiment [2]. Whereas its energy dependence has his property of universality, namely its power law behaviour $E_{cm}{}^{a}$, with a ≈ 1/3 is just like the case of hadron production by pp collisions and e^+e^- annihilations [7a,b].

Finally, we note in passing that the scaling property of *h* may be applied to estimate the production cross-section of a secondary using its measurement in the forward direction corresponding to $\eta \approx 0$, so that knowing $\rho_0 = (d\sigma/d\eta)_0$, one may deduce the cross-section by application of the scaling law (25).

Likewise, the energy of jets composed of a large number of secondary particles, as observed in the cosmic rays of very high energy may be estimated with (25), provided that the shape of each η-distribution can be determined with enough accuracy to get a reliable estimate of the specific height. In this regard, we note that the energy dependence of h, Eq. (25) holds for high energy p-nucleus as well as nucleus-nucleus collisions as reported before [7].

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References and Footnotes

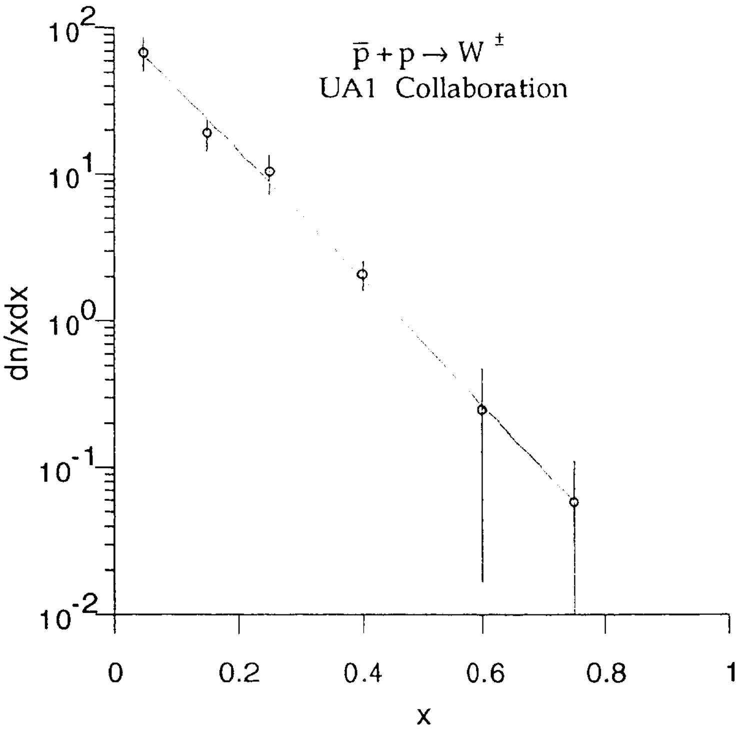
1. UA1 Collaboration, C. Albajar, et al. Zeit. Phys. C 44, (1989) 15
2. UA 5 Collaboration, G. J. R. E. Ansorge, et al. Phys. Lett. B 59 (!975) 229 and Almer, et al. Zeit. Phys. C 33, (1861) 1
3. T. F. Hoang, Phys. Rev. D 12 (1972) 296
4. UA1 Collaboration, C. Albajar, et al., Nucl. Phys. B 335 (1990)
5. R. Hagedorn, Riv. Nuovo Cimento, 6 (1983) n10
6. T. T. Chou, Chen Ning Yang and Ed. Yen, Phys. Rev. Lett. 54 (1985) 510
7. T. F. Hoang, (a) Phys. Rev. D 17 (1978) 927; (b) ibid. 38 (1988) 2779; (c) Zeit. Phys. C 62, (1994) 343 and (d) ibid. 61 (1994) 341
8. We have used the following pp data: $\sqrt{s}$ =1.8 TeV, F. Abe et al., Phys. Rev. D 41 (1990) 2330; 540 GeV, UA5 Coll. Ref. [2]; 13.7 GeV, Cavendish-Fermilab Coll., C. P. Wood et al., Nucl.Phys. B153 (1979) 299 and A. K. Naudi, et al., ibid. 169 (1980) 20; and Superkhov experiments $\sqrt{s}$ = 7.62 and 11.54 GeV, C. Poiret, et al., Zeit. Phys. C 11 (1981) 1 and J. Lemonnet, et al., ibid. 24 (1984) 103.

Table I - Parameters of the pseudorapidity distributions of $\bar{p} + p \to \pi, K^0$ at $\sqrt{s} = 540$ GeV of the UA5 Collaboration [2], according to the partition temperature model, Eq. (2)

	π	K^0
α	4.76 ± 0.23	3.15 ± 0.18
η^*	1.47 ± 0.04	1.45 ± 0.21
T_p (GeV)	3.41 ± 0.26	1.83 ± 0.87
K (GeV^{-2})	82.7 ± 9.0	22.6 ± 9.0
A	14.04 ± 1.51	5.61 ± 2.23
$1/h$	9.62 ± 0.31	9.43 ± 3.74

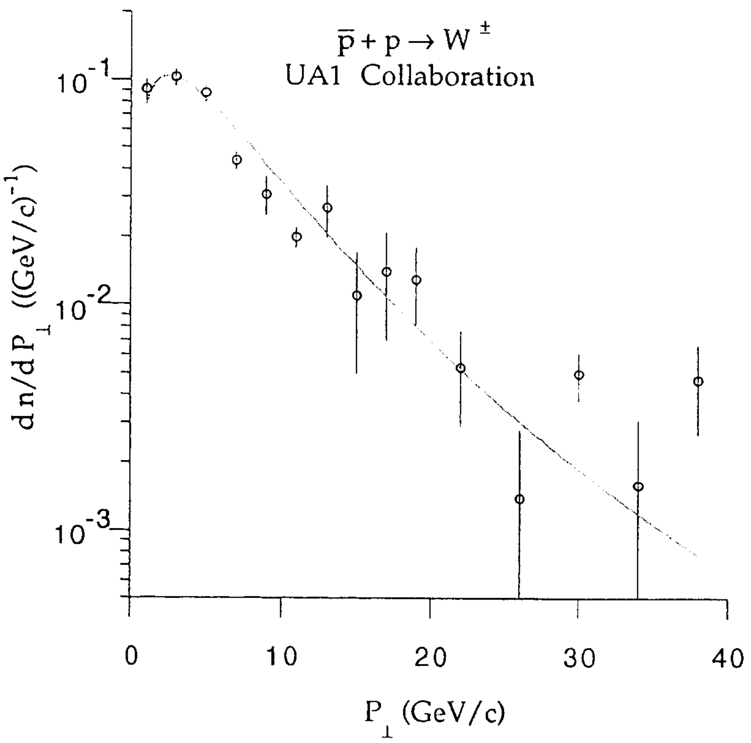
Figure Captions

1. The $x = 2P_{\parallel}/\sqrt{s}$ distribution of inclusive $W^{\pm}$ at $\sqrt{s} = 540$ GeV of the UA 1 Collaboration [1]. The straight line is a test of the scaling property in terms of the covariant Boltzmann factor. The temperature of W is estimated to be ~ 0.188 GeV.
2. The $P_{\perp}$ distribution of $W^{\pm}$ of the UA 1 Collaboration [1]. The curve is a no-free-parameters fit according to a power law inspired by QCD, Eq. (10), see text.
3. The angular distribution of $W^{\pm}$ in the cms of the colliding $\bar{p}$ and p, data of UA 1 Collaboration [1]. The curves represent the fits according to the Covariant Boltzmann factor, Eq. (15), with (solid line) and without (dashed line) the end point at $\cos\theta = 0.97$.
4. The pseudorapidity distributions of π and K^o from collisions at $\sqrt{s} = 540$ GeV of the UA 5 Collaboration [2]. The curves represent the fit according to the partition temperature model, Eq. (23), the parameters are listed in Table I.
5. The semi-log plot of the specific height of the pseudorapidity distribution of π, K and $\bar{\Lambda}$ vs. energy. The universality of the scaling is shown by the straight line representing the power law dependence of $1/h$ on the energy. Note that the W point (in cross) lies on the line of fit.



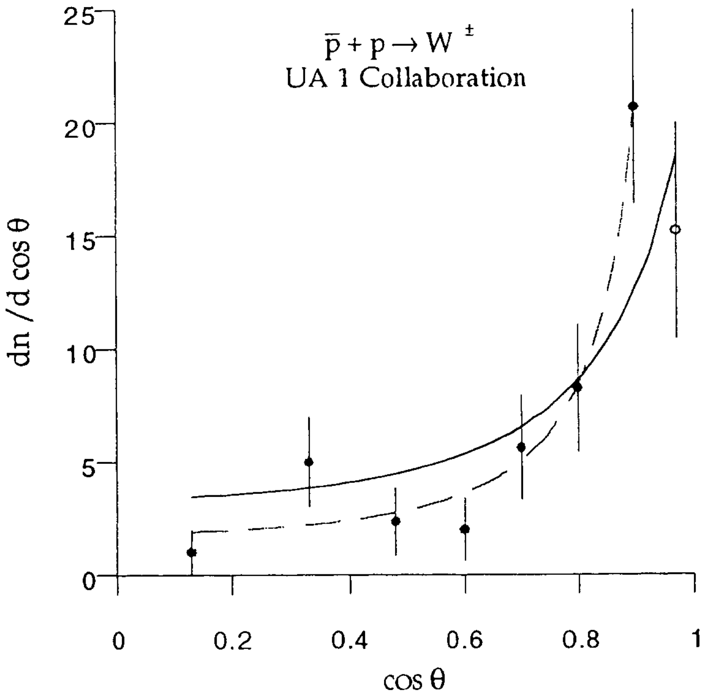
$\bar{p} + p \to W^{\pm}$
UA1 Collaboration
dn/xdx
10^2
10^1
10^0
10^{-1}
10^{-2}
0
0.2
0.4
0.6
0.8
1
x

Fig. 1



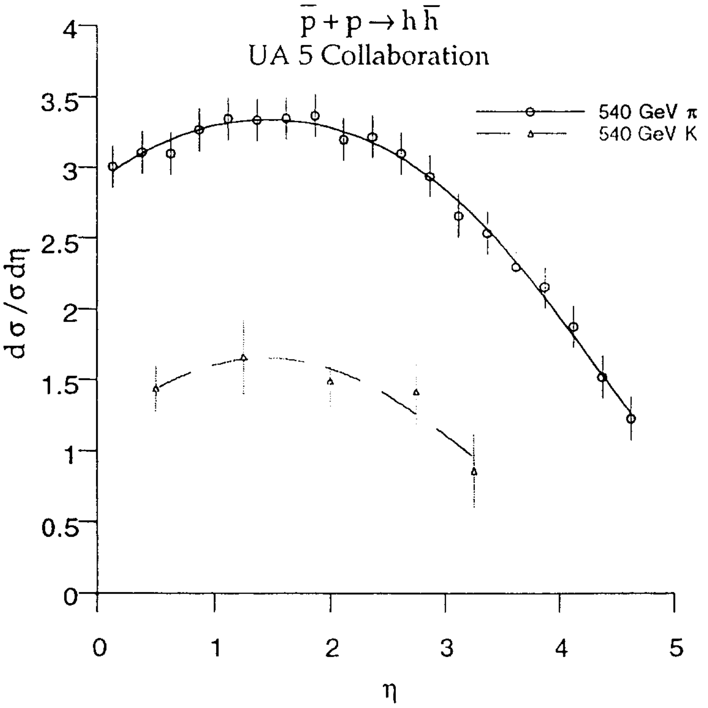
p̄ + p → W ±
UA1 Collaboration
10^-1
10^-2
10^-3
d n/d P⊥ ((GeV/c)^-1)
0
10
20
30
40
P⊥ (GeV/c)

Fig. 2



$\bar{p} + p \to W^{\pm}$
UA 1 Collaboration
25
20
15
10
5
0
dn /d cos θ
0
0.2
0.4
0.6
0.8
1
cos θ

Fig. 3



$\bar{p} + p \to h\bar{h}$
UA 5 Collaboration
540 GeV π
540 GeV K
d σ /σ dη
η
0
0.5
1
1.5
2
2.5
3
3.5
4
0
1
2
3
4
5

Fig. 4

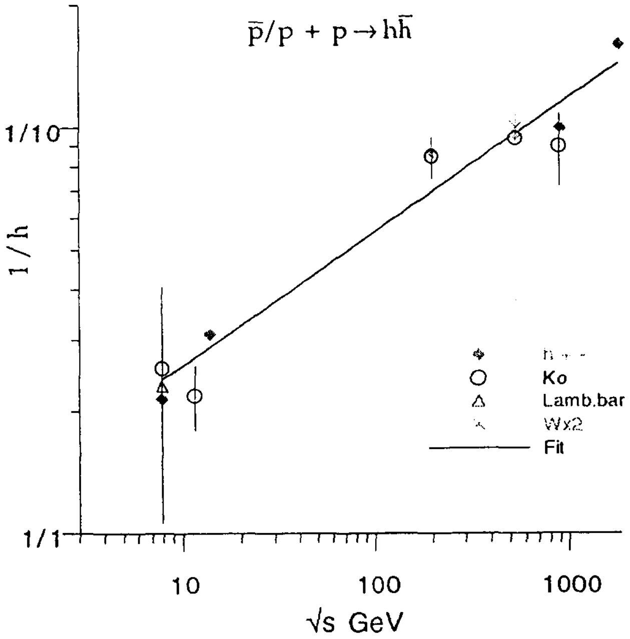


Fig. 5