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B and D Meson Decays in the Heavy Quark Effective Theory

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ABSTRACT

We study semileptonic and nonleptonic decays of both B and D mesons in the heavy quark effective theory (HQET) with $O(\bar{\Lambda}/m_Q)$ corrections ($\bar{\Lambda} \simeq O(\Lambda_{QCD})$) in order to investigate whether s quark can be treated as a heavy quark in the HQET or not. The factorization hypothesis is used for the analysis of the nonleptonic decays. B meson decay is again explained well for its overall semileptonic and nonleptonic decays even in this unified analysis. D meson decay is also described well in terms of the HQET with $O(\bar{\Lambda}/m_c)$ and $O(\bar{\Lambda}/m_s)$ corrections for $D^0 \to K^- \bar{l}\nu_l$, $D^0 \to K^-\pi^+$ and $K^-\rho^+$. By using the maximum likelihood method for the unified analysis of B and D mesons, the Isgur-Wise functions of linear and exponential type reproduce the data, while the pole type one does not work so well. Eventually, we can safely say that the s quark can be accommodated in the heavy quark effective theory.

1. Introduction

The heavy quark effective theory (hereafter HQET)[1] has proved to be a useful strategy for the study of weak decays of heavy flavored hadrons. The theory has the flavor and spin symmetry, which allows only a small number of form factors for transition matrix elements of those hadrons and reduces them to only one function, the so-called Isgur-Wise function in the symmetry limit $m_Q \to \infty$, Q denoting a heavy quark like b and c.

The HQET gives a tremendously simple description of weak decays of heavy quark hadrons [2-5], especially B mesons, compared to the bound state models [6]. The Cabibbo-Kobayashi-Maskawa matrix element V_{cb} for the $b \to c$ weak transition has been sucessfully extracted from B meson semileptonic decays for varous types of the Isgur-Wise function [7,4]. The nonleptonic decays of neutral B meson (B_d) are also studied at the lowest order in the HQET under the factorization hypothesis by several authors[2,9,10] and then at the order $\bar{\Lambda}/m_Q$ of corrections by us[11] to show that the hypothesis works well to describe the decays with the HQET ($\bar{\Lambda} \simeq O(\Lambda_{QCD})$).

Another recent concern about the HQET is whether s quark can be accommodated as a heavy quark to the HQET. Ito, Morii and Tanimoto have studied semileptonic D meson decays to show that $O(\bar{\Lambda}/m_s)$ corrections are significant for $D \to K^{(*)}\bar{l}\nu_l$ and the branching ratios of $D^0 \to K^-\bar{l}\nu_l$ and $D^+ \to K^{*0}\bar{l}\nu_l$ decays are consistent with the experiments [12]. On the other hand, Neubert and Rickert have obtained the behaviors of Isgur-Wise function as a function of $v \cdot v'$ (four-velocity transfer squared) from the form factors determined phenomenologically in the bound state model by Wirbel, Stech and Bauer[6] and shown that the universality among the hadronic form factors given by the HQET breaks down for all values of $v \cdot v'$ in the case of $c \to s$ transition, while it holds to an accuracy of 20 % for $b \to c$ transition [13].

In this paper, we investigate semileptonic and nonleptonic decays of both B and D decays in a unified way up to the $O(\bar{\Lambda}/m_Q)$ corrections ($Q = c, s$) to the HQET. Factorization hypothesis is assumed for the nonleptonic decays. We analyze the D decay, using the branching ratio data of semileptonic decay, $D^0 \to K^-\pi^+$,

$K^{*-}\pi^+$, $K^-\rho^+$ and $K^{*-}\rho^+$ as the nonleptonic decays and the recent beautiful data obtained by CLEO on the differential decay rates of $D^0 \to K^- \bar{l}\nu_l$[4] to show that s quark can be treated in the heavy quark effective theory, even though the s quark is in the transient region between light and heavy quarks.

In §2, the formalism for our analysis is given in the $O(\bar{\Lambda}/m_c)$ and $O(\bar{\Lambda}/m_s)$ corrections to the HQET. In §3, we use the maximum likelihood method to obtain the parameters in the theory for semileptonic and nonleptonic decays of B and D mesons and discuss the numerical results. Discussions and some comments are given in §4.

2. Formalism

Let us explain the formulas for the analysis. The matrix elements between D^0 (with velocity v_μ) and K^- or K^{*-} (with velocity v'_μ) are expressed as

$$\langle K^-(v')|\, V^\mu \,|D^0(v)\rangle = \sqrt{m_D m_K}[A(y)(v^\mu + v'^\mu) + B(y)(v^\mu - v'^\mu)],$$

$$\langle K^{*-}(v')|\, V^\mu \,|D^0(v)\rangle = i\sqrt{m_D m_{K^*}}C(y)\varepsilon^{\mu\nu\alpha\beta}\varepsilon^*_\nu v'_\alpha v_\beta,$$

$$\langle K^{*-}(v')|\, A^\mu \,|D^0(v)\rangle = \sqrt{m_D m_{K^*}}[D(y)\varepsilon^{*\mu} + E(y)(\varepsilon^*\cdot v)v^\mu + F(y)(\varepsilon^*\cdot v)v'^\mu], \quad (1)$$

where $y \equiv v\cdot v'$, ε^* is the polarization vector of K^*, and $A(y) \sim F(y)$ are given in the HQET by

$$A(y) = \xi(y)[1 + (\frac{1}{m_s} + \frac{1}{m_c})\bar{\Lambda}\rho_1(y)],$$

$$B(y) = -\xi(y)(\frac{1}{m_s} - \frac{1}{m_c})\bar{\Lambda}\rho_2(y),$$

$$C(y) = \xi(y)[1 + \frac{1}{m_s}\bar{\Lambda}(\rho_3(y) + \frac{1}{2}) + \frac{1}{m_c}\bar{\Lambda}(\rho_1(y) + \rho_2(y))],$$

$$D(y) = \xi(y)(y+1)[1 + \frac{1}{m_s}\bar{\Lambda}(\rho_3(y) + \frac{1}{2}R) + \frac{1}{m_c}\bar{\Lambda}(\rho_1(y) + \rho_2(y))],$$

$$E(y) = -\xi(y)\frac{1}{m_s}\bar{\Lambda}(2\chi_2^0 + \xi_+^0 - \frac{1}{2}),$$

$$F(y) = -\xi(y)[1 + \frac{1}{m_s}\bar{\Lambda}(\rho_3(y) - 2\chi_2^0 + \xi_+^0) + \frac{1}{m_c}\bar{\Lambda}(\rho_1(y) + \rho_2(y))], \quad (2)$$

up to $O(\bar{\Lambda}/m_s)$ and $O(\bar{\Lambda}/m_c)$ corrections, where the definition of $\bar{\Lambda}$ is given later and $R \equiv (y-1)/(y+1)$. Here $\rho_1(y)$, $\rho_2(y)$ and $\rho_3(y)$ are made up from the form factors $\chi_{1,2,3}(y)$ and $\xi_+(y)$ as follows,

$$\rho_1(y) = \chi_1(y) - 2(y-1)\chi_2(y) + 6\chi_3(y),$$

$$\rho_2(y) = (y+1)\xi_+(y) - \frac{1}{2}(y-2),$$

$$\rho_3(y) = \chi_1(y) - 2\chi_3(y), \quad (3)$$

where we use the same parametrizations for χ_1, χ_2, χ_3 and ξ_+ as we used in the analysis of B decay ;[11)]

$$\chi_1(y) = \chi_1^0(y-1), \quad \chi_2(y) = \chi_2^0, \quad \chi_3(y) = \chi_3^0(y-1), \quad \xi_+(y) = \xi_+^0, \qquad (4)$$

where χ_1^0, χ_2^0, χ_3^0 and ξ_+^0 are the constants to be determined.

For $\xi(y)$, we try the same forms often used as in refs. [2-5,7,10-12]as follows,

$$\begin{aligned}
\xi(y) &= 1 + \rho(1-y) \quad \text{(linear type)},\\
&= 1/[1-\rho(1-y)] \quad \text{(pole type)},\\
&= \exp[\rho(1-\mathrm{y})] \quad \text{(exponential type)} \qquad (5)
\end{aligned}$$

where ρ is also the constant to be determined.

Now we study the nonleptonic decays. The total width of $D^0 \to K^{(*)-}\pi^+(\rho^+)$ are expressed under the hypothesis of factorization[15)] in the HQET with $1/m_s$ and $1/m_c$ corrections as

$$\Gamma(D^0 \to K^-\pi^+) = \frac{1}{16\pi} G_F^2 C_{QCD}^2(\mu)|V_{cs}|^2|V_{ud}|^2 f_\pi^2 m_D m_K |P_K|$$

$$\times[(1-r)(y+1)A(y) + (1+r)(1-y)B(y)]^2,$$

$$\Gamma(D^0 \to K^{*-}\pi^+) = \frac{1}{16\pi} G_F^2 C_{QCD}^2(\mu)|V_{cs}|^2|V_{ud}|^2 f_\pi^2 m_D m_{K^*} |P_{K^*}|$$

$$\times(y^2-1)[D(y) + (y-r^*)F(y) + (1-yr^*)E(y)]^2,$$

$$\Gamma(D^0 \to K^-\rho^+) = \frac{1}{16\pi} G_F^2 C_{QCD}^2(\mu)|V_{cs}|^2|V_{ud}|^2 f_\rho^2 m_D m_K |P_K|$$

$$\times(y^2-1)[(1+r)A(y) - (1-r)B(y)]^2,$$

$$\Gamma(D^0 \to K^{*-}\rho^+) = \frac{1}{16\pi} G_F^2 C_{QCD}^2(\mu)|V_{cs}|^2|V_{ud}|^2 f_\rho^2 m_D m_{K^*} |P_{K^*}|$$

$$\times[2r_\rho^2(D^2(y)+(y^2-1)C^2(y)) + \{(y-r^*)D(y)+(y^2-1)F(y)+r^*(y^2-1)E(y)\}^2], \quad (6)$$

where the functions $A(y) \sim F(y)$ are the ones in eq.(2), $r \equiv m_K/m_D$, $r^* \equiv m_{K^*}/m_D$, $r_\rho \equiv m_\rho/m_D$, $y = (m_D^2 + m_{K^{(*)}}^2 - m_{\pi(\rho)}^2)/2m_D m_{K^{(*)}}$ and $|P_{K^{(*)}}|$ is the size of momentum of K or K^* in the D rest frame in the respective decay;

$$|P_{K^{(*)}}| = \frac{1}{2m_D}\sqrt{[(m_D + m_{K^{(*)}})^2 - m_{\pi(\rho)}^2][(m_D - m_{K^{(*)}})^2 - m_{\pi(\rho)}^2]}. \qquad (7)$$

The QCD leading-log correction factor for $\mu = m_s$ is ;[16)]

$$C_{QCD}(\mu = m_s) = [\frac{1}{3}(\frac{\alpha_{QCD}(M_W)}{\alpha_{QCD}(m_b)})^{-12/23}(\frac{\alpha_{QCD}(m_b)}{\alpha_{QCD}(m_c)})^{-12/25}$$

$$+\frac{2}{3}(\frac{\alpha_{QCD}(M_W)}{\alpha_{QCD}(m_b)})^{6/23}(\frac{\alpha_{QCD}(m_b)}{\alpha_{QCD}(m_c)})^{-3/25}](\frac{\alpha_{QCD}(m_c)}{\alpha_{QCD}(m_s)})^{a_L}, \quad (8)$$

where $a_L \equiv \frac{8}{27}(yr(y)-1)$, $r(y) = \frac{1}{\sqrt{y^2-1}}\ln(y+\sqrt{y^2-1})$. $\alpha_{QCD}(m_{b,c,s})$ are calculated from $\alpha_{QCD}(M_Z)$ =0.12 .[17]

Then we examine the semileptonic D decays, $D^0 \to K^-\bar{l}\nu_l$ and $K^{*-}\bar{l}\nu_l$. The corresponding differential widths are given in terms of the same functions for the nonleptonic decays, $A(y) \sim F(y)$ as follows,

$$\frac{\mathrm{d}\Gamma}{\mathrm{dy}}(D^0 \to K^-\bar{l}\nu_l) = \frac{G_F^2}{48\pi^3}C_{QCD}^2(\mu)|V_{cs}|^2 m_D^2 m_K^3\sqrt{y^2-1}(y^2-1)$$

$$\times[(1+r)A(y)-(1-r)B(y)]^2,$$

$$\frac{\mathrm{d}\Gamma}{\mathrm{dy}}(D^0 \to K_T^{*-}\bar{l}\nu_l) = \frac{G_F^2}{48\pi^3}C_{QCD}^2(\mu)|V_{cs}|^2 m_D^2 m_{K^*}^3\sqrt{y^2-1}$$

$$\times 2(1-2yr^*+r^{*2})[D^2(y)+(y^2-1)C^2(y)],$$

$$\frac{\mathrm{d}\Gamma}{\mathrm{dy}}(D^0 \to K_L^{*-}\bar{l}\nu_l) = \frac{G_F^2}{48\pi^3}C_{QCD}^2(\mu)|V_{cs}|^2 m_D^2 m_{K^*}^3\sqrt{y^2-1}$$

$$\times[(y-r^*)D(y)+(y^2-1)\{F(y)+r^*E(y)\}]^2, \quad (9)$$

where

$$C_{QCD}(\mu = m_s) = (\alpha_{QCD}(m_c)/\alpha_{QCD}(m_s))^{-6/27} \quad (10)$$

for $\Lambda_{QCD} = 0.1$ GeV .[14] We calculate the branching ratios, Γ_L/Γ_T and $\mathrm{d}\Gamma/(\Gamma \mathrm{dq}^2)$ $(D^0 \to K^-\bar{l}\nu_l)$, where

$$\Gamma = \int_1^{(m_D^2+m_K^2)/2m_D m_K} \frac{\mathrm{d}\Gamma}{\mathrm{dy}}(D^0 \to K^-\bar{l}\nu_l)\mathrm{dy},$$

$$\Gamma_{L,T} = \int_1^{(m_D^2+m_{K^*}^2)/2m_D m_{K^*}} \frac{\mathrm{d}\Gamma}{\mathrm{dy}}(D^0 \to K_{L,T}^{*-}\bar{l}\nu_l)\mathrm{dy}. \quad (11)$$

Next, we explain the formulas of the semileptonic and nonleptonic decays of B^0 meson. We get the formulas for $\bar{B}^0$ decays by replacing

1) D^0, K^- and K^{*-} with $\bar{B}^0$, D^+ and D^{*+} in eqs.(1), (6), (7) and eq.(9), respectively,

2) V_{cs} by V_{cb} in eqs.(6) and (9),

3) m_s by m_c and m_c by m_b in eq.(2).

Concerning the QCD correction factors $C_{QCD}(\mu)$ in eq.(8), we set the energy scale parameter μ as m_c with replacing m_c by E and m_s by m_c as follows,

$$C_{QCD}(\mu = m_c) = [\frac{1}{3}(\frac{\alpha_{QCD}(M_W)}{\alpha_{QCD}(m_b)})^{-12/23}(\frac{\alpha_{QCD}(m_b)}{\alpha_{QCD}(E)})^{-12/25}$$

$$+\frac{2}{3}(\frac{\alpha_{QCD}(M_W)}{\alpha_{QCD}(m_b)})^{6/23}(\frac{\alpha_{QCD}(m_b)}{\alpha_{QCD}(E)})^{-3/25}](\frac{\alpha_{QCD}(E)}{\alpha_{QCD}(m_c)})^{-6/25} \quad (12)$$

in eq.(8) where E is the total energy of the light quarks becoming together to form π or ρ. As to the B^0 semileptonic decay we set μ equal m_c with replacing m_c by m_b and m_s by m_c as follows,

$$C_{QCD}(\mu = m_c) = (\alpha_{QCD}(m_b)/\alpha_{QCD}(m_c))^{-6/25} \qquad (13)$$

in eq.(10).

3. Numerical analysis

In this section we investigate how far the HQET is able to reproduce the experimental results on the semileptonic and nonleptonic decays of B^0 and D^0 mesons, simultaneously.

In our analysis, we have six independent parameters. The first four parameters χ_1^0, χ_2^0, χ_3^0 and ξ_+^0 are the numerical coefficients in the form factors of $\bar{\Lambda}/m_Q$ corrections as shown in the previous section. The fifth one ρ is the constant factor contained in the Isgur-Wise function. Besides these parameters, we treat $|V_{cb}|$ as a free parameter in the B^0 decay. The value of $|V_{cb}|$ has been estimated through the Isgur-Wise function which could reprodece only the data of the q^2 dependence of the decay rate of $\bar{B}^0 \to D^{*+}l\bar{\nu}_l$: $\mathrm{dBr}/\mathrm{d}q^2(\bar{B}^0 \to D^{*+}l\bar{\nu}_l)$ [9,19] But it is thought that this procedure is not appropriate to estimate the correct value of $|V_{cb}|$ on account of the existence of large errors of experimental values near $y \sim 1$ region and the disregard of $1/m_Q$ corrections to the simmetry limit. We have many experimental data on B^0 and D^0 decays today beyond the q^2 distribution of $\bar{B}^0 \to D^{*+}l\bar{\nu}_l$ process. All of these six parameters are determined through a χ^2 minimum fit to the data of B^0 and D^0 decays.

In this analysis, we take into consideration the $O(\bar{\Lambda}/m_s)$ and $O(\bar{\Lambda}/m_c)$ corrections and drop the $O(\bar{\Lambda}/m_b)$. We neglect $O(\bar{\Lambda}^2/m_s^2)$ and $O(\bar{\Lambda}^2/m_c^2)$ contributions which appear in the squared terms of the eqs.(6) and (9). The value $\bar{\Lambda}$ is defined as $\bar{\Lambda} \equiv m_{B^{(*)}} - m_b \simeq m_{D^{(*)}} - m_c \simeq m_{K^{(*)}} - m_s$, and we can approximately take as $\bar{\Lambda} \simeq O(\Lambda_{QCD})$. Here for the size of $\bar{\Lambda}$, we take three different values $\bar{\Lambda} = 0.3$ GeV, 0.4 GeV and 0.5 GeV. The three different quark masses m_b, m_c and m_s are set to $m_b = 4.7$ GeV, $m_c = 1.5$ GeV and $m_s = 0.5$ GeV, respectively and $M_W = 80.22$ GeV in our calculation. The lifetimes of B^0 and D^0 mesons are given by $\tau_{B^0} = 1.50$

ps and $\tau_{D^0} = 0.415$ ps, respectively [20] Concerning the absolute values of the KM matrix element, we take $|V_{ud}| = 0.975$, and $|V_{cs}| = 0.974$. As to the π and ρ decay constants f_π and f_ρ, we use the values $f_\pi = 132$ MeV and $f_\rho = 208$ MeV.

We use the following data in the χ^2 fit.

For the B^0 semileptonic decays,
$\mathrm{Br}(\bar{B}^0 \to D^+l\bar{\nu}_l)$=1.9 ± 0.5 (%), which is given in ref.[20],
$\mathrm{Br}(\bar{B}^0 \to D^{*+}l\bar{\nu}_l)$=4.5 ± 0.5 (%) from ref. [21] and
$\Gamma(\bar{B}^0 \to D_L^{*+}l\bar{\nu}_l) \;/\; \Gamma(\bar{B}^0 \to D_T^{*+}l\bar{\nu}_l) = 1.105 \pm 0.26$ from ref. [22].
For the B^0 nonleptonic decays,
$\mathrm{Br}(\bar{B}^0 \to D^+\pi^-) = 0.30 \pm 0.04$ (%), $\mathrm{Br}(\bar{B}^0 \to D^{*+}\pi^-) = 0.26 \pm 0.04$ (%),
$\mathrm{Br}(\bar{B}^0 \to D^+\rho^-) = 0.78 \pm 0.14$ (%) and $\mathrm{Br}(\bar{B}^0 \to D^{*+}\rho^-) = 0.73 \pm 0.15$ (%), which are given in ref. [20].
As to $\mathrm{dBr}(B^0 \to D^{*+}l\bar{\nu}_l)/\mathrm{d}q^2$, we use the data from refs. [9], [14].

For the D^0 semileptonic decays the following data is used:
$\mathrm{Br}(D^0 \to K^-\bar{l}\nu_l) = 4.4 \pm 0.7$ (%) and $\mathrm{Br}(D^0 \to K^{*-}\bar{l}\nu_l) = 2.6 \pm 0.6$ (%),
which are obtained from the data of $\Gamma(D^0 \to K^-\pi)$ given by ref. [21]through the ratios: $\Gamma(D^0 \to K^-\bar{l}\nu_l)/\;\Gamma(D^0 \to K^-\pi^+) = 0.978 \pm 0.027 \pm 0.044$ and
$\Gamma(D^0 \to K^{*-}e^+\nu_l)/\;\Gamma(D^0 \to K^-e^+\nu_l) = 0.60 \pm 0.099 \pm 0.07$
given in ref. [14].
$\Gamma(D^0 \to K_L^{*-}\bar{l}\nu_l)/\Gamma(D^0 \to K_T^{*-}\bar{l}\nu_l) = 1.8 \pm 0.6$, which is given by ref. [23].

For the D^0 nonleptonic decays,
$\mathrm{Br}(D^0 \to K^-\pi^+) = 4.5 \pm 0.7$ (%), which is given by ref. [21].
$\mathrm{Br}(D^0 \to K^{*-}\pi^+) = 4.5 \pm 0.6$ (%), $\mathrm{Br}(D^0 \to K^-\rho^+) = 7.3 \pm 1.1$ (%),
$\mathrm{Br}(D^0 \to K^{*-}\rho^+) = 6.2 \pm 2.5$ (%), $\Gamma(D^0 \to K_L^{*-}\rho^+)/\;\Gamma(D^0 \to K_T^{*-}\rho^+)$
$= 0.9 \pm 0.7$ which are given in ref. [24].
As to $\mathrm{dBr}(D^0 \to K^-\bar{l}\nu_l)/\mathrm{d}q^2$, the data are given by ref.[14].

In our χ^2 minimum search we input the value zero for the parameters χ_1^0, χ_2^0, χ_3^0 and ξ_+^0, the value 0.5 for ρ and the value 0.045 for $|V_{cb}|$ as their starting points.

We show the numerical results of the χ^2 fit on B^0 and D^0 decays in Table 1, 2 and Fig. 1, 2. In Table 1, the values of first five parameters, $|V_{cb}|$ and the χ^2

minimum are shown for two types of the Isgur-Wise function, that is, the linear type and the exponential type. The linear type function gives us the smallest value of χ^2 minimum and the value devided by the number of data minus the number of parameters (χ^2/ndf) is equal to 32.6/(40-6)=0.96 for $\bar{\Lambda}$ =0.4 GeV, and the exponential type also gives us almost the same values of the χ^2 minimum. On the other hand, in the case of the pole type function the value of χ^2 minimum is larger than in the case of the linear one and it gives about twice of the linear one. Among the six parameters, the first four ones χ_1^0, χ_2^0, χ_3^0 and ξ_+^0 vary a certain extent according to the change of the value of $\bar{\Lambda}$. On the other hand, the fifth one ρ and $|V_{cb}|$ remain almost unchanged.

In Table 2, we show the numerical predictions together with their corresponding data on B^0 and D^0 decays. Among the data only the data of Γ_L/Γ_T on $D \to K^*\bar{l}\nu_l$ is for the D^+ decay. We restrict the data only to the B^0 and D^0 decays and do not use the data of the $B^\pm$ and $D^\pm$ decays except the above one on account that we cannot share the same formula between them without any further assumption on their decay rates of $b\bar{b} \to B^0\bar{B}^0$ and $b\bar{b} \to B^+B^-$. The comparison of our result with the data of $\mathrm{dBr}/\mathrm{d}q^2(\bar{B}^0 \to D^{*+}l\bar{\nu}_l)$ and $\mathrm{d}\Gamma/(\Gamma \mathrm{d}q^2)(D^0 \to K^-\bar{l}\nu_l)$ are shown in Fig. 1 and Fig. 2, respectively. The q^2-dependence of the $\bar{B}^0 \to D^{*+}l\bar{\nu}_l$ process is predicted by the exponential type of the Isgur-Wise function better than by the linear type one. In the case of $D^0 \to K^-\bar{l}\nu_l$ process our predictions are good for the q^2-dependence in both types of the Isgur-Wise function. In this process we do not take into consideration the data in the small region of q^2 below 0.40 (GeV^2) on account that we need to avoid the complicated structure in this region and do not give the undesirable effect to the χ^2 search in the small region of y. We also do not use the data at $q^2 = 1.15$ (GeV^2) on account of the large deviation as compared with the other data. We can conclude that the agreement between our predictions and data is good and our χ^2 fits are within the experimental errors for the $\bar{B}^0$ decays and $D^0 \to K^-$ decays. And we also obtain the fairly good results for the $D^0 \to K^{*-}$ decays.

Various parametrizations on the Isgur-Wise function have been performed in order to obtain the reasonable result on the value of $|V_{cb}|$.[19,25-27] We search for the

suitable Isgur-Wise functions which give us as small the χ^2 minimum (or χ^2/ndf) as possible and lead to the most probable value of $|V_{cb}|$. The results are shown in Table 3. In place of these parametrizations of the Isgur-Wise function we are able to obtain the small χ^2 minimum by multiplying the factor $\exp(-k(y-1)^2)$ to the form factors in the $\bar{\Lambda}/m_Q$ correction terms, where k is a constant. This method means that we can control the large contributions from the mass correction terms in the large y region beyond $y = 1$. The effects of this factor is also shown in Table 3. In this table, we show several types of the Isgur-Wise function including the ones already analyzed in the past together with their χ^2 minimum values and $|V_{cb}|$ and show the value of the parameter k when this factor is used together with the Isgur-Wise function. As seen in Table 3, we can obtain the reasonable results of the χ^2 fit and the value of $|V_{cb}|$ for the siutable parametrizations of the Isgur-Wise function or the appropriate multiplication factors.

4. Discussions

In the previous analysis we have clarified that the $\bar{\Lambda}/m_Q$ correction by the c quark is necessary and plays an important role to reproduce the experiments of the B^0 semileptonic and nonleptonic decays and to predict the value of $|V_{cb}|$. From our analysis of this time, we evaluate that the $\bar{\Lambda}/m_Q$ corrections by s and c quarks make the large contribution and play the important role for the D^0 semileptonic and nonleptonic decays as well as for the B^0 decays. Especially the contribution from the s quark is indispensable as seen in the fact that $\bar{\Lambda}/m_s$ is $O(1)$ and $\bar{\Lambda}/m_c$ is $O(1/3)$ for $\bar{\Lambda} = 0.5$ GeV compared with $\bar{\Lambda}/m_b \sim O(1/10)$.

From our analysis we can estimate the range of $|V_{cb}|$ about $0.0336 < |V_{cb}| < 0.0391$ for both the linear type and the exponential type Isgur-Wise functions and for $\bar{\Lambda} = 0.3 \sim 0.5$ GeV by using the lifetimes of B^0 and D^0 given in the previous section. The value of $|V_{cb}|$ follows not only from the data of the q^2 -dependence of the branching ratio of $\bar{B}^0 \to D^{*+}l\bar{\nu}_l$ process, but also all the data of the B^0 semileptonic and nonleptonic decays together with the same parametrization on the D^0 decays. Therefore it will be considered that our predictions of the value of $|V_{cb}|$ are more reliable than in the case of using the q^2-dependence of B^0 decay alone. But the value of $|V_{cb}|$ is changeable according to the parametrization of the Isgur-Wise function or the introduction of the multiplication factor in the $\bar{\Lambda}/m_Q$ correction terms. In order to restrict the range of variation of $|V_{cb}|$ more stringent, it will be necessary to obtain more reliable data especially on $D^0 \to K^*$ decays.

Incidentally, we show the comparison of the calculated branching ratio $\mathrm{dBr}(\bar{B}^0 \to D^{*+} l \bar{\nu}_l)/\mathrm{d}q^2$ with the corresponding data given recently[24)] in Fig. 3. The parameters are determined from the χ^2 minimum search for the combination of this data and the data for $\bar{B}^0$ decays in Table 2(a). Though our prediction is in good agreement as seen in Fig. 3, the predicted value $|V_{cb}|$ is fairly small, $|V_{cb}| \simeq 0.0314$ for the linear type Isgur-Wise function. This value of $|V_{cb}|$ is almost unchanged for the variation of $\bar{\Lambda}$.

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Table 1. The parameters and χ^2 minimum values for the linear (lin.) type and the exponential (exp.) type Isgur-Wise functions determined through the χ^2-minimum search for 40 data of the $\bar{B}^0$ and D^0 semileptonic and nonleptonic decays for $\bar{\Lambda}$ = 0.3, 0.4 and 0.5 GeV, respectively.

Type of I-W	$\bar{\Lambda}$	χ_1^0	χ_2^0	χ_3^0	ξ_+^0	ρ	$\lvert V_{cb}\rvert$	χ^2
lin.	0.3	-2.318	-0.321	0.592	-0.984	0.650	0.0336	32.91
	0.4	-1.959	-0.298	0.472	-0.799	0.648	0.0341	32.61
	0.5	-1.727	-0.284	0.388	-0.675	0.639	0.0342	32.86
exp.	0.3	-1.796	-0.135	0.749	-1.127	1.318	0.0387	34.98
	0.4	-1.506	-0.176	0.573	-0.902	1.321	0.0389	34.29
	0.5	-1.319	-0.207	0.469	-0.771	1.329	0.0391	33.95

Table 2(a). Predicted branching ratios (in %) of $\bar{B}^0 \to D^{(*)+}$ decays and Γ_L/Γ_T $(\bar{B}^0 \to D^{*+} l \bar{\nu}_l)$ together with their respective data. The results are for the linear (lin.) type and the exponential (exp.) type Isgur-Wise functions and for $\bar{\Lambda} = 0.3$, 0.4 and 0.5 Gev, respectively.

Process	Λ	Type lin.	Type exp.	Data
$\bar{B}^0 \to D^+ l \bar{\nu}$	0.3	1.777	1.683	1.9 ± 0.5
	0.4	1.820	1.697	
	0.5	1.833	1.706	
$\bar{B}^0 \to D^{*+} l \bar{\nu}$	0.3	4.451	4.787	4.5 ± 0.5
	0.4	4.475	4.795	
	0.5	4.451	4.787	
$\Gamma_L/\Gamma_T(D^* l \bar{\nu})$	0.3	1.262	1.252	1.105 ± 0.26
	0.4	1.275	1.256	
	0.5	1.285	1.258	
$\bar{B}^0 \to D^+ \pi^-$	0.3	0.350	0.306	0.30 ± 0.04
	0.4	0.359	0.307	
	0.5	0.361	0.308	
$\bar{B}^0 \to D^{*+} \pi^-$	0.3	0.280	0.270	0.26 ± 0.04
	0.4	0.280	0.278	
	0.5	0.278	0.268	
$\bar{B}^0 \to D^+ \rho^-$	0.3	0.829	0.730	0.78 ± 0.14
	0.4	0.848	0.734	
	0.5	0.853	0.736	
$\bar{B}^0 \to D^{*+} \rho^-$	0.3	0.755	0.727	0.73 ± 0.15
	0.4	0.750	0.725	
	0.5	0.742	0.718	

Table 2(b). Predicted branching ratios of $D^0 \to K^{(*)-}$ decays, $\Gamma_L/\Gamma_T(D^+ \to K^{*0} \bar{l} \nu_l)$ and $\Gamma_L/\Gamma_T(D^0 \to K^{*-} \rho^+)$ together with their respective data. The results are for the linear (lin.) type and the exponential (exp.) type Isgur-Wise functions and for $\bar{\Lambda} = 0.3$, 0.4 and 0.5 GeV, respectively.

Process	Λ	Type lin.	Type exp.	Data
$D^0 \to K^- \bar{l} \nu$	0.3	4.162	4.200	4.4 ± 0.7
	0.4	4.413	4.192	
	0.5	4.005	4.232	
$D^0 \to K^{*-} \bar{l} \nu$	0.3	3.850	3.931	2.6 ± 0.6
	0.4	3.727	3.881	
	0.5	3.627	3.861	
$\Gamma_L/\Gamma_T(K^* \bar{l} \nu)$	0.3	2.156	2.297	1.8 ± 0.6
	0.4	2.255	2.337	
	0.5	2.326	2.384	
$D^0 \to K^- \pi^+$	0.3	3.761	4.299	4.5 ± 0.7
	0.4	3.774	4.290	
	0.5	3.695	4.328	
$D^0 \to K^{*-} \pi^+$	0.3	3.691	3.416	4.5 ± 0.6
	0.4	3.749	3.550	
	0.5	3.810	3.654	
$D^0 \to K^- \rho^+$	0.3	7.777	7.496	7.3 ± 1.1
	0.4	7.730	7.477	
	0.5	7.424	7.551	
$D^0 \to K^{*-} \rho^+$	0.3	10.90	10.38	6.2 ± 2.5
	0.4	10.65	10.32	
	0.5	10.49	10.28	
$\Gamma_L/\Gamma_T(K^* \rho)$	0.3	1.361	1.500	0.9 ± 0.7
	0.4	1.427	1.537	
	0.5	1.472	1.574	

Table 3. Results for the various parametrizations of the Isgur-Wise function and the multiplication factor whch gives the χ^2 minimum value.

Isgur-Wise function	ρ	$\lvert V_{cb}\rvert$	χ^2
$\frac{2}{1+\rho(y-1)} - 1$	0.555	0.0372	32.15
$\frac{4\exp(-\rho(y-1))-1}{3}$	0.743	0.0364	32.02
$(\frac{2}{y+1})^{\rho}$ (ref. [19])	3.526	0.0412	40.03
$\frac{1}{(1+\rho(y-1))^2}$ (ref. [25])	0.970	0.0413	46.53
$\frac{2}{y+1}\exp(-\rho\frac{y+1}{y-1})$ (ref. [26])	3.184	0.0425	46.71
pole type with $k = 0.9$	1.502	0.0366	35.06
exponential type with $k = 0.3$	1.109	0.0363	33.24

Figure 1. Comparison of the calculated $\mathrm{dBr}(\bar{B}^0 \to D^{*+} l \bar{\nu}_l)/\mathrm{d}q^2$ with the corresponding data. The solid curve is for the linear type Isgur-Wise function and the dashed curve is for the exponential type function. The curves are drawn for $\bar{\Lambda}$ =0.4 GeV as an example, but similar curves are obtained for the other cases as well. The data of circle are given by ref. [9]and the data of triangle given by ref.[19].

Figure 2. Comparison of the calculated $\mathrm{d}\Gamma(D^0 \to K^- \bar{l} \nu_l)/\Gamma \mathrm{d}q^2$ with the corresponding data. The curve is obtained for the linear type Isgur-Wise function and $\bar{\Lambda}$ =0.4 GeV as an example. The curve for the exponential type is almost the same as the case of for the linear type in this q^2 region. The similar curves are obtained for the other cases of $\bar{\Lambda}$ as well as in Fig. 1. The data are given by ref. [14].

Figure 3. Comparison of the calculated $\mathrm{d}Br(\bar{B}^0 \to D^{*+} l \bar{\nu}_l)/\mathrm{d}q^2$ with the corresponding data. The data is given by ref. [28].

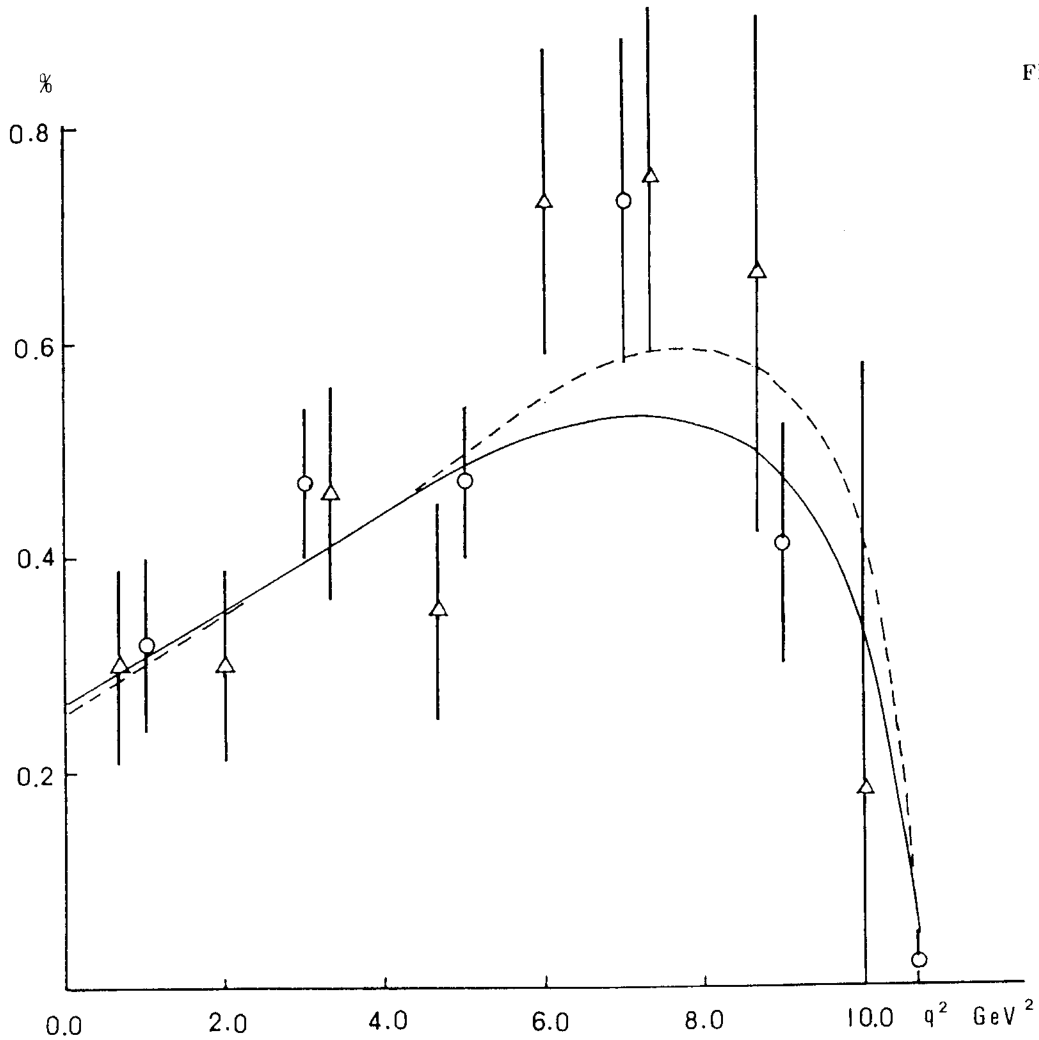

Figure 1.

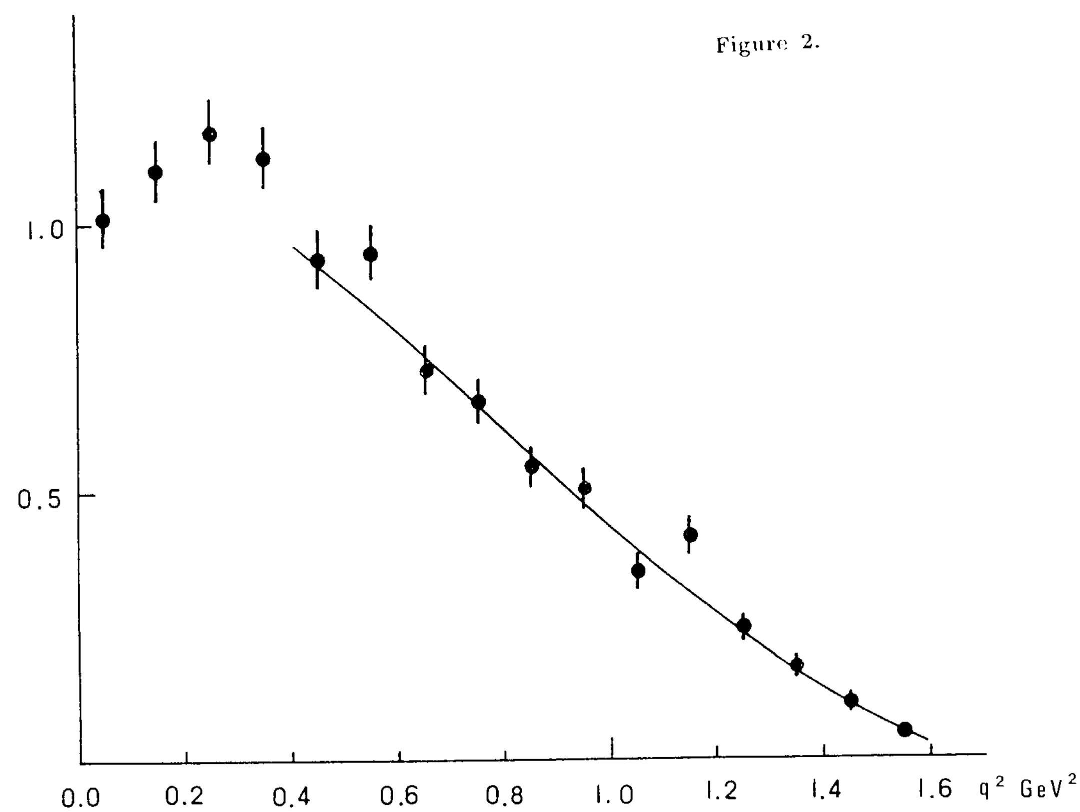

Figure 2.

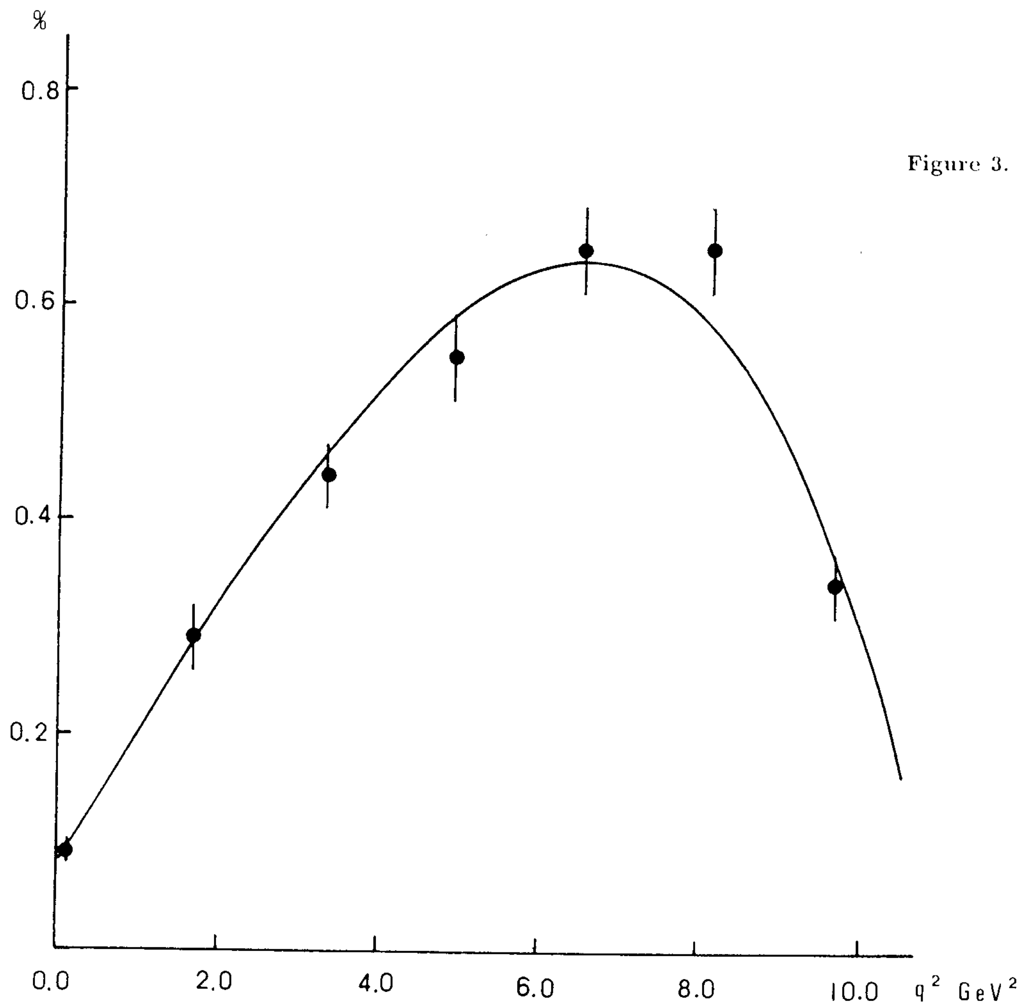

Figure 3.